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LIMITS OF THE MULTIPLIER ADJUSTMENT APPROACH TO CAPACITATED LOCATION PROBLEM

This contribution deals with a distribution system design problem, in which the located facilities satisfy all customer demands under limited abilities. The objective is to minimize the total costs, including both fixed charges and service costs. A special approach based on Lagrangean relaxation will be discussed here for its ability to cope with limited capacities of located facilities. Using the Lagrangean relaxation, the model of the original problem is rearranged to a model, which can be solved by exact algorithms for considerably large size. The capacities of the located facilities bring serious difficulties concerning solving technique, in the cases, where real-sized facility location problems are solved. In contrast to an uncapacitated facility location problem, which can be solved exactly in reasonable time for real-sized case containing hundreds of possible locations and thousands of customers.

In this paper, we shall discuss a transformation of the capacitated location problem into the uncapacitated location problem by means of Lagrangean relaxation of capacity constraints. To demonstrate the efficiency of the studied approach, numerical experiments were performed and their results are reported in the concluding part of this paper.

1. Introduction

A distribution system can be considered as a sort of a transportation system, which enables delivering goods from one or several sources to customers. This delivering can be either direct or with transhipment at several places (these are generally called terminals or facilities) – see Fig. 1.

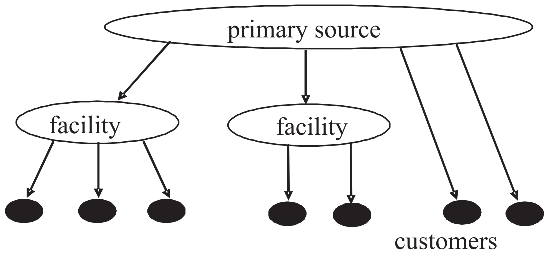

Fig. 1. Design of a distribution system

Determination of structure of a distribution system with transhipment belongs to the family of location problems. The associated location problem can be formulated as follows: given a set of possible facility locations and a set of customers, which are served from a finite number of located facilities, find which facilities should be used and which customers should be served from which facilities so that to minimise the total cost of serving all the customers and the fixed investment costs of building and maintaining the facilities.

In this paper, we shall investigate the capacitated location problem, with limited annual capacities of individual facilities. We shall devise the appropriate mathematical model and the solution method.

The capacitated location problem can be formulated as 0 – 1 linear programming problem, but the searching of its exact solution is very time-consuming or nearly impossible for large cases. One of possible approaches to a capacitated location problem can be based on its rearrangement to the uncapacitated location problem, solution of which can be found relatively fast, also for real-life size.

2. A Model of the Capacitated Location Problem

As preliminaries for a model construction we introduce the following notation of particular terms, which will be used throughout the whole paper.

Let J denote a finite set of customers and if a quantity of customer's demand can be expressed by a real number, then the demand of customer $j \in J$ is denoted by b_j.

Let I denote a finite set of possible facility locations, than the decision on a facility location at place $i \in I$ is modelled by zero-one variable $y_i \in \{0,1\}$, which takes value 1 if a facility should be located at i and it takes value 0 otherwise.

The model of the problem can be formulated as follows:

$$Minimize \quad \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} z_{ij} \tag{1}$$

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$$\text{Subject to} \quad \sum_{i \in I} z_{ij} = 1 \qquad \text{for } j \in J \qquad (2)$$

$$y_i - z_{ij} \geq 0 \qquad \text{for } i \in I, j \in J \qquad (3)$$

$$\sum_{j \in J} b_j z_{ij} \leq a_i \qquad \text{for } i \in I \qquad (4)$$

$$y_i \in \{0, 1\} \qquad \text{for } i \in I \qquad (5)$$

$$z_{ij} \in \{0, 1\} \qquad \text{for } i \in I, j \in J \qquad (6)$$

In this model f_i denotes the fixed investment and maintenance cost of the facility with location $i \in I$ in the considered time period.

c_{ij} is the cost of demand satisfaction of customer $j \in J$ via the facility $i \in I$. Size of the demand is b_j and cost c_{ij} can be modelled as follows:

$$c_{ij} = (e_0\, d_{si} + e_1\, d_{ij} + g_i)\, b_j$$

where d_{si} is the distance between a primary source s and a facility i, d_{ij} is the distance between a terminal (facility location) i and a customer j, e_1 denotes unit transportation costs on the link between the primary source s and the facility i, e_0 denotes unit transportation costs on the link between the facility i and the customer j and g_i denotes the unit cost for the reloading of goods in the facility location i.

We suppose that the capacity of a facility located at i is a_i.

Constraints (2) ensure that each customer demand must be satisfied from exactly one facility location and constraints (3) force out the placement of a facility at location i whenever a customer is assigned to this facility location. Constraints (4) ensure that the total demand satisfied via facility i does not exceed the given capacity a_i.

If constrains (4) are omitted, the problem (1) – (3), (5), (6) is known as the uncapacitated location problem.

3. A Solving method for the Capacitated Location Problem

For the exact solving of an uncapacitated facility location problem (1) – (3), (5) – (6), the suitable solution method is the branch-and-bound method with branching performed by fixing the chosen variable y_i to the value 0 or 1 in the depth-first search of the solution tree.

We shall use the branch-and-bound method for the exact solution of the uncapacitated location problem based on Erlenkotter's approach [Erlenkotter, 1978], which was modified as *BBDual* algorithm by [Janáček, Kováčiková, 1997]. The solution yields the number of used facilities, their spatial distribution and the association of the customers with the facilities.

Next, let us show how we can reduce the capacitated location problem to the uncapacitated one by means of Lagrangean relaxation. Let us introduce the vector u of non-negative Lagrangean multipliers, $u_i \geq 0$ for all $i \in I$ and rearrange the objective function (1) to the following form:

$$\sum_{i \in I} f_i y_i + \sum_{i \in I}\sum_{j \in J} c_{ij} z_{ij} + \sum_{i \in I} u_i \left(\sum_{j \in J} b_j z_{ij} - a_i\right) =$$

$$= \sum_{i \in I} f_i y_i + \sum_{i \in I}\sum_{j \in J} c_{ij} z_{ij} + \sum_{i \in I}\sum_{j \in J} u_i b_j z_{ij} - \sum_{i \in I} u_i a_i =$$

$$= \sum_{i \in I} f_i y_i + \sum_{i \in I}\sum_{j \in J} z_{ij} (c_{ij} + u_i b_j) - \sum_{i \in I} u_i a_i. \qquad (7)$$

By omitting the last term of expression (7), we obtain the following expression:

$$\sum_{i \in I} f_i y_i + \sum_{i \in I}\sum_{j \in J} \underline{c}_{ij} z_{ij} \qquad (8)$$

where $\underline{c}_{ij} = c_{ij} + u_i b_j$ for $i \in I, j \in J$.

The last term in (7) is a constant for fixed u_i, and so we can omit it when solving the reformulated problem. Substituting the optimal solution of (8), (2), (3), (5), (6) into (7), we obtain a lower bound of the objective function value of a so-far unknown optimal solution of (1) – (6). Moreover, if a solution of (8), (2), (3), (5), (6) satisfies constraints (4) and complementarity constraints (9), then this solution is the optimal solution of the capacitated problem.

$$u_i \left(\sum_{j \in J} b_j z_{ij} - a_i\right) = 0 \quad \text{for } i \in I \qquad (9)$$

In the other case, we can use this solution as a starting point of a dual heuristics.

To obtain an approximate solution of the capacitated location problem we start with the relaxed problem model, Lagrangean multipliers of which are set to zero. If the optimal solution of this problem obtained by *BBDual* algorithm satisfies capacity constrains (4), then the optimal solution of the original problem is found.

Otherwise (if capacities of some of the facilities are not sufficient for the given demands), we change multiplier values. We make the changes by choosing the facility sequentially with the maximal difference between capacity and total demand of the associated customers and solve the relaxed problem with adjusted multipliers by *BBDual* algorithm.

An increase of the multiplier associated with a given facility should result in decrease of the capacity deficit associated with the facility. The stopping criterion for our algorithm was that the summary deficit of all located facilities should be less than 5 % of the summary demand of all customers, i.e.:

$$\sum_{i \in I_k} \left(\sum_{j \in J} b_j z_{ij} - a_i\right) \leq \sum_{j \in J} b_j / 100 * 5 \quad \text{where } I_k \subseteq I,$$

$$I_k = \left\{ i \in I \sum_{j \in J} b_j z_{ij} > a_i \right\}. \qquad (10)$$

4. Experiments

Lagrangean relaxation of the capacitated location problem was implemented in Delphi and real data from the Slovak road network was used for numerical experiments. In the solved problems, 71 regional towns form the set of possible facility locations and 2907 villages form the set of customers. Demands of customers depend on the number of inhabitants in the villages. The costs connected with serving customers from a located facility are derived from distances in the real road network. The computations were performed with various values of fixed costs and capacities of facilities.

In most cases, a change in fixed costs causes a change in the number and locations of facilities, which can increase if the values of multipliers increase. This raise of the number of facilities may cause that all customer's demands are satisfied unless the facility capacities are exceeded (see Table 1).

The experiments showed that the change of the multiplier (increment by 1) could cause increase of the objective function (1), and we can obtain a new solution.

In Table 2 there is an example in which the sum of all customer demands is 53808 and the first selection of 10 facilities has a total capacity of 53900. Increase of u4 to 4 decreased the sum of all deficit demands from 7.98% to 3.22% and added 1 facility. Other changes of multiplier values in the above-mentioned way do not decrease the sum of deficit demands.

Location problem for various capacity of a facilities — Table 1

number of steps	located facilities	values of multiplier			objective function (1)	objective function (7)	not covered customer demands (max)	not covered customer demands (sum)	%
		4	47						
						Capacity of 2 facilities:		**59840**	
0	2	0	0		17865901	17865901	7795	7795	14.49
1	2	1			17865901	17873696	7795	7795	14.49
2	2	2			17866076	17881316	7620	7620	14.16
3	2	3			17866076	17888936	7620	7620	14.16
4	2	4			17986996	17986996	7592	7592	14.11
5	2		1		17866076	17896556	7620	7620	14.16
6	2	5			17986996	17994574	7578	7578	14.08
7	3		2		17943985	17886395	0	0	**0.00**
						Capacity of 2 facilities:		**49234**	
0	2	0	0		17865901	17865901	13098	13098	24.34
1	2	1			17865901	17878999	13098	13098	24.34
2	2	2			17866076	17891922	12923	12923	24.02
3	2	3			17866076	17904845	12923	12923	24.02
4	2	4			17986996	17986996	12895	12895	23.96
5	2		1		17866076	17917768	12923	12923	24.02
6	2	5			17986996	17999877	12881	12881	23.94
7	3		2		17943985	17912910	0	0	**0.00**

Location problem for 53900 capacity of 10 facilities — Table 2

number of steps	located facilities	values of multiplier			objective function (1)	objective function (7)	not covered customer demands (max)	not covered customer demands (sum)	%
		4	47						
0	10	0	0	0	11684833	11684833	2833	4295	7.98
1	10	1			11684837	11687666	2829	4291	7.97
2	10	2			11684843	11690489	2823	4285	7.96
3	10	3			11684876	11693312	2812	4274	7.94
4	11	4			11693307	11694387	1063	1732	3.22

Location problem for 48510 capacity of 9 facilities Table 3

number of steps	located facilities	values of multiplier			objective function (1)	objective function (7)	not covered customer demands (max)	not covered customer demands (sum)	%
		4	47	36					
0	9	0	0	0	12058775	12058775	4275	6772	12.59
1	9	1			12058798	12063050	4252	6772	12.59
13	9	13			12060284	12112440	4012	6772	12.59
14	9	14			12060479	12116437	3997	6765	12.57
20	10	20			12118942	12136922	1965	5276	9.81
21	10		1		12121718	12139598	1925	4492	8.35
22	10			1	12118944	12138887	1963	5294	9.84
23	10		2		12121618	12141523	1925	4500	8.36
24	10			2	12118990	12140804	1917	5253	9.76
31	10		6		12123534	12147239	1145	3766	7.00
32	10			6	12111701	12147163	1471	3940	7.32
33	10	21			12246944	12256388	1574	5007	9.31
34	10		7		12250594	12257464	1145	3550	6.60
35	10			7	12294238	12294238	2429	4630	8.60
		42							
36	10	1			12251266	12257363	871	2164	4.02

If we choose different input fixed costs in the testing instance, we can obtain 9 facilities with a total capacity of 48510. That is less than 90.16% of total demands. The first selection of the facilities caused total difference of 12.59%. Successive increase of multipliers decreased the total difference to 4.02% after 36 steps, but caused the raise of the number of facilities to 10 (see Table 3).

5. Conclusions

The Lagrangean relaxation seems to be an effective solving method for some cases of capacitated location problems with limited annual capacities of individual located facilities. If the capacity constraints need not be satisfied exactly and values of multipliers are successively increased for a located facility with the maximal number of deficit demands then the numerical experiments in most cases give acceptable results.

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