STABILITY OF THE TWO-LINK METROBUS

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Resume

It has been shown that a perspective type of passenger transport is a metro bus, which can be both two- and three-link. Mathematical model of an autotrain has been developed for a two-link metrobus with both an uncontrolled and a controlled trailer, with the help of which it is established that the speed of the metro bus is 10 m/s, the angular speed of links racking, lateral acceleration, the angles of axle wheels' diversion while manoeuvring of "steering wheel jerk" "Shift" and "turn" have a fading nature of oscillation, which ensures the stability of its motion. At the same time, the value of lateral accelerations in the center of masses of metrobus separate links with an uncontrolled trailer at a speed of 15 m/s, while performing various manoeuvres do not exceed 0.45 g, according to this characteristics, such metrobus is stable, instead of a metrobus with a controlled trailer, which at this speed loses its stability. This should be considered when developing the advanced two-link metrobus of increased overall length (24 m).

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1 Introduction

Design of city buses is a topic of interest in current research, both in terms of control [1] and sustainable mobility [2]. As for articulated buses, stability control is an issue of great practical importance, since the onset of unstable oscillations like "jack-knifing" can lead to loss of control and to dangerous accidents. Rapid bus, mainly Metrobus (Eng. Bus rapid transit, BRT) is the way to organize the bus (or trolleybus) connection, which has higher operational characteristics in comparison with regular bus routes (speed, reliability, transportation capacity). According to peculiar parameters (in particular, at speed) the system of high-speed bus transport can be compared to the light rail systems (high-speed tram) [3].

The BRT project involves the bus traffic on dedicated and often enclosed lanes. The main advantage of the metro bus is its complete isolation on the road from other types of transport. As a means of transportation, the next-generation articulated buses, equipped with engines up to 250 kW, have been selected. However, as in the metro and in metrobus salons places for standing are much more preferable. Due to this, only one articulated two-link bus carries up to 200 passengers [4].

The BRT system has a number of undeniable benefits:

- high passenger capacity and efficient payment systems provide low-cost travel;
- high traffic speed allows the metro bus to transport a significant part of passengers, which reduces the number of cars on the city roads and, thereafter, vehicle emissions have been reduced;
- a detailed information system informs passengers about schedules of the routes.

A state of the art study on directional performance issues in design of articulated heavy vehicles (mainly devoted to trucks) is presented in [5-7], with an overview of the models used in designing controllers. Some studies are specifically devoted to articulated buses (and in particular to pusher articulated buses), however presenting numerical simulations only of particular manoeuvres, using a single-track model [8-10].

Convenience, safety and improved traffic organization are not all that a new system of high-speed bus transport can offer to passengers. At the same time, it is very important to ensure the stability of the hinged-articulated vehicles while driving at high speed. Therefore, the aim of the work is to analyze the influence of mass, power and layout parameters of a two-link metrobus, as with uncontrolled and controlled trailer, on the stability of its traffic.

2 Methodology

The stability of traffic refers to properties of the automatic transport vehicles (ATV), which do not have a strict certainty in terminology, requirements, indicators and methods of assessment [11-16]. In practice, experimental characteristics that determine the stability of the ATV while driving have been used and in theory - direct and indirect indicators and their dependencies, among which the critical speed of traffic is the main one.

In famous nowadays studies the stability and controllability issues have been considered in two aspects [17]:

1. The study concerning the characteristics of all the elements of "driver-vehicle-road" system has been seen as a closed system of automatic control.

2. The study of own stability and controllability of a car (autotrain) in which the driver's influence has been excluded.

To ensure controllability and stability of the system of "driver-autotrain-road", it is necessary, though not sufficiently, to provide its own stability and control of the car and trailer parts, e.g. properties that lie in their design (without driver's motion correction via the instability). Thus, if the stability of the ATV is ensured, then the stability of the driver's system - ATV is also provided, but with a large margin. Therefore, it is enough to consider the traffic in the management of an open scheme, that is, the potential stability of the ATV itself.

Currently, the calculation methods for determining critical speeds are most fully developed for single cars, trailers and two-link sidecar autotrains. Much less studies have been devoted to the critical speed determination of the two-link trailer road trains, which include the two-link metrobus.

In works [18-20], when completing the differential equations of autotrains traffic, the following constraints have been taken into consideration: a flat calculation scheme, autotrain traffic is carried out on an equal supporting surface with constant speed along a linear or curvilinear trajectory set by the angle of rotation of the driven wheels of power unit.

In Equations (1-20) the following notations have been taken:

a - the distance from the front axle to the center of the bus mass;

b - the distance from the middle axis to the center of the bus mass;

bb - the distance from the back axle to the center of the bus mass;

bs - the distance from the center of mass to the axle of the rear suspension of the bus;

c - the distance from the center of the bus mass to the point of coupling with the trailer;

λ -the shift of the front driven wheels of the bus, due to the longitudinal inclination of the kingpin;

c_{o1} - the distance from the coupling point to the bus back position point;

θ - the angle of wheels rotation of the front axle of the bus;

b_1 - the distance from the center of the mass of the trailer to its front axle;

b_{11} - the distance from the center of the mass of the trailer to its back axle;

c_{11} - the distance from the coupling point to bus back overall point;

θ_2, θ_{21} -the rotation angle of the front and back axle of the trailer;

m, J - the mass and central moment of inertia of the bus;

v, u - the longitudinal and transverse speed vector projection of the center of mass on an axle connected with the bus;

ω - the angular speed of the bus according to the vertical axis;

m_1, J_1 - the mass and central moment of inertness of the control wheel module of the bus;

v_1, u_1 - the longitudinal and transverse projections of the speed vector of the center of mass of the control wheel module of the bus;

ω_1 - the angular speed of the control wheel module of the bus;

m_2, J_2 - the *mass and central moment of trailer inertness (second link)*;

v_2, u_2 - the longitudinal and transverse projection of the speed vector of the center of the mass of the trailer;

ω_2 - the angular speed of the trailer;

γ_1 - the angle assembly of the links of the bus;

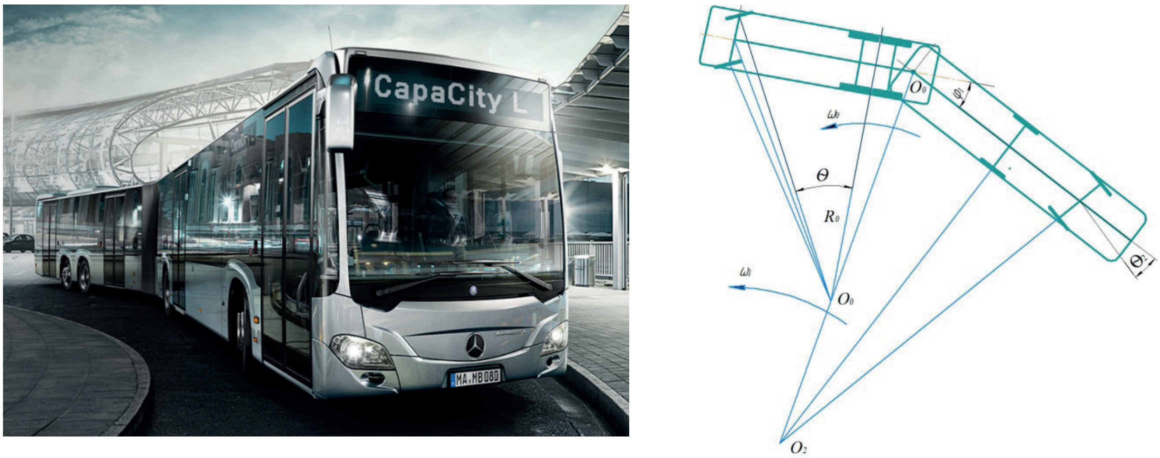
$X_1, X_2, X_{21}, X_3, X_{31}$ - the longitudinal forces on the axles of the bus and trailer;

Y_1, Y_2, Y_{21}, Y_3, Y_{31} - the transverse forces on the axles of the bus and trailer.

For such restrictions, the equation of two-link metrobus traffic has been written, Figure 1 [21]:

- for the controlling wheel module bus

$$-m_1(\dot{v}_1-\omega_1 u_1)-XA\cos\theta-X_1-YA\sin\theta=0\,,\quad(1)$$



a) b)

***Figure 1** Photograph (a) and calculation scheme of a metrobus (b)*

$$-m_1(\dot{u}_1 - \omega_1 \nu_1) - XA\sin\theta - YA\cos\theta + Y_1 = 0\,. \quad (2)$$

- for the basic of the bus

$$m(\dot{\nu} - \omega u) + X_2\cos\theta_1 + X_{21}\cos\theta_{11} - Y_2\sin\theta_{11} - \\ - XA + XB\cos\gamma_1 - YB\sin\gamma_1 = 0, \quad (3)$$

$$m(\dot{u} - \omega\nu) + Y_2\cos\theta_1 + Y_{21}\cos\theta_{11} - X_2\sin\theta_{11} - \\ - YA + YB\cos\gamma_1 - XB\sin\gamma_1 = 0. \quad (4)$$

- for a trailer

$$-m_2(\dot{\nu} - \omega_2 u_2) + XB - X_3\cos\theta_2 + Y_3\sin\theta_2 - \\ - X_{31}\cos\theta_{21} + Y_{31}\sin\theta_{21} = 0, \quad (5)$$

$$-m_2(\dot{u} - \omega_2\nu_2) + YB - X_3\sin\theta_2 + Y_3\cos\theta_2 + \\ + X_{31}\sin\theta_{21} + Y_{31}\cos\theta_{21} = 0. \quad (6)$$

The common solution of Equations (1) - (6) allows to find the reactions at points of connection of the control wheel module with the basic of the bus XA and YA, the basic of the bus and the trailer XB and YB.

$$XB = m_2\dot{\nu}_2 - m_2\omega_2 u_2 + X_3\cos\theta_2 - Y_3\sin\theta_2 + \\ + X_{31}\cos\theta_{21} - Y_{31}\sin\theta_{21}, \quad (7)$$

$$YB = m_2\dot{u}_2 - m_2\omega_2 u_2 + X_3\sin\theta_2 - Y_3\cos\theta_2 + \\ + X_{31}\sin\theta_{21} - Y_{31}\cos\theta_{21}. \quad (8)$$

$$XA = m_1\dot{u}_1\sin\theta + m_1\omega_1\nu_1\sin\theta - Y_1\sin\theta - \\ - m_1\dot{\nu}_1\cos\theta + m_1\omega_1 u_1\cos\theta - X_1\cos\theta; \quad (9)$$

$$YA = -m_1\dot{u}_1\cos\theta - m_1\omega_1\nu_1\cos\theta + Y_1\cos\theta - \\ - m_1\dot{\nu}_1\sin\theta + m_1\omega_1 u_1\sin\theta - X_1\sin\theta. \quad (10)$$

After determining the reactions at points of the autotrain links connection, taking into account the equations of the traffic rotation of each link of the autotrain, a system of differential equations has been written in the following form:

$$m(\dot{\nu} - u\omega) = -X_2\cos\theta_1 - X_{21}\cos\theta_{11} + \\ + Y_2\sin\theta_1 + XA - XB\cos\gamma_1 + YB\sin\gamma_1; \quad (11)$$

$$m(\dot{u} - \nu\omega) = Y_2\cos\theta_1 + Y_{21}\cos\theta_{11} + \\ + X_2\sin\theta_1 + YA - YB\cos\gamma_1 + XB\sin\gamma_1. \quad (12)$$

$$I\dot{\omega} = aYA - b(Y_2\cos\theta_1 + X_2\sin\theta_1) - \\ - bb(Y_{21}\cos\theta_{11} + X_{21}\sin\theta_{11}) + \\ + c(YB\cos\gamma_1 + XB\sin\gamma_1) + M_1 + M_2; \quad (13)$$

$$I_1\dot{\omega}_1 = -YA\lambda\cos\theta + XA\lambda\sin\theta - M_1 = 0; \quad (14)$$

$$I_2\dot{\omega}_2 = d_1 YB - b_1(Y_3\cos\theta_2 + X_3\sin\theta_2) - \\ - b_{11}(Y_{31}\cos\theta_{21} + X_{31}\sin\theta_{21}) + \\ + c_1(YC\cos\gamma_2 + XC\sin\gamma_2) - M_3. \quad (15)$$

After substituting the values of reactions at the points of connection of autotrain individual parts, a system of differential equations relative to the variables $(u, \omega, \theta, \dot{\theta}, \gamma_1, \dot{\gamma}_1)$ has been written in the form:

$$m(\dot{\nu} - u\omega) = -\dot{\omega}[m_1\sin\theta\lambda + m_2 d_1\sin\varphi_1] - \\ - \ddot{\theta}m_1\sin\theta \times \lambda - \omega^2[m_1 \times \lambda\cos\theta + m_2 d_1\cos\varphi_1 - \\ - m_1 a + m_2 c] - \dot{\theta}^2 m_1 \times \lambda\cos - \dot{\varphi}_1^2 m_2 d_1\sin\gamma_1 - \\ - \dot{\omega}_1 m_2 d_1\sin\varphi_1 - 2\omega\dot{\theta}m_1 \times \lambda\cos\theta + \varphi 1\omega \times \\ \times[-2m_2 d_1\cos\varphi_1] + \omega u(m_1 + m_2) - X_1\cos\theta - \\ - Y_1\sin\theta - X_2 - X_{21} - X_3\cos(\varphi_1 + \theta_2) + \\ + Y_3\sin(\varphi_1 + \theta_2) - X_{31}\cos(\varphi_1 + \theta_{21}) + \\ + Y_{31}\sin(\varphi_1 + \theta_{21}); \quad (16)$$

$$m(\dot{u} - \omega\nu) = -\dot{u}2m_2\cos^2\varphi_1 + \dot{\nu}m_2\sin 2\varphi_1 - \dot{\omega} \times \\ \times\{m_1(a - \lambda\cos\theta) - m_2[\cos\varphi_1(2c_1 + d_1) - c] + \\ + \cos\varphi_1 \times (2c \times \cos\varphi_1 + c_1 + d_1 + c)\} + \ddot{\theta}m_1 \times \\ \times\cos\theta - \omega\nu(m_1 + m_2) \times (1 - 2\cos^2\varphi_1) - \\ \omega u m_2\sin 2\varphi_1 - \omega^2[m_1\lambda\sin\theta - m_2 \times (d_1\sin\varphi_1 + \\ + c\sin 2\varphi_1)] - 2\omega\dot{\varphi}_1 m_2 d_1\sin\varphi_1 - \dot{\varphi}_1^2 m_2 d_1 \times \\ \times\sin\varphi_1 + m_2 d_1\cos\varphi_1 + Y_1\cos\theta - X_1\sin\theta + Y_2 + \\ + Y_{21} + Y_3\cos(\varphi_1 + \theta_2) - X_3\sin(\varphi_1 + \theta_2) + \\ + Y_{31}\cos(\gamma_1 + \theta_{21}) - X_{31}\sin(\gamma_1 + \theta_{21})]; \quad (17)$$

$$I_1\ddot{\theta} = -\dot{\omega}(I_1 + m_1 a\lambda\cos\theta - m_1\lambda^2) - \dot{v}m_1\lambda\sin\theta - \dot{u}m_1\lambda\cos\theta + \ddot{\theta}m_1\lambda^2 + \omega u m_1\lambda\sin\theta + \omega v m_1\lambda\cos\theta - Y_1\lambda + M_1 + M_2; \quad (18)$$

$$I\dot{\omega} = aYA - b(Y_2\cos\theta_1 + X_2\sin\theta_1) - bb(Y_{21}\cos\theta_{11} + X_{21}\sin\theta_{11}) + c(YB\cos\gamma_1 + XB\sin\gamma_1) + M_1 + M_2; \quad (19)$$

$$I_2\dot{\omega}_1 = -\dot{\omega}[I_2 + m_2 d_1(c\cos\varphi_1 + d_1)] - \dot{\omega}_1 m_2 d_1^2 - \dot{v}m_2 d_1\sin\varphi_1 + \dot{u}m_2 d_1\cos\varphi_1 + \omega v m_2 d_1\cos\varphi_1 + \omega u m_2 d_1\sin\varphi_1 + \omega^2 m_2 c d_1\sin\varphi_1 - Y_3(d_1 + b_1)\times \cos\theta_2 - X_3(d_1 + b_{11})\cos\theta_{21} - X_{31}(d_1 + b_{11})\sin\theta_{21}. \quad (20)$$

To integrate the equations of the metrobus, it is necessary to determine the longitudinal, transverse forces on its axes, as well as the moments of inertia of the metrobus links. Longitudinal forces on the wheels of the autotrain have been determined as:

$$X_1 = f\cdot Z_i\,, \quad (21)$$

where f - coefficient of autotrain wheels rolling resistance, Z_i - the load on the axis of autotrain individual sections, which have been determined by using the following dependencies:

$$\sum F_Z = 0;\ \sum mom_{oy}F_i = 0,, \quad (22)$$

$$Z_1 = m_1 g + mgbs/l;\ Z_2 0.5mga/l;\ Z_3 = 0.5m_2 g;, \quad (23)$$

where $bs=(b+bb)/2;\ l=a+bs-\lambda$.

The moments of the metrobus links inertia have been determined in accordance with the study of Podrygalo and Volkov [22], where a quite precise probabilistic method for determining the radii of inertia of a car (trailer, semitrailer) has been proposed. It is based on two assumptions: firstly, the moment of inertia of the car (trailer, semitrailer) depends on the law of its mass distribution within the limits of the railway track, base and height; and secondly, the distribution density of the moment of inertia has been subordinated to the normal distribution law.

The most probable values of the radius of inertia relative to the vertical axis have been given by the formula

$$\rho_z = \sqrt{\frac{1}{2}ab + \frac{B^2}{12} \pm \frac{1}{6}ab}\,, \quad (24)$$

relative to the longitudinal and transverse axes of the formula

$$\rho_x = \sqrt{\frac{1}{2}(H-h)h + \frac{B^2}{12} \pm \frac{1}{6}(H-h)}\,, \quad (25)$$

$$\rho_y = \sqrt{\frac{1}{2}ab + \frac{1}{3}(H-h)h \pm \frac{1}{6}ab}\,, \quad (26)$$

where B - the track of the car (trailer), H - the height of the car (trailer), h - the distances from the center of mass to the road's plane.

The moment of inertia of the metrobus links are determined by the known formula

$$I_i = \rho_i m_i\,. \quad (27)$$

The moment of resistance in the hinge between the links of the metrobus has been determined as [23]

$$M_{0i} = \frac{2}{3}Z_{0i}\mu\frac{R_i^2 - r_i^2}{R_i^2 + r_i^2},, \quad (28)$$

where Z_{0i} - vertical load in the support-coupling device, μ - the coefficient of friction in the support-coupling device, μ=0.15...0.20, R_i, r_i - bigger and smaller radii of the support-coupling device, respectively.

Nowadays, there are several analytical approximations of the dependence of the lateral reaction applied in the spot of the wheel contact, from the angle of lead, namely [24]:

$$Y_i = k_i \times arctg(c_i\delta_i),$$
$$Y_i = k_i \times th(c_i\delta_i), \quad (29)$$
$$Y_i = \frac{k_i\delta_i}{\sqrt{1 + k_i\times(\varphi^2 G_i^2)^{-1}\times\delta_i^2}},$$

where δ_i, Y_i - abduction angles and lateral forces, φ - coefficient of transverse linkage between the tire and the supporting surface (has been considered as a constant value of this study), k_i - coefficient of resistance to lateral withdrawal, G_i - vertical load on the wheel.

The general requirement for all the functions given in the Equation (29) states that, function $f(\delta)$ is the sum of an alternating series

$$Y = k\delta - k'\delta^3 + k''\delta^5 - + \ldots. \quad (30)$$

In the further calculations, in mathematical modelling, the last Y in Equation (29) dependence is used. The need to take into account nonlinearity is explained by the fact that only in a very narrow range the dependence between the forces acting on the axis and the angles of the axis drawdown is close to the linear one, while with other values of the angles of withdrawal, this dependence is nonlinear and the lateral force cannot exceed the Y^* adhesion forces. As the lateral force approaches to its maximum value, a partial slip in the lateral direction begins and then a full slide starts. Accordingly, the maximum value of the lateral force $Y = Y^*$ can be found, based on the fact that

$$\lim_{\delta\to\infty} Y(\delta) = \frac{k}{\chi} = Y^*, \Rightarrow \chi = \frac{k}{Y^*}, Y^* = \varphi G. \quad (31)$$

If the coefficient of resistance of the lateral carriage is indicated in the absence of the longitudinal forces acting on the wheel, through k_0, then k is defined as [24]

$$k = k_o\frac{\sqrt{1-(X/\varphi G)^2}}{1 + 0.375X/G}\,, \quad (32)$$

where X - the longitudinal force acting on the wheel, the value of which has been determined by the ratio $X=M/r$, if $M/r < \varphi G$ and $X = \varphi G$, if $M/r \geq \varphi G$.

In the absence of the traction (brake) moment on the wheels of the autotrain axes, the lateral forces on the wheels of its axes have been determined by the dependence [24]

$$Y_i = \frac{k_i \delta_i}{\sqrt{1+\chi_i^2 \delta_i^2}}, \quad (33)$$

$$\chi_i = \frac{k_i}{\varphi Z_i}. \quad (34)$$

Integration of the equations system in (20) was performed with the software Maple 14. Equation (20) in "the machine form" due to the additional symbols should be written, where:

- *v*, *u* - longitudinal, transverse projection of the velocity vector of the center of mass on an axle connected with the tractor;
- *omega* - angular speed of the tractor, relative to the vertical axis;
- *v1*, *u1* - longitudinal and transverse projection of the velocity vector of the center of the masses of the steering wheel module;
- *omega1* - angular speed of the steering wheel module;
- *v2*, *u2* - longitudinal and transverse projection of the velocity vector of the center of the masses of the second link;
- *omega2* - angular velocity of the second link;
- *U* - derivative of the lateral velocity of the center of the masses;
- *Ω* - derivative of the angular speed of the tractor relative to the vertical axis;
- Θ - the change speed of the rotation angle (θ) of the controlled module;
- *TT* - angular acceleration of the controlled module;
- *Phi1* - the change speed of the rotation angle ($\varphi 1$) of the second link;
- *PPT1* - angular acceleration of the second link;
- *V* - acceleration in the longitudinal direction.

Taking into account the accepted symbols of the autotrain motion Equation (6) in the "machine type" for steady motion ($\dot{\nu} = 0$) has been presented in the form:

- by variable u:

e1:=-m(U+omega*v)+Y2+Y21*cos(theta11)+X21*-sin(theta11)-m1*U-X1*sin(theta)+ cos(theta)*Y1+-cos(phi1)*X31*sin(theta21)+sin(phi1)*X31*cos(theta21)+m2*U+ cos(phi1)*X3*sin(theta2)-sin(phi1)*Y3*sin(theta2)+-cos(phi1)*Y31*cos(theta21)-2*m2*cos(phi1)^2*U+-cos(theta)*m1*lambda*Omega+m2*omega*v+2*m2*-cos(phi1)^2*c* Omega-2*m2*omega*v*cos(phi1)^2-m1*sin(theta)*lambda*Theta^2+cos(theta)*m1*lambda*TT-m1*sin(theta)*lambda*omega^2-m1*omega*v-m1*a*Omega-sin(phi1)*Y31*sin(theta21)+cos(phi1)*m2*d1*PT1+cos(-phi1)*Y3*cos(theta2)-2*cos(phi1)*m2*omega*-sin(phi1)*u-m2*c*Omega+cos(phi1)*m2*d1*Omega+2*cos(phi1)*m2*V*sin(phi1)-+2*cos(phi1)*m2*sin(phi1)*c*omega^2+sin(phi1)*X3*+-sin(phi1)*m2*d1*omega^2+2*sin(phi1)*m2*omega*d1*Phi1+sin(phi1)*m2*d1*Phi1^2;*

- by variable ω:

*e2:=-J*Omega+a*cos(theta)*Y1-c*cos(phi1)*Y31*-cos(theta21)-c*cos(phi1)*X3*sin(theta2)-c*cos(-phi1)*Y3*cos(theta2)-cos(phi1)*X31*sin(theta21)+c*sin(phi1)*X3*cos(theta2)-2*a*m1*sin(theta)*omega*lambda*Theta+a*cos(theta)*m1*lambda*Omega-a*m1*sin(theta)*lambda*omega^2-a*m1*sin(theta)*lambda*Theta^2-a*X1*sin(theta)+a*cos(theta)*m1*lambda*TT+c*sin(phi1)*Y31*sin(theta21)+ c*sin(phi1)*X31*cos(theta21)+2*c*sin(phi1)*m2*omega*d1*Phi1+c*sin(phi1)*m2*d1*Phi1^2+ c*sin(phi1)*m2*d1*omega^2-c*cos(phi1)*m2*d1*PT-1-c*cos(phi1)*m2*d1*Omega-bb*Y21-m2*c^2*Omega-a*m1*U-m1*a^2*Omega+c*m2*U-a*m1*omega*v+c*m2*omega*v+kk1*(theta-theta0)+h1*Theta+kk2*(phi-phi10)+h2*Phi1;*

- by variable θ:

e3:=-J1(Omega+TT)+lambda*(-Y1+cos(theta)*m1*U-m1*lambda*Omega-m1*lambda*TT-m1*sin(theta)*V+m1*sin(theta)*omega*u+cos(theta)*m1*a*Omega+cos(theta)*m1*omega*v+ m1*sin(theta)*a*omega^2)-kk1*(theta-theta0)-h1*Theta;*

- by variable φ_1:

e4:=-J2(Omega+PT1)-m2*d1^2*Omega-m2*d1^2*PT1-d1*X31*sin(theta21)-b1*Y3*cos(theta2)-b1*X3*sin(theta2)-b11*Y31*cos(theta21)-b11*X31*sin(theta21)-d1*m2*V*sin(phi1)+d1*m2*cos(phi1)*U+d1*m2*omega*v*cos(phi1)+d1*m2*omega*sin(phi1)*u-d1*Y31*cos(theta21)-d1*X3*sin(theta2)-d1*Y3*cos(theta2)-c*cos(phi1)*m2*d1*Omega-kk2*(phi1-phi10)-h2*Phi1+kk3*(phi2-phi20)+h3*Phi2.*

3 Results

Integration of motion equations has been made by such initial data:
L_a=12 m; L_m=22 m; a =3.9 m; b=1.45 m; bb=2.75 m; c=3.8 m; λ=-0.0023 m; co_1=1.2 m; B=2.0 m; b_1=-0.65 m; b_{11}=0.65 m; DL2=4.8 m; c_{11}=7.8 m; V:=0; X1:=0; X2:=0; X21:=0; X3:=0; X31:=0; m1:=600; m:=17400; kf:=0; m2:=16000; k1:=160000; k2:=252000; k21:=252000; k3:=165000; k31:=165000; kk1:=2600; kk2:=300; h1:=30; h2:=30; kappa1:=0.8; kappa2:=0.8; kappa3:=0.8; kf:=0; v:=10; phi10:=0; theta:=0; theta2:=0; theta21:=0; J1:=12.22; J:=288395; J2:=271189;
Z1:=70000; Z2, Z21:=0.5*m*g*a/l=80000; Z3,

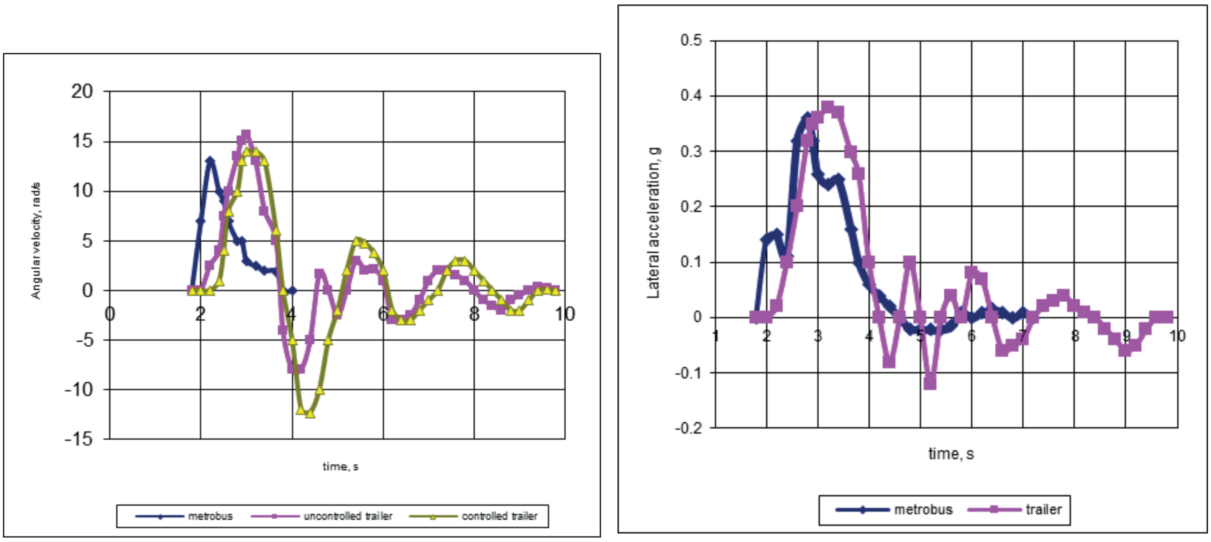
Z31:=0.5*m*g*a/l=55000;
Y1:=k1*delta1/sqrt(1+(k1*delta1/(kappa1*Z1))^2);
Y2:=k2*delta2/sqrt(1+(k2*delta2/(kappa2*Z2))^2);
Y21:=k2*delta21/sqrt(1+(k2*delta21/(kappa2*Z2))^2);
Y3:=k3*delta3/sqrt(1+(k3*delta3/(kappa3*Z3))^2);
Y31:=k3*delta31/sqrt(1+(k3*delta31/(kappa3*Z3))^2).

The results of calculating the angular velocity of the metrobus linkage, lateral acceleration, angles of wheel alignment of axes, the angle of the links folding during the "steering wheel jerk" manoeuvre for a metrobus with an uncontrolled and a controlled trailer for direct control over the axle and gear ratio of about 0.5 has been shown in Figures 2-5.

The fading character of the oscillation of the angular velocity of the bus and the trailer rushing demonstrates the stable character of the metrobus links movement during this manoeuvre (Figure 2). Thus, it should be noted that the absolute values of the rush speed for an uncontrolled and controlled trailer are almost identical, which can lead to a breakdown of stability when increasing the speed of the metrobus.

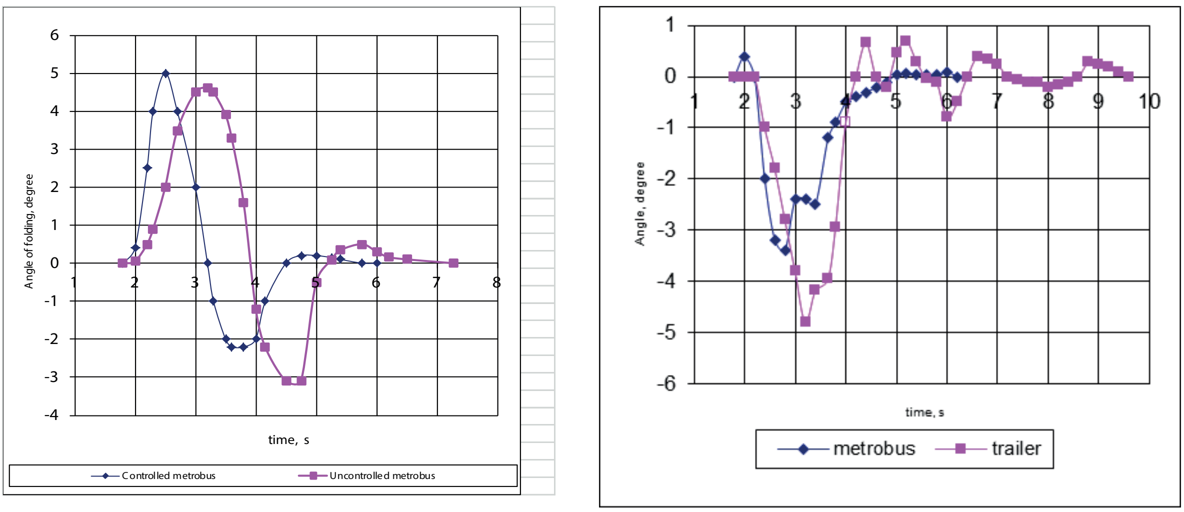
The stability of motion, to a greater extent, can be judged by the magnitude of lateral accelerations, operating in the center of masses of individual links. According to literary sources, the stability of motion can be considered satisfactory if the transverse accelerations in the center of masses do not exceed 0.45 g. This condition corresponds to the metrobus with a controlled trailer (more unstable in comparison to the uncontrolled one) (Figure 1).

In addition, the fading character of fluctuations also indicates the stability of the metrobus. As in the



a) b)

Figure 2 *Angular rushing velocity (a) and lateral acceleration of the metro bus (b) during the manoeuvre "steering wheel jerk"*



a) b)

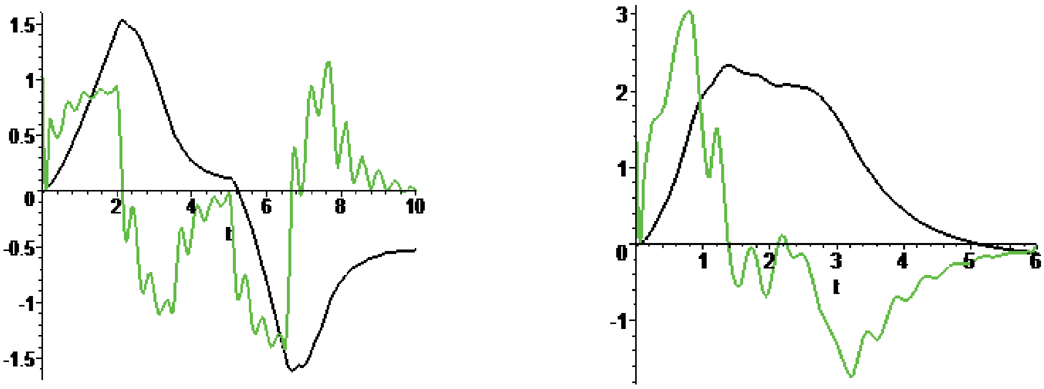
Figure 3 The angle of folding *(a) and separation of the centers of the masses links (b) of the metrobus during the manoeuvre of "steering wheel jerk"* preceding case, the accelerations in the center of the trailer mass are somewhat higher than the acceleration of the bus, so the limiting factor, when performing the manoeuvre of "steering wheel jerk", is either accelerating the center of the mass of the trailer.

According to values of the lateral accelerations, the lateral forces acting in the center of the masses of individual links have been determined and behind them the angle of folding, Figure 3a and the angles of links separation, Figure 3b.

It should be noted that bigger angles of allocations outlets are inherent in the trailer. The variable character of lateral acceleration and lateral forces leads to oscillations of the links lateral displacement angles and the greater amplitude of oscillations takes place at the center of the trailer masses. That can lead to superfluous rotation of the metrobus and the violation of stability when increasing the speed of motion. From this follows an important practical conclusion - the pressure in the trailer tires, as one which largely determines the resistance coefficient of the drive, should be chosen in such a way that does not lead to excessive twist of the metrobus.

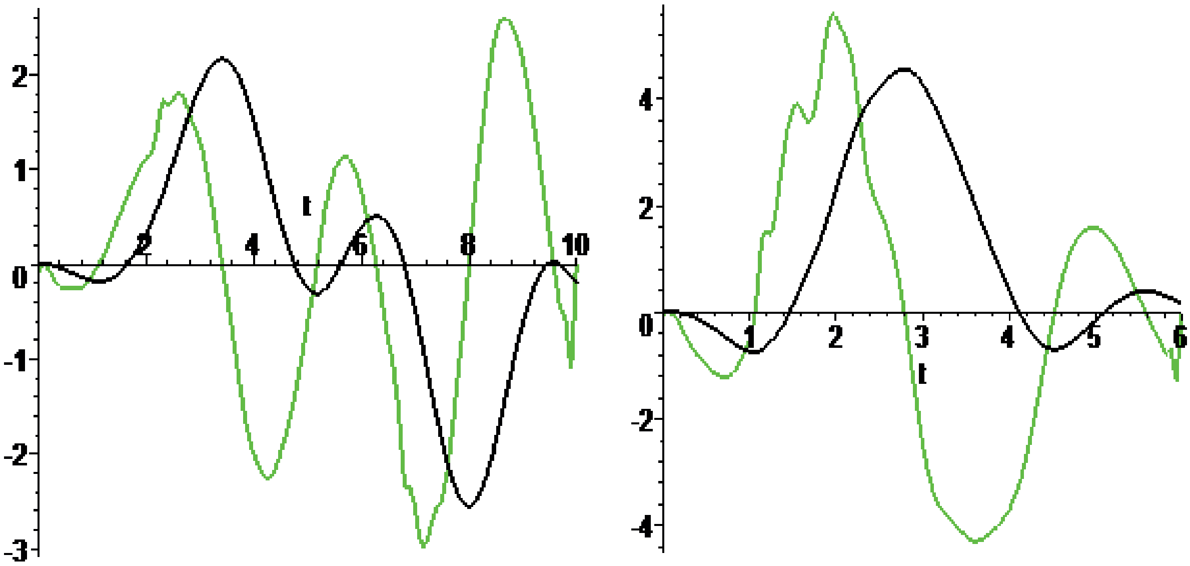
While performing a "reset" manoeuvre, similar calculations have been made. The qualitative results of calculating the main parameters of the autotrain motion for the implementation of this manoeuvre, to a large extent, coincide with the results of the previous "steering wheel jerk" manoeuvre, but the absolute values of the parameters are significantly lower.

In addition to the above-discussed manoeuvres of "steering wheel spin" and "reset", other modes of motion



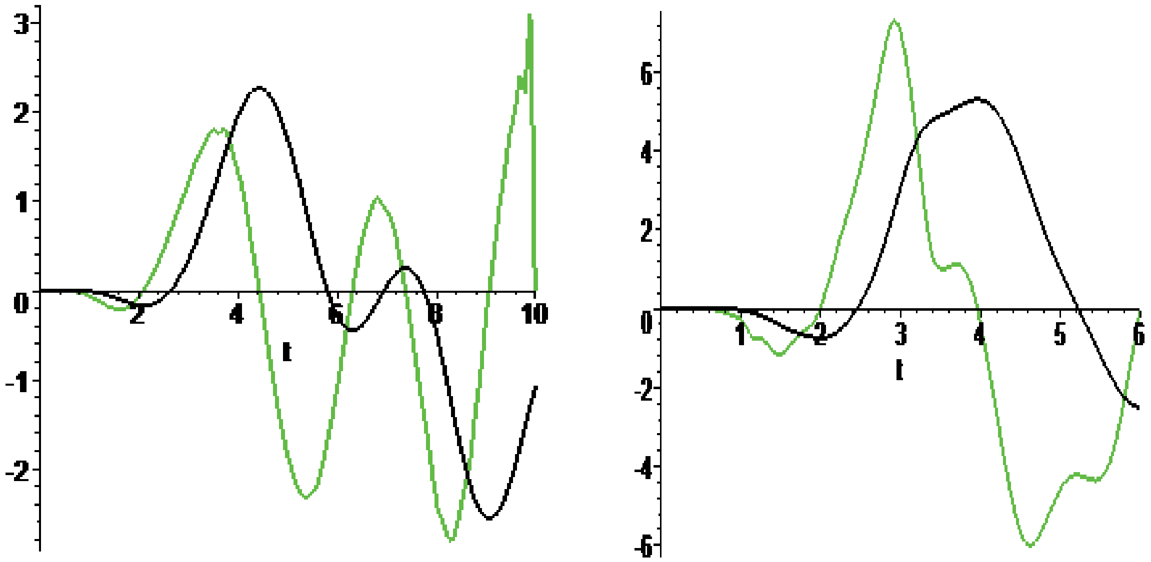
a) lateral speed *b) lateral acceleration*

Figure 4 *Lateral speed and lateral acceleration of the center of the bus with a controlled trailer mass during the manoeuvre "turn" at speeds of 10 m/s (a) and 15 m/s (b)*



a) lateral speed *b) lateral acceleration*

Figure 5 *Lateral speed and lateral acceleration of the center of the bus with an uncontrolled trailer mass during the manoeuvre "turn" at speeds of 10 m/s (a) and 15 m/s (b)*



a) lateral speed *b) lateral acceleration*

Figure 6 *Lateral speed and lateral acceleration of the center of the controlled trailer metrobus mass during the manoeuvre "turn" at speed 10 m/s (a) and 15 m/s (b)*

were also considered, namely the 90-degree rotation, S-shaped rotation, ISO manoeuvre, straight-line motion [25].

The results of calculation of the lateral speed and lateral acceleration of uncontrolled and controlled autotrain trailer links during 90-degree rotation manoeuvre are given in Figures 4-6, where a limiting factor in performing various manoeuvres is a controlled metrobus trailer, acceleration of which is 25 to 36 % higher than acceleration of an uncontrolled metrobus trailer.

4 Conclusions

The mathematical model of the metrobus has been developed, both with an uncontrolled and controlled trailer, by means of which it is established, that the speed of the metrobus 10 m/s, the angular velocity of the links, the lateral acceleration, the angles of the wheel axle during the manoeuvre of "steering wheel jerk", "reset" and "turn" have a diminishing character of oscillations. In this case, values of lateral accelerations in the center of the masses of individual links of the metrobus with an uncontrolled trailer when performing various manoeuvres do not exceed 0.45 g. So, according to this characteristic such metrobus is stable, while a metro bus with a controlled trailer at the speed of 15 m/s loses its stability. This should be taken into account when developing the advanced two-link metrobus with increased overall length (24 m).

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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