Propeller shafts of the vehicle’s drive transmit a torque to relatively large distances. The shafts are basically long and slender and must be dimensioned not only in terms of torsional stress, but it is also necessary to monitor their resistance to lateral vibration.

In the paper, a simple model (of the solved problem) is constructed by the method of physical discretization, which is evident from the nature of the centrifugal force fields’ influence on the spectral properties of the shaft. An analytical solving of speed resonances prop shafts test model (whose aim is to obtain values for verification subsequently processed models based on the transfer-matrix method and the finite element method) is performed.

Keywords: Hook’s joint, shaft, vibration, mathematical and physical model, transfer matrix, Finite element method

1. Introduction

Problem of modelling and description of propeller shafts is quite old [1]. Propeller shafts of drive vehicles are evolutionary systems. Evolutionary system means the parameters of the shaft are changing in time. The shafts are long and slender. For this reason, they are affected by torsional stress and also by lateral and transversal vibrations [2]. Due to the continuous operations, the shafts have to operate in subcritical speed. Results of previous works, which were also compared to experiments, showed that the propeller shafts represent strong evolutionary systems (increasing the angular velocity of rotation significantly reduces the spectrum of natural frequency relative lateral vibrations) and in practical calculations it is necessary to respect this influence. For that reason, it is not possible to model the shafts using procedures that are commonly reported in the literature, but it is necessary to formulate a model that allows for this effect to be respected. Due to results of previous works and experiments, it is not possible to model the shafts using procedures that are commonly reported in the literature [3], [4], but it is necessary to formulate a model that allows respecting that with increasing the angular velocity of rotation the spectrum of natural frequency relative lateral vibrations is significantly reduced.

Propeller shafts are in a steady state stressed by excitation bending moment’s harmonics and their vectors are perpendicular to the rotating plane of a relevant fork Hooks joints. The drive torque mentioned in a steady state is generated due to the transmission flow through Hooks joints and causes lateral oscillations of the propeller shaft in its rotating space. In formulating a mathematical model, it is necessary to start from the assumption of existence of the relative spatial bending vibration in the shaft system $(x,y,z)$, which rotates at an angular speed $\omega$. If one neglects the Coriolis force and gyroscopic moments acting on the element of the shaft, one can solve the problem in the rotating plane $0(x,y)$. The instantaneous state of the element is determined by the velocity and the angular velocities. This article aims to build a mathematical model of a coupling shaft to calculate spectral and modal properties of the connecting shaft with respect to the field of centrifugal forces that is causing the addition of natural frequencies of bending vibrations relative to the angular velocity of the shafts rotation.

2. Formulation of the problem

Propeller shafts are in a steady state stressed by excitation bending moments harmonic, and their vectors are perpendicular to the rotating plane of a relevant fork Hook’s joints (Figure 1). A model was built on an assumption of existence of the relative spatial bending vibration in the shaft system $(x,y,z)$, which rotates at an angular speed $\omega$. The dimensionality of the problem can be reduced from 3D to 2D by neglecting the Coriolis force and gyroscopic moments acting on the element of the shaft. Then, one can solve the problem in the rotating plane $(x,y)$. The instantaneous state of the element is determined by the angular velocity $\omega_0$, the velocity $\dot{v}$, and the angular velocity $\omega_0$. A mathematical model of a coupling shaft was built in order to calculate the spectral and modal properties of the connecting shaft, including the natural frequency of bending oscillations.

3. Physical discretization

The drive shaft, shown in Figure 1 (consider solid bearings), is replaced by a discrete mechanical system with only one degree of freedom. This system is divided into two equal halves, which represent an intangible spring (Figure 3) having the rigidity $k$. 

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where \( m_k \) is the natural frequency of relative undamped oscillations. By modification of this equation one obtains:

\[ m_k^2 \Omega^2 + \omega^2 = \frac{k}{m} \]  

which is equation of the circle with origin at \( O(\Omega, \Omega) \) and radius \( \frac{k}{m} \).

4. The test model

The test model parameters are chosen of the prototype car – Skoda 781. This choice comes from the cooperation with the industry. Parameter of this test model are:

\[ r = 0.0105 \text{ m}, \quad l = 0.65 \text{ m}, \quad E = 2.1 \cdot 10^5 \text{ Pa} \quad \text{and} \quad \rho = 7.8 \cdot 10^3 \text{ kg/m}^3. \]

Using these parameters the following was obtained:

\[ J = 9 \cdot 10^{-4} \text{ m}^4, \quad S = 3.46 \cdot 10^{-4} \text{ m}^2, \quad k = 3.3 \cdot 10^5 \text{ Nm}^{-1}, \quad m = 0.88 \text{ kg} \quad \text{and} \quad \Omega(0) = 591.9483 \text{ rad/s} \]

Figure 1 Model of propeller shafts

Figure 2 Coordinate system of the shaft

Figure 3 Replacing of the drive shaft by divided system of one degree of freedom

Figure 4 Modelling of the shaft by the spring

\[
E_i = \frac{1}{2} m \dot{y}^2 + \frac{1}{2} m (\dot{\phi} \dot{\phi})^2 \quad \quad E_f = \frac{1}{2} k \dot{y}^2
\]

In addition, equations of motion are written by formulas:

\[
m \ddot{y} + (k - m \omega^2) y = 0, \quad M - 2my\omega = 0
\]

Equation of relative oscillating movement in rotating plane can be rewritten in the form:

\[
\ddot{y} + \Omega^2 y = 0
\]
6. The Transfer-matrix method

For calculation using the transfer-matrix method, the model of the shaft is treated as one dimensional continuum \[11\] of constant circular cross section. It is defined by:

- **Geometrical parameters**
  - \( l [m] \) - the length of the one dimensional continuum
  - \( r [m] \) - radius of cross section of the shaft.

- **Material constants**
  - \( \rho [mkg^{-3}] \) - the material density
  - \( E [Pa] \) - the modulus of elasticity in tension or compression.

- **Operating parameter**
  - \( \omega [rads^{-1}] \) - the angular velocity of rotation of the plane \( O(x,y) \) around the axis \( x \).

The solution is sought in the form of

\[ Y(x,t) = \sum_{i=1}^{n} q_i(t) \phi_i(x) \]

where \( \phi_i(x) \) are the 3rd order polynomials. For more details see \[6\]. One can take this model and join it multiple times together to create graduated shaft (Figure 9). The graduated shaft means that the shaft is divided into several parts, which are mathematically described separately. These descriptions of part are linked in the model through the boundary conditions of the parts.

Using the finite element method \[7\], the whole shaft is then described by a matrix and each part as a sub-matrix. For more details see \[6\], \[8\], \[9\], \[10\].
The mathematical model, based on the physical discretization was defined and used for solving the problem of finding the critical speed of rotations. This model was programmed as a script in the GNU Octave. In addition, there is a possibility of using the analytical solution derived from Equation (6). Both of these solutions give the circular dependency of $\Omega$ and $\omega$ (Equation (5) and Equation (7)). The use of Finite element method for modelling the propeller shaft and for calculation of critical speed was attempted. However, this needs a little bit more of programming and calculation. The use Transfer-Matrix method was tried, as well. Using this method should enable modeling of a shaft composed of different parts with different properties. The initial goal is to prepare the scripts for testing. After that, the program should be written, which will be more user friendly than the scripts.

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**References**


