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LEGENDRE POLYNOMIALS USED FOR THE APPROXIMATION OF CYLINDRICAL SURFACES

The paper deals with a concept of application of orthogonal Legendre polynomials for the approximation of cylindrical surfaces. It presents fundamentals of a mathematical model of such approximation. The theoretical considerations are supported by the results of the experimental verification of the developed concept.

1. Introduction

Today's metrological tasks make use of various methods and strategies of measurement. The discrete data representing an analyzed form profile, however, are frequently insufficient. In other words, it is impossible to know all the profile values, as measurement can be conducted only for a certain, limited number of points. In order to obtain the entire profile, approximation is necessary.

In the case of cylindrical surfaces, the approximation involves:
– obtaining the view of the whole surface being measured; this will allow visual assessment of the form errors,
– filtering the obtained profile,
– specifying the parameters of the cylinder under measurement,
– comparing the profiles of cylindricity.

2. The Legendre/Fourier cylindricity profile approximation

Suppose that a current cylindricity profile is described by means of the function

$$R(\varphi, z), \ 0 \le \varphi < 2\pi, \ 0 \le z \le H \qquad (1)$$

(*It would be good to specify R, φ, z, H by the words!!!!*)

Assume also that by applying a certain strategy, we obtain a number of measuring points with the coordinates

$$\varphi_k, z_k, R_k \quad \text{for} \quad k = 1, 2, ..., N. \qquad (2)$$

Let $\Psi(\varphi, z, a)$ be a certain parametrized class of functions, where is a vector of the parameters. The problem of approximation of the measured points with a selected class of functions is commonly formulated using the least squares principle. Define the approximation quality by means of the index

$$J = \sum_{k=1}^{N} (R_k - \Psi(\varphi, z, a))^2. \qquad (3)$$

The aim is to select values of the parameters a so that the value of the coefficient J is minimized. If the approximating function is a linear combination of certain linearly independent base functions $\psi(\varphi, z)$, i.e.

$$\Psi(\varphi, z, a) = \sum_{j=1}^{M} a_j \psi_j(\varphi, z), \qquad (4)$$

then the problem of approximation by the least squares method can be solved analytically. Indeed, thus

$$J = (R - \Psi a)^T (R - \Psi a), \qquad (5)$$

where R is a column vector with the length N containing the values of the measured profile, and Ψ is the matrix with the size $M \times N$, the elements being equal to $\psi_j(\varphi_k, z_k)$, $j = 1, 2, ..., M$, $k = 1, 2, ..., N$. If $N \le M$ and the columns of the matrix Ψ are linearly independent, then the vector of the parameters a, for which the coefficient reaches a minimum is equal to

$$a = (\Psi^T \Psi)^{-1} \Psi^T R. \qquad (6)$$

The set of basic functions should be complete in such a sense that by selecting an appropriately great order of approximation M, we are able to approximate an arbitrary function of two variables with predetermined accuracy. Since, for the fixed z, the function $R(\varphi, z)$ is a periodic function of the variable φ with a period of 2π, we shall assume that for the fixed z, the approximating function is a partial sum of a trigonometric Fourier series. The profile $R(\varphi, z)$ as a function of the variable z is certainly not a periodic function. Thus, assume that for the fixed value of the angle φ, the approximating function is a polynomial. For many reasons it is favorable to represent the polynomial as a sum of orthogonal polynomials. Accordingly, consider the following class of functions approximating a cylindrical area:

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$$\Psi(\varphi, z, a) = \sum_{m=0}^{M_z} L_m(z) \left(\frac{1}{2} a_{m0} + \sum_{n=1}^{M_\varphi} (a_{mn} \cos(n\varphi) + b_{mn} \sin(n\varphi)) \right), \qquad (7)$$

$$L_m(z) = \overline{L}_m \left(\frac{2z}{H} - 1 \right), \qquad (8)$$

where $\overline{L}_m$ is a Legendre polynomial of the m degree. Since Legendre polynomials are orthogonal in the interval $(-1, 1)$ [2], a set of polynomials $L_m(z)$ is orthogonal in the range of variability of the variable z of the cylindrical surface. With so defined an approximating function, the vector of the parameters a includes all the coefficients a_{mn} and b_{mn} occurring in equation (7), while the basic functions have the form

$$L_m(z)/2, \; L_m(z) \cos(n\varphi) \text{ and } L_m(z) \sin(n\varphi). \qquad (9)$$

3. Experimental verification of the proposed concept

In order to experimentally verify the method of approximation of cylindrical surfaces using the Legendre orthogonal polynomials, it was necessary to apply the mathematical apparatus discussed in Section 2. The analyzed cylindrical element was measured with the method of cross-sections. The graph in Fig. 1a was prepared on the basis of the results of measurement of a cylindrical element with the mean cylinder axis in the assumed position.

The element was measured at 1024 points located in the cylinder cross-section. Due to some calculation problems, it was essential to filter the data, i.e. reduce the number of measuring points. Eventually, the number of points per circuit amounted to 128. With the data having been filtered, the next step was to determine the matrixes of Legendre approximation coefficients. In the experiment, we assumed that the maximum degree of the approximation polynomial was one less than the number of the measured cross-sections. The number of profile harmonics to be analyzed was reduced and ranged 0-25 owing to calculation problems.

Figure 1b) shows a graph produced after the approximation of the measured surface using the proposed method.

In Fig. 2, on the other hand, there is a spatial representation of the approximation error, i.e. the difference between the initial and the approximated values of profiles of the analyzed cylindrical surface. The analysis of Fig. 2 shows that the approximation error of the cylindrical surface obtained when using the suggested method is negligible, its maximum value being 0.4 μm. The average relative error of approximation in this case (i.e. when applying the suggested method) is 1.01 %.

4. Conclusions

Analyzing the graphs in the Section 3, we can notice that the proposed method assuming the application of Fourier series and Legendre orthogonal polynomials enables us to approximate cylindrical surfaces with high precision. Since the orthogonality of Legendre polynomials allows obtaining better-conditioned matrixes, the method appears to be more useful and does not cause such calculation problems as other methods, for instance, those using Chebyshev polynomials described in Refs. [5, 6]. However, if there are no calculation problems, and it is not necessary to reduce the number of analyzed profile harmonics, this method can provide us with even more precise information. The results confirm the rightness of the proposed concept of approximation. Thus, it can be used in practice, for example, for determining the parameters or filtering the profiles described by means of a small number of discrete measuring data or for comparing cylindricity profiles obtained with the aid of various measuring devices and strategies.
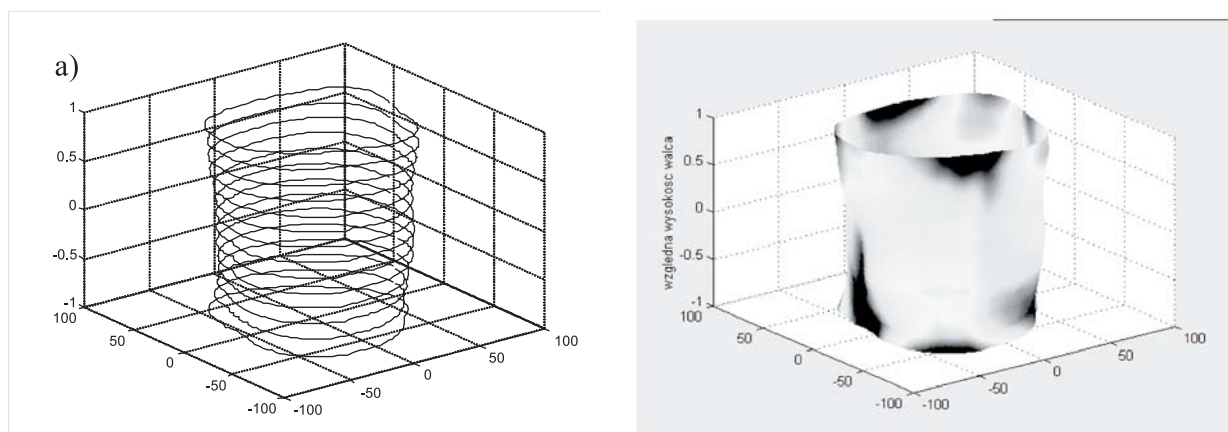

Fig. 1. Representing a measured cylindrical element using: a) measuring discrete data; b) approximation coefficients
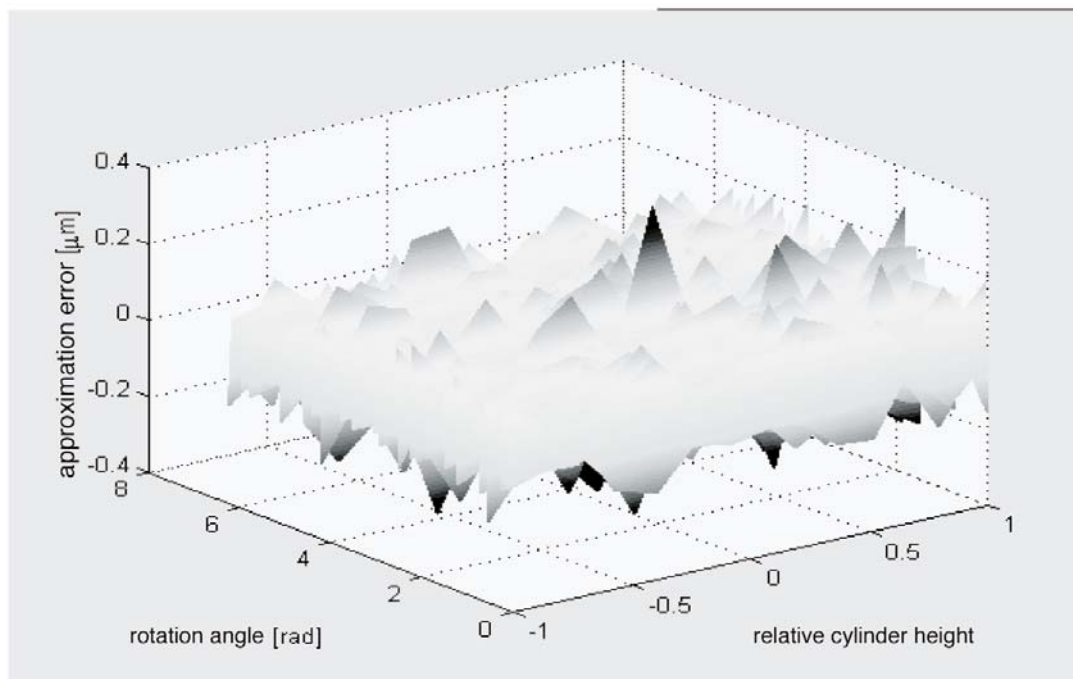
Fig. 2. Approximation error

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