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ADJUSTMENT OF CODE RANGING OF GNSS OBSERVATIONS

Gauss-Jacobi combinatorial algorithm is a very useful method to adjust code ranging of global navigation satellite observations to find the systematic errors caused by radio waves passing through the atmosphere. The method is used as alternative adjustment technique to the least square method. While, the least square adjustment requires the linearization of nonlinear functions, combinatorial approach seems to be simplier in direct processing the nonlinear models. The diagnostics of observation errors consists in comparison of an evaluated partial positional norm determined in the partial mathematical models with the global one determine by robust estimation.

Keywords: *Code ranging, combinatorial algorithm, observation errors, positional norm.*

1. Introduction

Global navigation satellite system (GNSS) involves two observation techniques to measure the range (pseudorange) between satellite and receiver. Code pseudorange measurement, in short code ranging, is used to measure the time difference between the received code and generated replica code. Phase pseudorange measurement, phase ranging, is based on measurement of phase difference between the received and generated signal. The received carrier is Doppler shifted due to the mutual motion of satellite and receiver [1]. The accuracy of pseudorange obtained by the code or phase ranging depends on random and systematic influences. The whole system error involves the particular errors of determination of satellite time and position, signal transition errors caused by using the incorrect atmospheric model comprising the influence of ionospheric and stratospheric refraction and topography errors known as multipath effect. The great part of system error is generated by a receiver failure, which comprises a variation of phase centre and clock errors. The paper is devoted to a numerical analysis of code ranging especially to diagnose influence of systematic errors by applying the Gauss-Jacobi algorithm [2, 3 and 4].

2. Numerical Approaches of Code Ranging

Gauss-Jacobi algorithm is often used to estimate unknown parameters in nonlinear models [5 and 6]. In geodesy, it is applied as an alternative method to nonlinear Gauss-Markov model, which uses Taylor series for linearization. Application of Gauss-Jacobi combinatorial algorithm in diagnostics of systematic errors in code ranging assumes to arrange the vector of receiver position, which involves the differences between system time t and time delay τ_i of a signal which originates from a satellite position as follows [7, 8 and 9]:

$$\mathbf{r}^T(t-\tau_i) = (r_1(t-\tau_1), r_2(t-\tau_2), ..., r_n(t-\tau_n)). \quad (1)$$

Vector of measurements, which is represented by the pseudorange values $d_i(t)$ and observed in time t can be defined by the formula:

$$\mathbf{y}^T = (d1(t), d_2(t), ..., d_n(t)). \quad (2)$$

Providing $\Delta t_u(t)$ is correction of the system time of a receiver, $\Delta t_i(t-\tau_i)$ is correction of the system time of a satellite and c is signal career, the basic formula for code ranging is as follows:

$$d(t) = c\tau_i + c \cdot \Delta t_u(t) - c\Delta t_i(t-\tau_i), \quad (3)$$

where $c\tau_i = D(t)$ represents the real range, $c\Delta t_u(t) = b(t)$ is the unknown parameter because the system time varies in each satellite and correction of system time of a receiver is an unknown value. The last quantity of an equation (3) $c \cdot \Delta_i(t-\tau_i) = b(t-\tau_i)$ is corrected range, which involves

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the time correction of satellites acquired from almanac. Then, the function of code pseudorange can be rewritten into a common formula:

$$\overline{d}(t) = d(t) + b(t - \tau_i) = D(t) + b(t). \qquad (4)$$

3. Gauss-Jacobi Combinatorial Algorithm

In conformity with the previously described observation model (4), the mathematical model of code ranging is demonstrated according to formulas [3 and 4] as follows:

$$\mathbf{y} = \sqrt{(\mathbf{r}(t) - \mathbf{r}(t-\tau_i))^T(\mathbf{r}(t) - \mathbf{r}(t-\tau_i))} + \\ + \mathbf{b}(t) + \mathbf{e} \qquad (5)$$

where the vector of receiver position **r**(t), vector of clock corrections **b**(t) and vector of residuals **e** belong to the unknown parameters of model, which are estimated by Gauss-Jacobi combinatorial algorithm. It is a very useful method to ensure both procedures, to estimate the appropriate pseudorange and to identify the possible systematic influence on mathematical model (5). The principle of this method is in creating the minimal number of partial combinations of the model, which satisfies the combinatorial number:

$$C_k = \binom{n}{u} = \frac{n!}{u!(n-u)!}. \qquad (6)$$

The rank of the particular matrices of the p-models depends on the number of unknown parameters u. The solution of the combinatorial algorithm consists in estimating the introductory positional parameters β_i^p from the i-th equation of p-model as follows

$$\beta_i^p = \begin{pmatrix} \beta_1^p \\ \beta_2^p \\ \vdots \\ \beta_u^p \end{pmatrix} = \mathbf{A}_i^{p^{-1}} \mathbf{y}_i^p \qquad (7)$$

and in defining the partial matrices:

$$\mathbf{G}_i^p = \mathbf{A}_i^{p^T} \mathbf{P}_i^p \mathbf{A}_i^p, \qquad (8)$$

which have to be positive definite and regular because of their inversion. The unknown parameters of the mathematical model (5) are represented by the weighted averages which are estimated from the equation:

$$\overline{\beta} = \begin{pmatrix} \overline{\beta}_1 \\ \overline{\beta}_2 \\ \vdots \\ \overline{\beta}_u \end{pmatrix} = (\mathbf{G}_i^p + \mathbf{G}_{i+1}^p)^{-1} (\mathbf{G}_i^p \mid \mathbf{G}_{i+1}^p) \begin{pmatrix} \beta_i^p \\ \beta_{i+1}^p \end{pmatrix} \qquad (9)$$

The proper identification of the systematic influence in the model consists in comparison of a partial combinatorial positional norm estimated from the i-th model:

$$K_i^p = \sqrt{x_i^2 + y_i^2 + z_i^2} \qquad (10)$$

with the median positional norm K_{med} in the case of robust estimation or with the "global" positional norm $K_{\overline{\beta}}$ estimated average from the whole model.

4. Verification of Efficiency of Combinatorial Algorithm

The diagnostics of systematic effect on final pseudorange belongs to the fundamental approaches of each GNSS developer and, therefore, there are a lot of hardware and software solutions in engineering practice. Differential GNSS seems to be one of the best methods to avoid this effect by using it in the process of determining the precise receiver position in geodesy applications [1]. However, the actual ionospheric and tropospheric model, precise time delay and satellite ephemerids are always important to know. The efficiency of the Gauss-Jacobi combinatorial algorithm was verified on real raw data arranged in "rinex" format, which were obtained by the static GNSS method. Inter alia, the "rinex" data file contains the number of visible satellites (G4, G31, G29,...) and the appropriate carrier phases (L1, L2), pseudo ranges (P1, P2) and Doppler frequency, as can be seen in Fig. 1.

The precise satellite's position needed for pseudorange calculations is defined by Cartesian coordinates in Global Reference Frame IGS08 and is accessible as a product of NASA and ESA in "sp3" format. For n = 13 satellites and u = 4 unknown parameters (x, y z, $bias$), we calculated the minimal number of satellite combinations $C_{k,\min} = 715$.The unknown parameters of the partial models were estimated according to the equation (7) to calculate the partial positional norms according to the equation (10):

$$K_i^p = \sqrt{\mathbf{r}_i(t-\tau)^T \mathbf{r}_i(t-\tau)}. \qquad (11)$$

The significant difference between the positional norm in model average

$$K_{\overline{\beta}} = \sqrt{\mathbf{r}_{\overline{\beta}}(t-\tau)^T \mathbf{r}_{\overline{\beta}}(t-\tau)} \qquad (12)$$

and the positional norm in median:

$$K_{med} = \sqrt{\mathbf{r}_{med}(t-\tau)^T \mathbf{r}_{med}(t-\tau)} \qquad (13)$$

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3953782.5851  1342642.0241  4805763.0479              APPROX POSITION XYZ
      1.4800       0.0000       0.0000            ANTENNA: DELTA H/E/N
   1    1                                   WAVELENGTH FACT L1/2
   6   C1   L1   D1   P2   L2   D2                # / TYPES OF OBSERV
 2011    10    27    7   34   0.000000              TIME OF FIRST OBS
 2011    10    27    8   36   35.000000             TIME OF LAST OBS
  15                                        LEAP SECONDS
  16                                        # OF SATELLITES
      C1   L1   D1   P2   L2   D2                COMMENT
  G 1    0    0    0    0    0    0             PRN / # OF OBS
  G 2   752   752   752   752   752   752              PRN / # OF OBS
  G 3    0    0    0    0    0    0             PRN / # OF OBS
  G 4   413   413   413   413   413   413              PRN / # OF OBS
  G 5    0    0    0    0    0    0             PRN / # OF OBS
11 10 27  8  0  0.0000000   0 13 G4G31G29G25G9G2G12R21R11R10R9R20R19
 24176494.240   127048293.21747   -3076.746   24176496.700   98998657.71443    -2397.473
 22845397.200   120053379.65348    2584.396   22845394.980   93548087.05046     2013.815
 21045890.540   110596911.88649    2022.561   21045888.680   86179407.38048     1576.021
 20193365.920   106116851.24949     153.863   20193365.780   82688451.81948      119.893
 24765729.440   130144710.41947   -4214.593   24765734.440   101411442.73744   -3284.112
 21905710.820   115115306.17649    -510.444   21905707.740   89700211.52647     -397.750
 21143870.200   111111779.52349   -1865.957   21143867.720   86580590.79747    -1453.995
 20892791.820   111801573.66808    3003.423   20892794.780   86956794.58608     2335.996
 20260218.720   108264459.41508    2510.455   20260223.840   84205707.26906     1952.575
 19495945.680   103924354.91709   -1781.693   19495952.600   80830062.14807    -1385.763
 22953129.380   122568379.89907   -4255.016   22953141.980   95330990.20607    -3309.467
 19217751.780   102765958.73908    -721.196   19217756.680   79929086.45208     -560.932
 22289583.060   119234200.24408   -3799.061   22289588.120   92737710.35006    -2954.832
```

Fig. 1 Demonstration of raw GNSS data in "rinex" format

gives the view of error influence on observation data. The receiver position calculated from the code ranging obtained from satellites R11 and R19 seems to be influenced by observation error, as we can see from the positional norm differences displayed in Table 1. In practice, the differences from the positional norm demonstrate the pseudorange error, which can be caused by the multipath influence or improper satellite geometry. While, the first reason is easy to eliminate by an appropriate software, the second one is registered as PDOD value in a receiver.

Differences between partial and global positional norms Table 1

Combination number	Partial combinations of satellites	Positional Norm in km	Deviations	
			Average norm	Median norm
1	G4 G31 G29 G25	6366.334	-0.019	-0.013
2	G4 G29 G25 G9	6366.320	-0.032	-0.026
3	G4 G25 G9 G2	6366.302	-0.050	-0.044
4	G4 G9 G2 G12	6366.328	-0.024	-0.019
5	G4 G2 G12 R21	6366.371	0.019	0.025
6	G4 G12 R21 R11	6366.347	-0.005	0.000
7	G4 R21 R11 R10	6366.352	0.000	0.006
8	G4 R11 R10 R19	6366.589	0.237	0.243
9	G4 R10 R19 R20	6366.181	-0.171	-0.166
10	G4 R19 R20 R9	6366.383	0.031	0.036
11	G2 G4 G31 G29	6366.345	-0.007	-0.002
12	G2 G31 G29 G25	6366.336	-0.016	-0.010
13	G2 G29 G25 G9	6366.335	-0.016	-0.011
14	G2 G25 G9 G12	6366.376	0.024	0.029
15	G2 G9 G12 R21	6366.346	-0.006	0.000
16	G2 R12 R21 R11	6366.351	0.000	0.005
17	G2 R21 R11 R10	6366.138	-0.214	-0.209
18	G2 R11 R10 R19	6365.988	-0.363	-0.358
19	G2 R10 R19 R20	6366.358	0.006	0.012
20	G2 R19 R20 R9	6366.397	0.045	0.050
21	G31 G29 G25 G9	6366.352	0.000	0.005
22	G31 G25 G9 G2	6366.342	-0.010	-0.004
23	G31 G9 G2 G12	6366.346	-0.006	-0.001
24	G31 G2 G12 R21	6366.346	-0.005	0.000
25	G31 G12 R21 R11	6366.346	-0.006	0.000

The differences of the partial norms from the global positional norm in median can also be illustrated in Fig. 2.

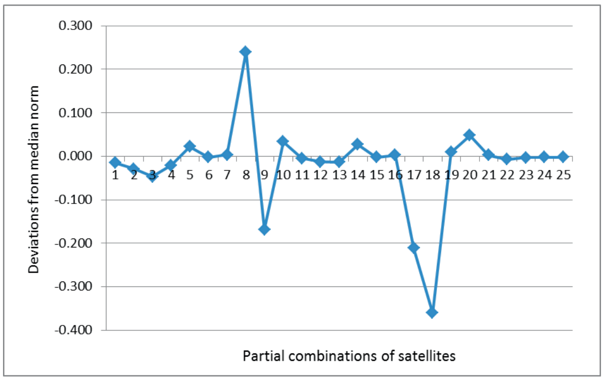

Fig. 2 Comparison of partial positional norm with the global one in median

5. Conclusion

The primary purpose of using Gauss-Jacobi combinatorial algorithm was to amend the endless iterative methods of parameter estimation with the creation of the minimal number of mathematical combinations of observed data. The practical application of this method demonstrates its strength to estimate unknown parameters in nonlinear mathematical models with current diagnostics of observation errors with systematic effect. Comparing the global positional norm with a partial one estimated in *i*-th partial model refers to code ranging error existence probably caused by multipath effect. Gauss-Jacobi combinatorial algorithm enables us to identify improper satellite position in a given system time.

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