USE OF THE BEZIER CURVES FOR A VEHICLES DRIVING CYCLES' MODELING

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Resume

While performing the research of the vehicles operating characteristics in real operating conditions, a question of creation or correction of a movement driving cycle always appears. An absence of a mathematical model rather complicates the research process and makes a quick correction of movement conditions impossible. To solve the given problem, it was proposed to model a driving cycle using the Bezier curves. As a result of this research, it is stated that the most advantageous is to use the Bezier curves of the second and third degrees. During the analysis of results of driving cycles modeling and their comparison to the standardized and real movement cycles, a satisfactory coincidence of results was obtained. The conducted thematic research confirms the previous results, directed to creation of a universal dynamic model of a vehicle's driving cycle.

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1   Introduction

In spite of an intensive development of vehicles with an electric drive, during their exploitation many questions, concerning an electric energy consumption by such vehicles in concrete operating conditions, appear. The mentioned question is particularly acute during the exploitation of commercial transport, namely the buses that run the city routes. Very often, there is a problem of considerable deviation of an electric energy consumption by the same vehicle in different operating conditions. Besides, the electric energy consumption, declared by the manufacturers, noticed in official documents, is usually rather lower than the real one. In addition, these differences do not allow unique evaluating of the electric drive vehicles exploitation efficiency. Appearance of such arguments and non-conformities is associated with the difference between the driving cycle applied during the test and the real operating conditions. In such a way, a question arises of creation of such a driving cycle that could be easily adapted to the necessary operating conditions. Its application should allow getting the real figures of electric energy consumption and evaluating efficiency of one or another vehicle. This problem is rather actual and the researches that are proposed to be made within the framework of a programme Horizon 2020, give evidence to this [1].

Analysis of the previously conducted researches, connected with the research and modeling of a driving cycle and concerning the energy consumption by vehicles according to driving cycles, testify to a great interest of scientists for solving the given problem. In the research work [2], the authors have investigated multi-factors model of movement and have made the corresponding experimental research with the aim to minimize the electric energy consumption, taking into account the concrete vehicles operating conditions. A driving cycle for a passenger car in Mashhad city, Iran, compared to the standard driving cycles in Europe and America, showed a significant distinction about the speed of movement [3].

A great number of the driving cycles, offered in literature, are corrected to the concrete operating conditions and do not allow to adapt them to the other conditions or to change the cycle parameters [4-10]. Besides a problem of creating a driving cycle for the concrete operating conditions and the movement modes, also appears a problem of changeability of a driving cycle in the same locality depending on the time of day [11].

The authors in a research work [12] have investigated different driving cycles that are used to find a consumption of fuel, electric energy and the levels of the vehicles exhaust gases emissions. All the investigated driving cycles have significant differences in time, movement speed and movement modes. Along with that, the authors draw attention to necessity of matching a driving cycle for concrete cities, regions.

A similar situation also occurs with the driving cycles for buses, besides it becomes complicated due to absence of a unique standardized cycle with help of which it would
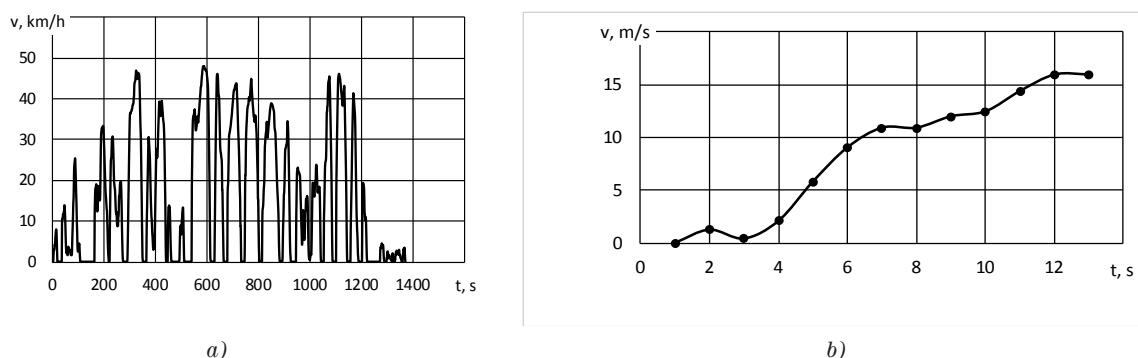
a)

b)

Figure 1 *Driving cycle of urban bus movement, obtained by experiment in Lutsk city (a); speed-up section (b)*

Table 1 *Equations of Bezier curves and level of validity of their approximation for a real vehicle movement cycle on the example of Lutsk city*

Bezier curve	equation	validity level
linear, $n = 1$	$V = 1.3333t - 1.3333$	1
quadratic, $n = 2$	$V = -0.0285t^2 + 1.821t - 1.7689$	1
cubic, $n = 3$	$V = -0.0031t^3 + 0.0212t^2 + 1.6207t - 1.9949$	0.9987
quartic, $n = 3$	$V = -0.0033t^4 - 0.1174t^3 + 1.3298t^2 - 3.5439t + 2.3598$	0.9966

be possible to compare the research results. That is why many researches in separate cities and regions in order to determine a representative driving cycle were conducted [13-15]. A driving cycle is determined as a numeric series of speeds and a scheme of movement is reproduced by a number of parameters and operation characteristics [16].

2   Research methods

A problem of the driving cycles' creation is being solved for a long time. There are four main methods of the driving cycles' creation, namely: microtrip-based, segment-based, pattern classification and modal cycle construction. Each of them has its own advantages and disadvantages and discerns by different complexity [17-19].

It is a common knowledge that a driving cycle in a graphic interpretation is a set of segments that include an acceleration of a vehicle, a movement with constant speed and braking (deceleration). The researches of real driving cycles, previously published [17, 20-24], are based on determination of extreme indices of the elementary sections of a cycle and of their deviation from standardized driving cycles.

A great problem today is still a possibility to build a driving cycle according to given data, the main of which can be the acceleration of speed-up and braking, the durability of every elementary section, the number of sections of speed-up, braking, acceleration. Results of analysis of the standardized and experimental driving cycles, described in literature sources, [20-24] demonstrate that any driving cycle can be shown as a set of curves that describe some of its section. It was proposed to take the Bezier curves as a base for building the driving cycles. The analysis of literature sources testifies about an effective usage of Bezier curves during the modeling of a vehicle mechanical

trajectory [25-26], in the automatized vehicles control systems [27-28], modeling and prediction of the route of vehicle movement and avoidance of collision for pilotless vehicles [29].

3   Research results

During the creation of a driving cycle it is advantageous and the most optimal to use:

1) for modeling of constant movement sections - linear Bezier curves, at $n = 1$,

$$B(t) = (1 - t)P_0 + tP_1, \; t \in [0;1], \tag{1}$$

where
P_0 - position of the zero point,
P_1 - position of the finishing point,
t - the time parameter.

2) for modeling of sections of speed-up and braking; - quadratic Bezier curves, at $n = 2$,

$$B(t) = (1 - t)^2 P_0 + 2t(1 - t)P_1 + t^2 P_2, \; t \in [0;1], \tag{2}$$
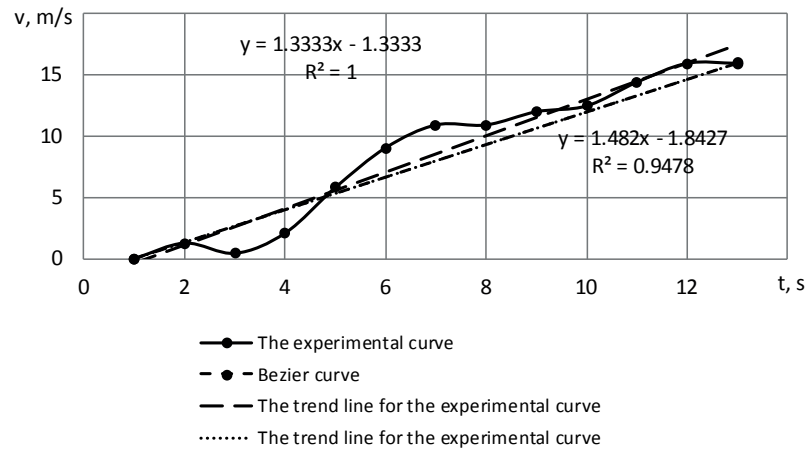
where
P_0 - position of the zero point,
P_1 - position of the intermediate point,
P_2 - position of the finishing point.
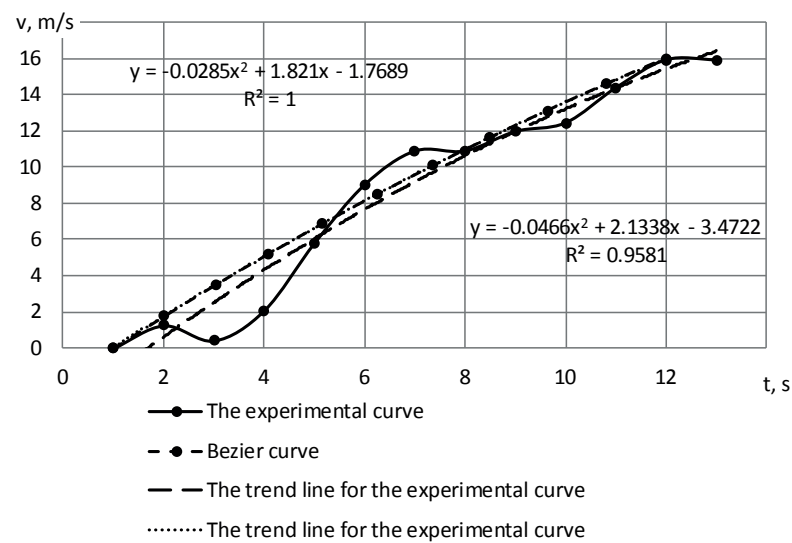 - cubic Bezier curves, at $n = 3$,

$$B(t) = (1 - t)^3 P_0 + 3t(1 - t)^2 P_1 + \\ + 3t^2(1 - t)P_2 + t^3 P_3, \; t \in [0;1], \tag{3}$$
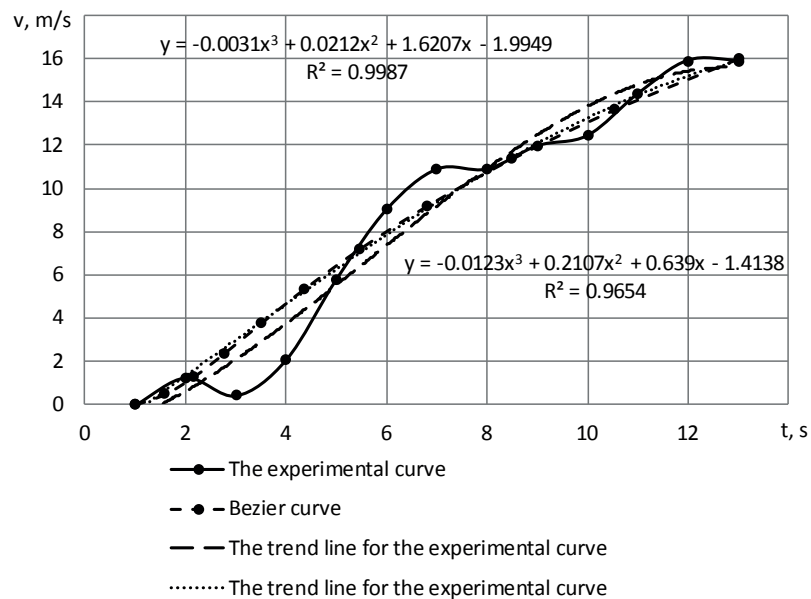
where
P_0 - position of the zero point,
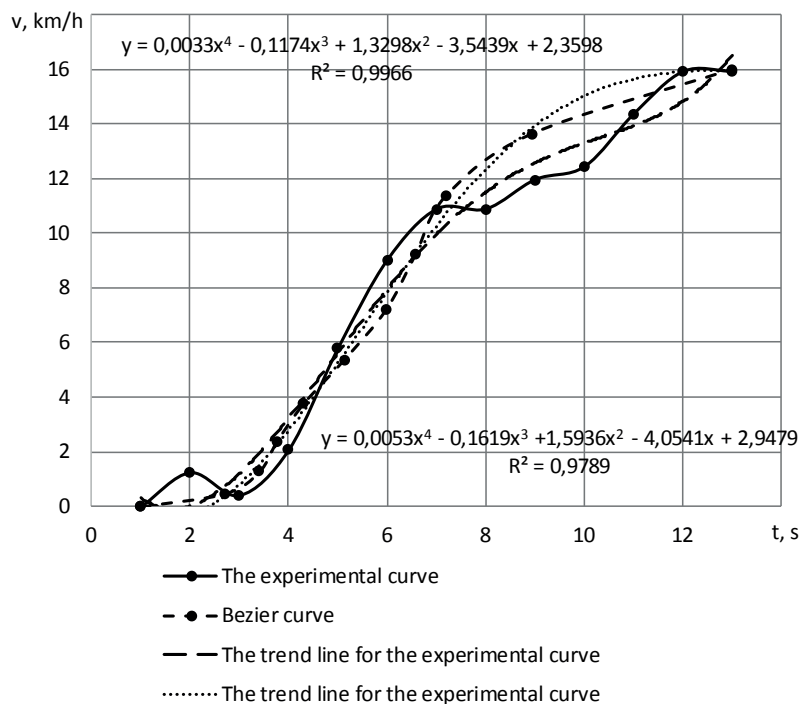P_1, P_2 - position of the intermediate points,

a) The linear Bezier curve

b) The quadratic Bezier curve

c) The cubic Bezier curve

d) The quartic Bezier curve

Figure 2 *Results of the driving cycle speed-up section modeling*

Table 2 *Equations of a real driving cycle and a level of validity of their approximation*

Bezier curve	equation	validity level
linear approximation	$V = 1.482t - 1.8427$	0.9478
polynomial curve of second degree	$V = -0.0466t^2 + 2.1338t - 3.4722$	0.9581
polynomial curve of third degree	$V = -0.0123t^3 + 0.2107t^2 + 0.639t - 1.4138$	0.9654
polynomial curve of fourth degree	$V = -0.0053t^4 - 0.1619t^3 + 1.5936t^2 - 4.0541t + 2.9479$	0.9789

P_3 - position of the finishing point.

     - quartic Bezier curves, at $n = 3$,

$$B(t) = (1 - t)^3 P_0 + 3t(1 - t)^2 P_1 + 3t^2(1 - t) \cdot$$
$$\cdot P_2 + 4t^3(1 - t)P_3 + t^4 P_4, \ t \in [0;1], \qquad (4)$$

where

P_0 - position of the zero point,

P_1, P_2, P_3- position of the intermediate points,

P_4 - position of the finishing point.

A speed-up section, extracted from a driving cycle of urban bus movement, obtained by experiment in Lutsk city is shown in Figure 1.

While building the driving cycles, a speed is measured in m/s, because a change of speed by time is an acceleration. It was made with an aim to get the unique results. A vehicle speed on the graphs that demonstrate the driving cycles, is given in km/h, according to the common rules, including standards.

Using the Bezier curves of the first, second, third and fourth degrees, made a modeling of the speed-up section for

an experimental driving cycle, shown at Figure 1. In Table 1 are given the appropriate equations and a level of validity of their approximation. The corresponding graphs are shown at Figures 2 (*a*) to (*d*).

In Table 2 there are shown the curves of approximation and the levels of validity for a real vehicle movement cycle graph on the example of Lutsk city.

To compare the obtained dependences, the appropriate calculations were conducted and their results are shown in Table 3. In the course of calculations, values of speeds were determined according to dependences shown in Tables 1 and 2. After that, the obtained data were compared to real speeds recorded directly during the movement.

The minimum deviation of speed, according to Table 3, is seen in the course of using the Bezier curves of the second and third degrees.

As far as an average acceleration is concerned, another indicator of the driving cycle, analogical calculations were conducted, as well, for values of average acceleration. Results are shown in Table 4.

In Tables 3 and 4 there are some inaccuracies that are caused by external influence of the environment and by the vehicle's operating conditions, for example: speed

Table 3 *Deviations of modeling results from their real driving cycle for speed*

real speed (m/s)	approximation							
	linear		curve of second degree		curve of third degree		curve of fourth degree	
	approximation by real dependence	Bezier curve	approximation by real dependence	Bezier curve	approximation by real dependence	Bezier curve	approximation by real dependence	Bezier curve
0.00	-1.84	1.33	3.47	1.77	1.41	1.99	-2.95	-4.47
1.25	-2.08	1.25	2.63	1.23	1.83	1.61	0.92	1.14
0.42	-4.39	-0.92	-0.19	-1.34	-0.19	-0.89	0.41	1.04
2.08	-4.21	-0.58	-0.43	-1.36	0.01	-0.89	0.90	1.18
5.80	-1.97	1.80	1.48	0.74	2.08	1.17	2.58	2.27
9.03	-0.22	3.69	3.00	2.40	3.52	2.78	3.44	2.64
10.88	0.15	4.22	3.23	2.75	3.53	3.06	2.99	2.03
10.88	-1.33	2.88	1.70	1.30	1.72	1.56	1.03	0.34
11.96	-1.74	2.63	1.34	0.98	1.07	1.22	0.64	0.62
12.43	-2.75	1.77	0.48	0.12	0.00	0.38	0.14	1.05
14.36	-2.30	2.36	1.16	0.77	0.62	1.13	1.50	3.29
15.91	-2.24	2.57	1.55	1.09	1.17	1.63	2.62	4.88
15.91	-3.72	1.24	0.48	-0.07	0.57	0.76	1.99	3.74

and direction of wind, road cutting in a plan, its roughness and vehicle suspension, driver's behaviour. Those factors sometimes have a great influence [30]. The least deviation of the vehicle's acceleration, according to Table 4, is noticed during application of the Bezier curves of the second and third degrees.

In such a way it was determined that it would be optimal to use the Bezier curves of the second and third degrees, with the aim to model and build the movement driving cycle. Along with that, using of the Bezier curves of the first degree is advantageous on the sections of a cycle with constant speeds.

During the building of the Bezier curves of the second and third degrees that describe the movement driving cycle, the biggest problem was determination of the intermediate points' P_1, P_2, positions. At first, it is necessary to investigate a simpler version, namely searching of point P_1 positions for the quadratic curve. Since a point P_1 on intervals $[P_0; P_1]$ characterizes the linear Bezier curves, its position would be the position of intersection of the corresponding lines. As the initial and final points of a vehicle movement process are known, so the equation of a line could be written out using a formula:

$$\frac{t - t_1}{t_2 - t_1} = \frac{v - v_1}{v_2 - v_1}, \tag{5}$$

where

t_1, t_2 - time of the beginning and the end of movement correspondingly, s,

v_1, v_2 - initial and final vehicle speed, m/s.

From that dependence, equations for lines of the speed-up v_a and braking v_b were obtained:

$$v_a = \frac{v_2(t - t_1) - v_1(t - t_2)}{(t_1 - t_2)}, \tag{6}$$

$$v_b = \frac{v_4(t - t_3) - v_3(t - t_4)}{(t_3 - t_4)}. \tag{7}$$

After the dependences 6 and 7 were compared, positions of lines' intersection point, which correspond to the positions of point P_1, were obtained as:

$$\frac{t(v_1 - v_2) + v_2 t_1 - v_1 t_2}{(t_1 - t_2)} = \\ = \frac{t(v_3 - v_4) + v_4 t_3 - v_3 t_4}{(t_3 - t_4)}, \tag{8}$$

$$t_{P_1} = \frac{\begin{array}{c}v_2 t_1 t_3 - v_1 t_2 t_3 - v_2 t_1 t_4 + v_1 t_2 t_4 - \\ - v_4 t_1 t_3 + v_3 t_1 t_4 + v_4 t_2 t_3 - v_3 t_2 t_4\end{array}}{\begin{array}{c}v_2 t_3 - v_1 t_3 - v_2 t_4 + v_1 t_4 - \\ - v_4 t_1 + v_3 t_1 + v_4 t_2 - v_3 t_2\end{array}}. \tag{9}$$

Position v_{P_1} of a point P_1 is calculated by expression:

$$v_{P_1} = \frac{t_{P_1}(v_1 - v_2) + v_2 t_1 - v_1 t_2}{(t_1 - t_2)}, \tag{10}$$

or

$$v_{P_1} = \frac{\left(\frac{v_2 t_1 t_3 - v_1 t_2 t_3 - v_2 t_1 t_4 + v_1 t_2 t_4 - v_4 t_1 t_3 + v_3 t_1 t_4 + v_4 t_2 t_3 - v_3 t_2 t_4}{v_2 t_3 - v_1 t_3 - v_2 t_4 + v_1 t_4 - v_4 t_1 + v_3 t_1 + v_4 t_2 - v_3 t_2}\right)(v_1 - v_2) + v_2 t_1 - v_1 t_2}{(t_1 - t_2)}, \tag{11}$$

Table 4 Deviations of modeling results from their real driving cycle for acceleration

real acceleration (m/s²)	Approximation							
	linear		curve of second degree		curve of third degree		curve of fourth degree	
	approximation by real dependence	Bezier curve	approximation by real dependence	Bezier curve	approximation by real dependence	Bezier curve	approximation by real dependence	Bezier curve
1.25	-0.23	-0.08	-0.84	-0.54	0.41	1.61	3.87	5.61
-0.83	-2.32	-2.17	-2.83	-2.57	-2.02	-2.14	-0.51	-0.11
1.67	0.18	0.33	-0.23	-0.01	0.21	-1.31	0.48	0.15
3.72	2.24	2.39	1.91	2.10	2.06	-0.91	1.68	1.09
3.23	1.74	1.89	1.51	1.66	1.44	-3.03	0.86	0.37
1.86	0.37	0.52	0.23	0.35	0.02	-5.97	-0.44	-0.61
0.00	-1.48	-1.33	-1.53	-1.45	-1.82	-9.33	-1.96	-1.69
1.08	-0.40	-0.26	-0.36	-0.32	-0.64	-9.66	-0.40	0.28
0.47	-1.01	-0.86	-0.87	-0.86	-1.08	-11.58	-0.50	0.43
1.93	0.45	0.60	0.68	0.65	0.62	-11.30	1.35	2.25
1.54	0.06	0.21	0.39	0.32	0.55	-12.73	1.12	1.58
0.00	-1.48	-1.33	-1.06	-1.17	-0.60	-15.15	-0.63	-1.14
average acceleration values, m/s²								
1.32	-0.16	-0.01	-0.25	-0.16	-0.07	-0.11	0.41	0.91
average acceleration values, %								
100	-12.12	-0.76	-18.94	-12.12	-5.30	-8.33	31.06	68.94
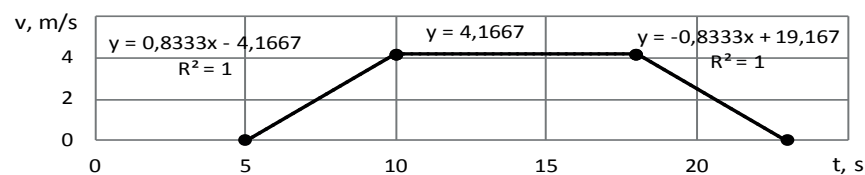
Figure 3 Elementary driving cycle

$$v_{P_1} = \frac{2v_1^2 t_2 t_4 - 2v_1^2 t_2 t_3 - v_1 v_4 t_1 t_3 + v_1 v_3 t_1 t_4 + v_1 v_4 t_2 t_3 - v_1 v_3 t_2 t_4 - 2v_1 v_2 t_2 t_4 + v_2 v_4 t_1 t_3 - v_2 v_3 t_1 t_4}{(t_1 - t_2)} -$$
$$- \frac{v_{P_1} v_2 v_4 t_2 t_3 + v_2 v_3 t_2 t_4 - v_2 v_4 t_1^2 + v_2 v_3 t_1^2 + v_2 v_4 t_1 t_2 - v_2 v_3 t_1 t_2 - v_1 v_4 t_1 t_2 + v_1 v_3 t_1 t_2 + v_1 v_4 t_2^2 - v_1 v_3 t_2^2}{(t_1 - t_2)}. \tag{13}$$

Equations (9) and (12) give a possibility to find the positions of an intermediate point P_1 to build a Bezier curve.

If one builds a driving cycle with help of lines, so in a line equation $v = at + b$, the angular coefficient a stands for a value of average deceleration on a given segment of a driving cycle. This regularity should be taken into consideration during the building of driving cycles with the help of Bezier curves. Thus, a graph of a section «speed-up - braking», for a standardized driving cycle, shown in [31] (Figure 3) is obtained.

Taking into consideration that in the real driving cycles the sections of movement with a constant speed are actually absent, modeling of a part of the driving cycle, shown in Figure 3, was performed with help of the Bezier curves of the second degree. Results are shown in Figure 4.

Deviations of speed of the modeled driving cycle from the standardized one are shown in Table 5.

The maximum deviation is observed in points of passage from the speed-up to a constant speed and from the constant speed to braking. Along with that, taking into account that in real conditions of movement, the passages from one mode to another are made smoothly without abrupt changes, the given non-conformity can be neglected.

4 Discussion of results

The Bezier curves that are used for the driving cycles modeling (Figure 4) does not completely represent the necessary characteristics of a driving cycle. That is why it is advisable to build a driving cycle by parts, marking out the sections of the speed-up, deceleration and constant speed, using the quadratic Bezier curve during that the whole process. According to the given
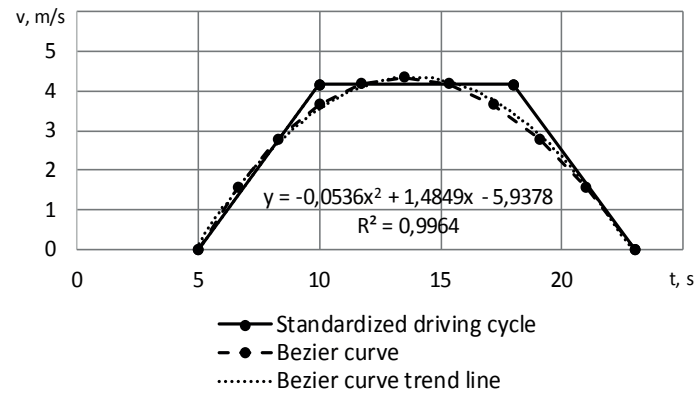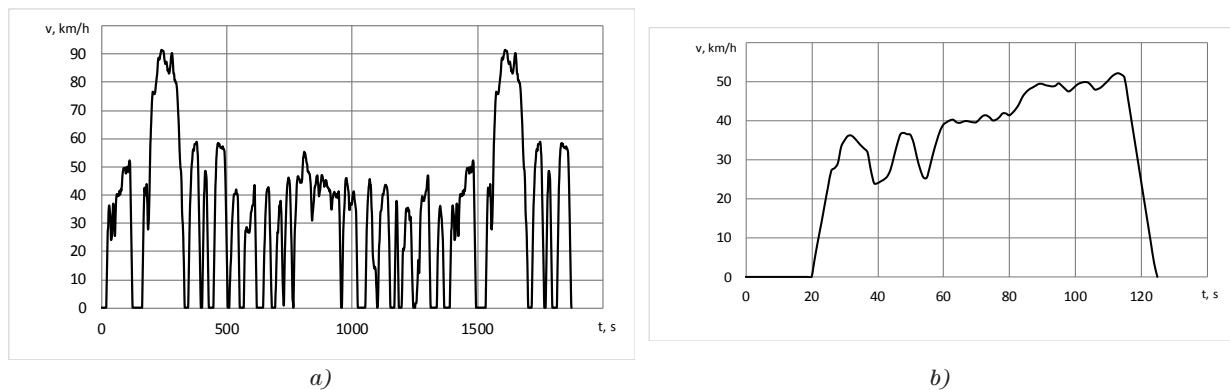
Figure 4 *Elementary driving cycle*

| a) | b) |

Figure 5 *Standardized driving cycle (a) and its section that was modeled using the Bezier curves (b)*

Table 5 *Deviations of the modeled driving cycle from the standardized one*

| time, s	speed of driving cycle (m/s)		deviations of a modeled driving cycle	
	standardized	modeled	m/s	km/h
5	0.000035	0.1092	-0.10925	-0.3933
6	0.83328	0.988	-0.15472	-0.55699
7	1.66661	1.7566	-0.08999	-0.32396
8	2.49994	2.415	0.08494	0.305784
9	3.33327	2.9632	0.37007	1.332252
10	4.1666	3.4012	0.7654	2.75544
11	4.1666	3.729	0.4376	1.57536
12	4.1666	3.9466	0.22	0.792
13	4.1666	4.054	0.1126	0.40536
14	4.1666	4.0512	0.1154	0.41544
15	4.1666	3.9382	0.2284	0.82224
16	4.1666	3.715	0.4516	1.62576
17	4.1666	3.3816	0.785	2.826
18	4.1676	2.938	1.2296	4.42656
19	3.3343	2.3842	0.9501	3.42036
20	2.501	1.7202	0.7808	2.81088
21	1.6677	0.946	0.7217	2.59812
22	0.8344	0.0616	0.7728	2.78208
23	0.0011	-0.933	0.9341	3.36276
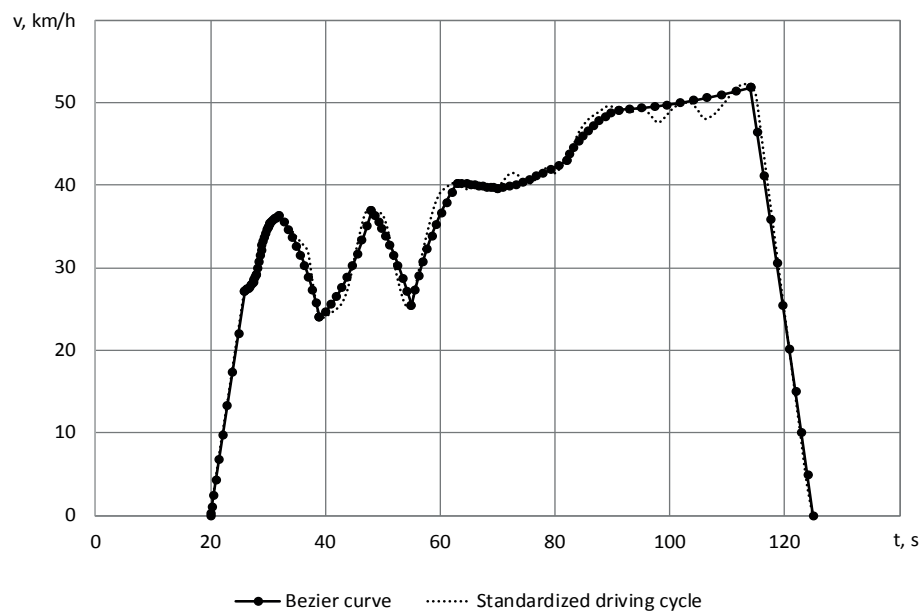
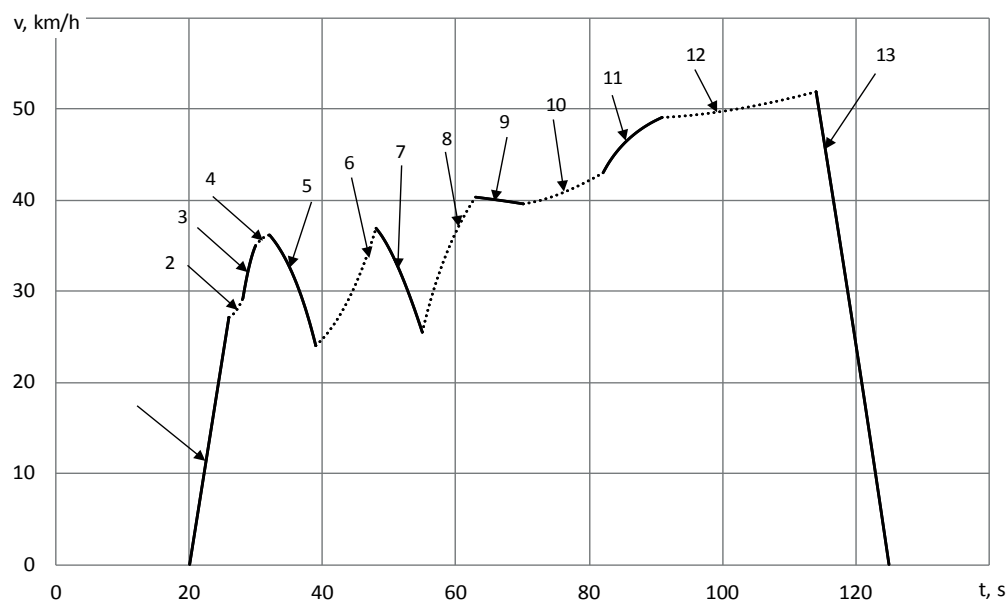**Figure 6** *Modeling of the standardized driving cycle by Bezier curves*

Figure 7 *Elementary parts of the modeled driving cycle*

method, a section of the driving cycle FTP was built, (Figures 5, 6).

While building a section of a driving cycle, for the initial point is taken a point of the vehicle movement beginning, while the final point, which at the same time is the initial point of the next section, has positions that correspond to the extreme value of speed (minimum and maximum).

During the modeling of a driving cycle, an FTP part was divided into 13 segments; each of them is in a model presented by the quadratic Bezier curve, shown in Figure 7. If in the quadratic Bezier curve the points P_1 and P_2 have the same positions, the Bezier curve of the second degree turns into a line. Some non-conformities of the modeled driving cycle with a standardized one

can be eliminated by dividing these sections into smaller parts.

In Table 6 is given an extension for each part and the maximum speed deviation on a given part.

As a result of the research, the maximum deviation of speed between the standardized driving cycle and its model was found to be about 10 % (elementary parts 5 and 12). Along with that, taking into consideration that a sign of speed's difference between the given driving cycles can be either positive or negative, an average deviation by a part of a cycle is about 3 %. To decrease the speed deviation o from the standardized driving cycle, a part 12 was additionally divided in three elementary parts (Figure 8). As a result of this, the maximum deviation for the elementary part 12 was
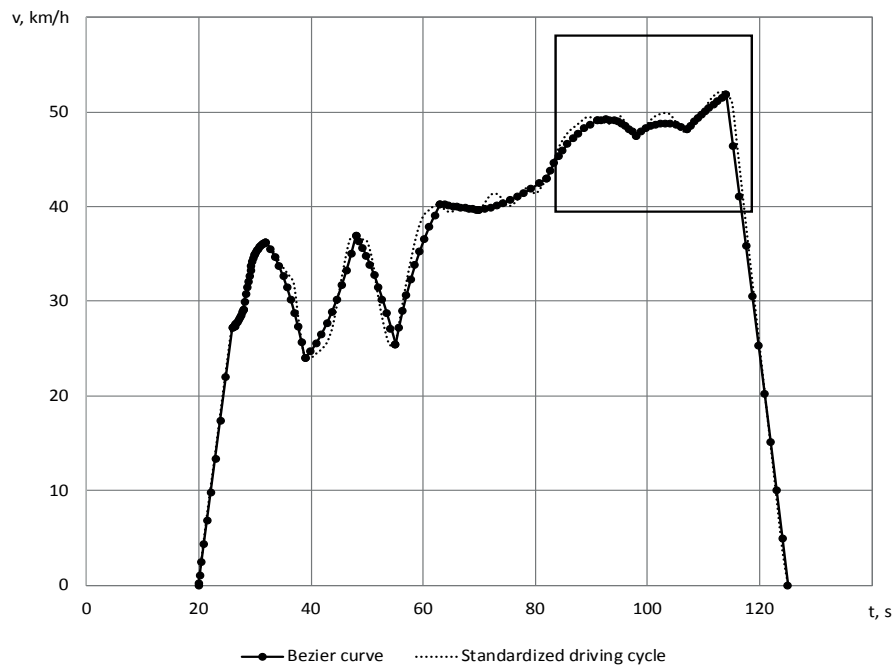
Figure 8 *Division of a part of the 12th driving cycle on the segments to get higher precision*

Table 6 *Extension for each part and the maximum speed deviation on an elementary part of the 12th driving cycle*

number of an elementary part of a cycle	extension of an elementary part of a cycle (s)	maximum deviation by speed (%)
1	6	0
2	2	0.011
3	2	1.926
4	2	0.670
5	7	9.629
6	11	7.297
7	7	4.475
8	8	6.012
9	7	1.464
10	12	3.196
11	9	2.603
12	23	9.101
13	11	0

2.2 %. In such a way, it was found that for increasing the length of an elementary part, the corresponding increasing of deviation is observed. A value of deviation would primarily depend on the length of an elementary part and a character of a curve on this part. As it was shown before, the polynomial curves of the second and third degrees gave the most accurate results.

An accuracy level, with the help of which a modeling was being made, should correspond to the accuracy level of the experimental research, i.e. the deviations of a modeled driving cycle should correspond to deviations that appear during the experimental research of vehicles using the given driving cycle.

5 Conclusions

The research objective was searching and investigation of possibilities for the vehicles driving cycles' modeling. To achieve this objective it was proposed to use the Bezier curves. The performed theoretic research has shown a possibility to realize the tasks of driving cycles' modeling using the mentioned curves. In addition to this, the optimal is to use the Bezier curves of the second degree and together with them - the linear Bezier curves.

A character of a proposed method of the driving cycle's modeling is a possibility to choose the desirable accuracy of the model, while using different lengths. As a result of

this research, the maximum and minimum differences of speed between the real driving cycle and a proposed model were determined, that amount to 10% and 3%, respectively. It was also set that decreasing of elementary parts increases accuracy of the model. Dividing one part into three smaller parts decreased the difference between cycles from 10% to 2.2 % of elementary driving cycle sections.

It is certain that this process of modeling is rather difficult and the higher is the accuracy, the more difficult would be to model, however, the created model would be rather dynamic and can be easily changed in a case of change of some external factors. The performed thematic research confirms the previous results, aimed at a creation of a universal dynamic model of vehicles' driving cycle.

References

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