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COMMUNICATIONS

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THE COLUMN GENERATION TECHNIQUE FOR PUBLIC TRANSPORT LINE PLANNING BY IP-SOLVER

The paper deals with an application of the column generation method to the public transport line planning making use of a common optimization environment. The authors focus on the opportunities offered by the optimization software for the column optimization and for man-machine approach to the process of line planning. The paper resumes the former approach to the transport line planning based on line selection from a large set of all sensible lines and, on the contrary to the former approach, introduces the column generation method for a new route design. A case study from practice is used to compare both approaches and to point out their advantages and disadvantages.

1. Introduction

The public transport line planning is a crucial deal for each designer involved. In general, the planning process comprises several stages which start with a line route design and finish with a driver scheduling [2], [4], [7]. We shall concentrate on the first stage only where a system of line routes is designed. Even at this stage, various objectives to the design can be applied to make the transportation system more attractive for potential passengers. Depending on the chosen objective, various sorts of information about origin-destination passenger flows are required and the fact must be taken into account that any specific information need not be available at the beginning of the planning process.

We mention here an approach which needs only information about demand for transportation on the links of an underlying transportation network. The objective of line system designing is to improve service of the link with the worst demand satisfaction. The link demand satisfaction is evaluated as the ratio of the number of passengers who want to travel along the link in one hour and the total number of seats which are at disposal on the link within one-hour period. The resulting system of line routes must comply with a constrained number of vehicles which can be assigned to individual lines.

In this paper we come out from the approach called "PRIVOL" [3], [5], [8] which starts with a large redundant set of pre-designed lines covering all links of the considered transportation network. A linear program is formulated in this original approach where disposable vehicles are assigned to individual lines and the worst relative link demand satisfaction is maximized. This linear problem is solved to optimality and some lines are subsequently discharged from the starting set based on a number of assigned vehicles. This process of line elimination can be repeated several times until a convenient system of line routes is obtained. The major disadvantage of this approach consists in the necessity to design the huge starting line set in advance. Furthermore, it must be taken into account that extent of the line set does not assure that it contains the most convenient lines for the system. This approach does not design a line really, but it only selects some lines from the starting set. To enable a link route generation under the above-mentioned objective, we suggest here an approach based on the column generation method [1], [6]. We will show how to use the standard optimization environment for the method implementation so that a common designer who is not supported by a team of programmers and other informatics professionals can use this approach.

2. Model of the original line elimination approach

The simplest case of the line elimination approach assumes that there is a number c of vehicles which form a homogenous fleet. Each vehicle has the same capacity K (number of seats). The graph of transportation network consists of a set of nodes U and a set of links H. An hour's demand for transportation on link h is described by intensity q_h. The symbol t_h denotes the time which is necessary for traversing the link h by a vehicle of the considered homogeneous fleet. Let the symbol L denote the starting set of line routes where each route is given by some collection of links from H. In the following models we shall use notation L_h and H, where L_h denotes the set of line routes serving the link h and H_l denotes the set of all links forming the line route l. The symbol N_l denotes so-called frequency of vehicles assigned to the line route l which is the number showing how many times a vehicle is able to run the route within one hour. It is obvious that the equation $N_l = 1/T_l$ holds where T_l denotes the time necessary for traversing the route l. In our case we assume that (1) holds.

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$$T_l = \sum_{h \in H_l} t_h \tag{1}$$

The original line elimination approach [4] formulates the linear model using integer variable x_l modeling the decision on a number of vehicles which should be assigned to the individual line route *l*.

Let variable *y* represent a lower bound of the relative link demand satisfaction and let coefficient K_h denote the ratio K/q_h, then a model of the vehicle assignment problem can be formulated as follows:

$$\textit{Maximize} \;\; y \tag{2}$$

$$\textit{Subject to} \;\; \sum_{l \in L} x_l \le c \tag{3}$$

$$y - \sum_{l \in L_h} K_h N_l x_l \le 0 \;\; \textit{for } h \in H \tag{4}$$

$$x_l \in Z^+ \;\; \textit{for } l \in L \tag{5}$$

$$y \ge 0 \tag{6}$$

Constraint (3) of the model (2)–(6) ensures that the number of assigned vehicles does not exceed the limit *c*. The constraints (4) link up the lower bound *y* to the individual link ratios (7),

$$\frac{K \sum_{l \in L_h} N_l x_l}{q_h} \tag{7}$$

where the narrator expresses the number of seats which are offered to passengers on the link *h* within one hour period and the denominator corresponds with the number of passengers who want to travel along the link *h* in one hour. The LP-relaxation of the model (2)÷(6) is solved as a part of the original approach [4] and the link route with minimal value of x_l is discharged from the set *L*.

3 The column generation method

In general, the column generation method is used in connection with such problems where the most of columns of the associated linear model has the same structure. Furthermore, the solved problem must be formulated so that each addition of a column to the model enlarges the set of feasible solutions. Then a better solution can be found in the enlarged set than in the previous one. In addition, such method must be derived which enables to evaluate possible improvement of the given objective function or to find that no improvement can be reached by addition of any column. The problem (2)–(6) has these necessary properties. The column connected with the variable x_l corresponds with the unique line route and any addition of a new line route to the set *L* enlarges the set of feasible solutions.

Now we derive a possible method of column evaluation using the theory of duality [6]. Let *L* denote an original set of line routes. Let us denote the LP-relaxation of the model (2)–(6) as the primal problem. Introducing dual variable v_h for each constraint *h* of the constraint system (4) and dual variable *w* for the constraint (3), a dual problem of the primal one can be formulated as follows:

$$\textit{Minimize} \;\; cw \tag{8}$$

$$\textit{Subject to} \;\; \sum_{h \in H} v_h \ge 1 \tag{9}$$

$$w - \sum_{h \in H_l} K_h N_l v_h \ge 0 \;\; \textit{for } l \in L \tag{10}$$

$$v_h \ge 0 \;\; \textit{for } h \in H \tag{11}$$

$$w \ge 0. \tag{12}$$

For the optimal solutions (x^*, y^*) and (v^*, w^*) of the primal and dual problems respectively, the equation (13) holds:

$$cw^* = y^*. \tag{13}$$

Now, let us consider a new line route *r* given by the links of H_r. Let us define coefficients of the associated column A_r in the following way:

$$a_{hr} = K_h N_r \;\; \textit{if } h \in H \;\; \textit{and } a_{hr} = 0 \;\textit{otherwise} \tag{14}$$

The primal enlarged problem (2)–(6) can be reformulated for the enlarged set $L \cup \{r\}$ of line routes as follows:

$$\textit{Maximize} \;\; y \tag{15}$$

$$\textit{Subject to} \;\; \sum_{l \in L} x_l + x_r \le c \tag{16}$$

$$y - \sum_{l \in L_h} K_h N_l x_l - a_{hr} x_r \le 0 \;\; \textit{for } h \in H \tag{17}$$

$$x_l \ge 0 \;\; \textit{for } l \in L\{r\} \tag{18}$$

$$y \ge 0 \tag{19}$$

For any feasible solution of (15)–(19) and for the optimal solution (v^*, w^*) of (8)–(12), we can derive the following inequality:

$$\begin{aligned} y &\le y + w^*\Big(c - \sum_{l \in L:} x_l - x_r\Big) + \sum_{h \in H} v_h^* \Big(\sum_{l \in L_h} K_h N_l x_l + a_{hr} x_r - y\Big) = \\ &= y + cw^* - \sum_{l \in L} w^* x_l - w^* x_r - \sum_{h \in H} v_h^* y + \sum_{l \in L} \sum_{h \in H_l} v_h^* K_h N_l x_l + \sum_{h \in H} v_h^* a_{hr} x_r = \\ &= cw^* + y\Big(1 - \sum_{h \in H} v_h^*\Big) + \sum_{l \in L} x_l \Big(-w^* + \sum_{h \in H_l} K_h N_l v_h^*\Big) + x_r \Big(-w^* + \sum_{h \in H} a_{hr} v_h^*\Big) \end{aligned} \tag{20}$$

Since the expressions in brackets of the second and third items of the resulting right-hand-side of (20) are negative (see dual constraints (9) and (10)), the following inequality holds:

$$y \le cw^* + x_r\left(-w^* + \sum_{h \in H} a_{hr} v_h^*\right) \tag{21}$$

Taking into consideration the equation (13), we obtain:

$$y - y^* \le x_r\left(-w^* + \sum_{h \in H_r} K_h N_r v_h^*\right) \tag{22}$$

Thus we can find that the inequality (23) is necessary for the case when the addition of a column A_r is able to improve the objective function y.

$$w^* \le \sum_{h \in H_r} K_h N_r v_h^* \tag{23}$$

The inequality (23) can be rearranged to the form:

$$w^* T_r \le \sum_{h \in H_r} K_h v_h^*, \tag{24}$$

which is

$$w^* \sum_{h \in H_r} t_h \le \sum_{h \in H_r} K_h v_h^*. \tag{25}$$

The inequality (25) can be rewritten as

$$0 \le \sum_{h \in H_r} \left(K_h v_h^* - t_h w^*\right). \tag{26}$$

After these preliminaries, we can assert that a new line route r can improve the objective function (15) only if the right-hand-side of (26) is positive. To find if such a line route exists we formulate the problem of finding the line route which maximizes the right-hand-side of (26). For this purpose we introduce a set of binary variables $z_h \in \{0,1\}$ for $h \in H$ where the variable z_h takes the value of 1 if and only if the link h belongs to the generated line route. If a network sub-graph fulfilling the conservative constraints (28) for each node of the network is accepted as a feasible line route then the line route generation problem can be stated as follows:

$$\textit{Maximize} \quad \sum_{h \in H} \left(K_h v_h^* - t_h w^*\right) z_h \tag{27}$$

$$\textit{Subject to} \quad \sum_{h \in H^{in}(u)} z_h = \sum_{h \in H^{out}(u)} z_h \quad \textit{for } u \in U \tag{28}$$

$$z_h \in \{0,1\} \quad \textit{for } h \in H \tag{29}$$

The link sets $H^{in}(u)$ and $H^{out}(u)$ denote a set of all the links coming into the node u and a set of all the links going out of the node u respectively.

If an optimal solution z^* of the problem (27)-(28) has a positive value of the objective function (27), then the solution forms an improving line route. In the opposite case, no improving line route exists at all.

4. Exploitation of XPRESS-IVE tool

The theoretical derivation of the improving line route generation problem is only one step on the way to the usable tool of a line route system design. As for the developed approach is addressed to a common line system designer, we rearranged the models so that the common commercial optimisation environment [9], [10] can be used. Furthermore, we realize that a route designed by solving the model (27)-(29) need not be a feasible route in the concrete real transportation network. Because of this fact we followed the idea that a designer should have a possibility to evaluate the designed route-column and depending on its practical feasibility or unfeasibility, he should be able to accept or refuse the route for the next processing. That is why we implemented the column generation method in two programs written in the programming language Mosel and we decompose the method into a series of individual steps. Each step is represented by one run of the first program consisting of two optimisation processes where the first process solves LP-relaxation of the vehicle assignment problem (2)-(6) and the second one solves the column generation problem (27)-(29) for the shadow prices (optimal values of dual variables) corresponding with the first problem.

If the solving process of the second problem gives an improving column, then the generated column enlarges the starting LP-relaxation problem and the enlarged problem is saved to a unified file. This way, the process of the column generation method can be interrupted after each step and, if necessary, the generated column can be modified in the file. The scheme of the first program is depicted in Fig. 1.

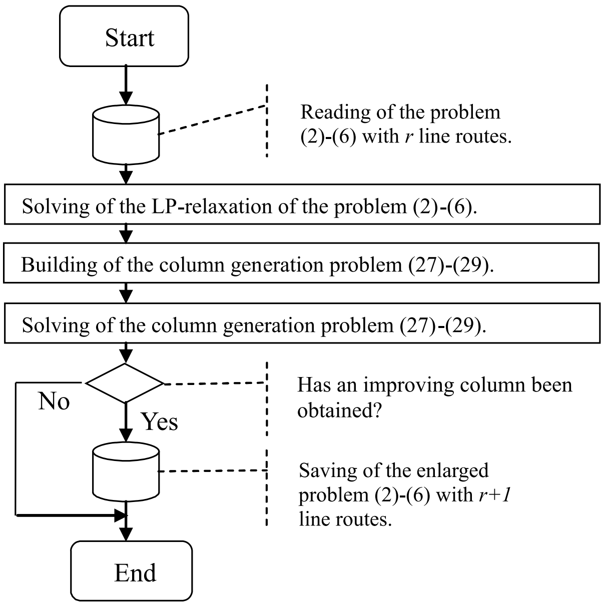

Fig. 1 Scheme of the first program

The second program solves the integer programming problem (2)-(6). This program can be applied on each version of the unified file in which data describing the vehicle assignment problem are saved. It means that after each step (column generation) it is possible to evaluate quality of the current system of line routes.

5. Case study

To verify the developed method, we made use of the real transportation network of Frydek-Mistek in the Czech Republic. The transportation network consists of 17 nodes and 46 links. The intensities q_h and travel times t_h for each link of the transportation network were also taken from the real traffic situation of the town. The vehicle fleet consists of 23 vehicles.

For our experiments we have a system of 9 real line routes at disposal. This real system of routes is reported as RealS09 hereafter in reported study. One of the line routes serves all the links of the transportation network. The biggest line route was taken as a basis for forming a generated system of 12 line routes denoted as GenS12. The generated system was formed by a series of column generation program runs which formed 22 line routes. As some of the generated routes were too small or they did not fulfil some local condition of feasibility, the generated system was revised, some routes were connected and some were discharged. Thus the system GenS12 of twelve feasible routes was obtained.

The numerical experiments were done with both starting line route systems to verify the column generation program and reveal trends of associated parameters. Eleven steps of column generation were performed with each starting system which means that systems with a number of lines varying from 10 to 20 were obtained for RealS09 and systems with a number of lines varying from 13 to 23 were obtained for GenS12. Solving the problem (2)–(6) corresponding with the enlarged system followed each column generation. The following parameters were observed:

y_{LP} – optimal value of the lowest relative link demand satisfaction of LP-relaxation of the problem (2)–(6),

y_I – optimal value of the lowest relative link demand satisfaction of integer solution of the problem (2)–(6),

NoUL – number of used links in optimal integer solution of the problem (2)–(6).

The results for RealS09 and GenS12 are reported in Table 1 and Table 2 respectively. The number of line routes denoted as NoL is used in the tables to distinguish the evaluated systems of line routes.

Optimal values of parameters obtained by the series of experiments starting with RealS09 — Table 1

NoL	9	10	11	12	13	14
y_{LP}	1.773	1.820	1.873	1.883	1.895	1.977
y_I	1.673	1.673	1.740	1.740	1.771	1.777
NoUL	9	10	10	10	11	12
NoL	15	16	17	18	19	20
y_{LP}	1.988	2.011	2.029	2.045	2.088	2.096
y_I	1.811	1.863	1.863	1.863	1.917	1.917
NoUL	12	13	13	13	12	12

Optimal values of parameters obtained by the series of experiments starting with GenS12 — Table 2

NoL	12	13	14	15	16	17
y_{LP}	1.700	1.835	1.910	1.994	2.025	2.115
y_I	1.587	1.728	1.752	1.862	1.862	1.907
NoUL	8	9	10	11	11	11
NoL	18	19	20	21	22	23
y_{LP}	2.159	2.169	2.178	2.185	2.186	2.188
y_I	1.907	1.914	1.914	1.914	1.914	1.933
NoUL	11	13	13	13	13	15

All experiments were performed on a notebook with parameters Core 2 Duo 2GHz 2GB RAM. The individual computing times are not reported here, but we can assert that no time of program run exceeded one second.

6. Conclusions

We have developed the column generation approach for a special case of a line route design problem where the worst relative link demand satisfaction is optimized. The resulting method was suggested so that it can be easily implemented in the common commercial optimization environment XPRESS-IVE and man-machine approach can be included to the process of individual route generation. We verified the method by performing the suggested column generation process with two starting sets of line routes (GenS12 and RealS09). From the results reported in tables 1 and 2 we found that each column generation brings some improvement of the objective function of an optimal solution obtained for LP-relaxation. Nevertheless, an improvement of LP-relaxation solution need not be accompanied by any improvement in an optimal integer solution. Thus a next improving line can be obtained after several column generations. The experiments also showed that even if the process starts from the better starting line route system (compare y_I for NoL = 9 in Table 1 with y_I for NoL = 12 in Table 2), the same number of column generations could give a worse result (compare y_I for NoL = 20 in Table 1 with y_I for NoL = 23 in Table 2). In general, we can state that the column generation process performed on XPRESS-IVE environment together with a man-machine approach represents a valuable tool for a line route designer.

Further research in this field can be focused on the following directions. First, similar approaches could be developed for other criteria which are used for the construction of an objective function. Next, more complex models for line route generation can be studied to produce more convenient routes and avoid their local infeasibilities.

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