1. Introduction

The medical emergency system design is a crucial task for each responsible designer due to the interaction of two opposite demands on the system performance. On the one hand, the designer is forced not to exceed a given number of located facilities – ambulance vehicles and to solve a large facility location problem. On the other hand, he must ensure the accessibility of the service for potential patients. This accessibility is usually given by a fixed time limit, in which some ambulance vehicle should reach an arbitrary located potential patient [2], [12], [13].

This last demand is hard to meet, because of random travel time on a real road network. In addition, when an accident occurs, an ambulance starts its trip to the accident location to provide the service, which consists of first aid to causalities and their transportation to a hospital. Within this service, the facility (ambulance vehicle) cannot perform any service of other demands. It means that if some other accident occurs simultaneously in the area of this vehicle, then some other ambulance must serve it, or service of the later accident must be considerably postponed. This way, the service system works like a queuing system [11]. Under these circumstances, the access condition cannot be fulfilled absolutely, but only with some probability. Due to the impossibility to include means of the queuing theory into analytical models of the location problem, there are used various surrogate criteria such as an average or total travel time from the ambulance location to potential patients, which belong to the ambulance servicing area [6], [7], [8]. Another type of criterion (covering criterion) is that a maximal travel time from the nearest ambulance location to a customer should not exceed a given value. Designers face the above-mentioned ambulance occupation by using so called double coverage criterion, which is formulated so that the number of potential patients, which lie within a given time radius of two or more facilities should be maximal [2].

Usage of each of these criteria leads to a particular model of mathematical programming [5]. An exact method applied to the particular model has its specific demands for computational time and memory. In the next sections, we present an overview of these criteria, report about preliminary computational experiments and perform a comparison of them.

2. The emergency service system design problem and quality criteria formalization

Within the scope of this paper we confine ourselves to the problem, in which a medical emergency service system is designed. In contrast to the private service systems, the objective of this sort of public service system should stress equity of a “customer” in access to the provided service.

The emergency service system design belongs to the family of location problems [1], in which it must be decided on centre locations, where ambulance vehicles should be placed, because an effective satisfaction of the potential patient demands is possible only if the corresponding service provider concentrates its sources at several places of the served area and provides the service from these places only. The served area consists of dwelling places placed in nodes of a road network. These dwelling places form a finite set \( J \). The number of inhabitants of dwelling place \( j \in J \) will be denoted as \( b_j \). The emergency service system design problem can be formulated as a decision about location of at most \( p \) emergency centers at some places from a larger set \( I \) of possible center locations so that the value of chosen criterion is minimal. The question, which must be answered first, is: “How to estimate the time of access to a customer?” Let \( j \) be customer’s location and \( i \) be a centre of the service provider. Both the locations are nodes of a road network, which consists of links and nodes. Based on the link quality, each link belongs to a class from a finite classification...
system. In accordance to this system, an average speed is assigned to each link. This way, an estimation of the necessary traversing time for each link can be obtained from the link length and the average speed corresponding to the link class. Using this time instead of the link length, the accessibility time \( t_j \) can be enumerated as the time length of the shortest path in the network connecting \( i \) and \( j \). Time \( t_j(v) \) is a function of vector \( v = (v_1, v_2, ... , v_p) \) of the speeds, which corresponds to the particular link classes. Nevertheless, the average speeds are not constant, but they depend on weather, traffic volume and other dynamically changing conditions. Considering this condition variability, no system design ensures full satisfaction of the estimated time and each further developed criterion is enumerated in accordance to a given speed scenario. Let \( \phi(v, j) \) represent the located centre, which is the time-nearest one to \( j \) considering the link speeds given by \( v \). Further, let \( l_j \subseteq I \) denotes the set of places, in which an ambulance vehicle is located.

After these preliminaries, we formulate the particular criteria. The first family of “allocation criteria” is represented by the total travel time from the ambulance location to potential patients. This criterion can be described by the following expression:

\[
\sum_{i,j} b_i t_{i0,i}(v) \tag{1}
\]

This criterion doesn’t reflect equity of a “customer” in access to the provided services at all. The original requirement of the concerned public is that each inhabited place must be reachable to the provided services at all. The original requirement of the criterion can be described by the following expression:

\[
\text{Minimize} \sum_{i,j} b_i (t_{i0,i}(v) - T^{\max}) \tag{2}
\]

The second family of “covering criteria” [2] is represented by the criterion, which simply counts the potential patients, which are out of the time limit:

\[
\sum_{i,j} b_j (t_{i0,i}(v) > T^{\max}) \tag{3}
\]

The third family of “double coverage criteria” [2] is represented by the criterion, which counts the potential patients not covered at least from two ambulance locations. It is said that a patient is covered if the distance to the nearest ambulance station is less than a given limit \( T^{\max} \). Let \( \phi(v, j) \) represent the second time-nearest station to \( j \) considering the link speeds given by \( v \); \( \phi(v, j) \) belongs to the set \( l_j \subseteq I \) of places, in which an ambulance vehicle is located. The formulation of the last criterion can be as follows:

\[
\sum_{i,j} b_j (t_{i0,i}(v) > T^{\max}) \tag{4}
\]

The expressions (1)-(4) are to be minimized subject to the constraint that the number of located facilities must not exceed the given number \( p \).

The next generalization of these criteria may issue from observation of possible scenarios of the vehicle speeds. The family of the scenarios constitutes finite set \( V \) of possible speed vectors \( v_q \), \( q = 1, ... , m \) and each scenario may be weighted by coefficient \( h_q \). The weights can be set proportionally to the empirical frequencies or arbitrary else to reflect the necessity to keep the accessibility condition at a sensible level. The further generalization can be obtained by optimising a linear combination of criteria, where particular criteria are weighted according to their importance.

3. Models and solving techniques for the emergency service system design problem

The mathematical programming approach to the emergency system design comes out from the assumption that the ambulance vehicles are allowed to be located only at some places from the finite set \( I \) of possible locations. The decision on placing or not placing an ambulance vehicle must be done for each candidate location \( i \in I \). This decision can be modelled by the variable \( y_i \), which takes the value 1 if a vehicle is placed at location \( i \) and it takes the value 0 otherwise. The case, in which it is possible to place more than one vehicle at one location, can be rearranged to the considered zero-one decision problem by duplication or triplification of the relevant locations.

The emergency system design problem with the criterion (1) cannot be described only by the location variables \( y_i \), due to the fact that the individual contribution to the objective function value depends on the distance between the customer and the nearest located ambulance. To be able to describe this sort of relations, we introduce zero-one variables \( z_{ij} \) for each pair \( (i, j) \) of a possible location and a customer. Using these variables, the assignment of each customer to some ambulance location can be easily described. If we denote \( c_{ij} = b_j t_{i0,i}(v) \), then the following model describes the emergency system design problem with the criterion (1).

\[
\text{Minimize} \sum_{i,j} c_{ij} z_{ij} \tag{5}
\]

Subject to \( \sum_{i,j} z_{ij} = 1 \) for \( j \in J \) \( \tag{6} \)

\( z_{ij} \leq y_i \) for \( i \in I, j \in J \) \( \tag{7} \)

\( \sum_{j} y_i \leq p \) \( \tag{8} \)

\( y_i \in [0,1] \) for \( i \in I \) \( \tag{9} \)

\( z_{ij} \in [0,1] \) for \( i \in I, j \in J \) \( \tag{10} \)

The expression (5) corresponds to the sum of the real access times multiplied by numbers of afflicted inhabitants. The constraints (6) ensure that each dwelling place (customer) is assigned to the
exactly one of possible locations. The constraints (7) are so called
binding constraints, which force the variable \( y_i \) take the value 1,
whenever a customer is assigned to location \( i \). The constraint (8)
puts the limit \( p \) on the number of located vehicles.

The model (5)–(10) describes also the emergency system
design problem with the criterion (2). It is sufficient to denote
\( c_q = b_s(t_q(v) - T^{\text{max}}), \) if \( t_q(v) > T^{\text{max}} \) and \( c_q = 0 \) otherwise.

The problems connected with criterion (3) can be modelled
using a set of the auxiliary zero-one variables \( x_j \), which express by
the values 1 or 0, whether the demand of customer \( j \) is or is not
satisfied. To be able to recognize, whether customer \( j \) is or is not
accessible from location \( i \), we introduce zero-one constant \( a_{ij} \)
for each pair \((i,j) \in I \times J\). The constant \( a_{ij} \) equals 1
if and only if customer \( j \) can be reached from location \( i \) in
the access time \( T^{\text{max}} \), i.e. \( t_q(v) \leq T^{\text{max}} \).
Otherwise, the constant \( a_{ij} \) equals 0. Then we can formulate the problem as:

Minimize \[ \sum_{j \in J} b_j \left( 1 - x_j \right) \] (11)

Subject to \[ \sum_{i \in I} a_{ij} y_i \geq x_j \] \text{ for } j \in J \] (12)

\[ \sum_{i \in I} y_i \leq p \] (13)

\[ y_i \in \{0,1\} \] \text{ for } i \in I \] (14)

\[ x_j \in \{0,1\} \] \text{ for } j \in J \] (15)

The objective function (11) gives the volume of uncovered
demands. The constraints (12) ensure that the variables \( x_j \)
are allowed to take the value 1, if and only if there is at least one
ambulance vehicle located in the access time \( T^{\text{max}} \) from
the customer location \( j \). The constraint (13) puts the limit \( p \) on the number
of located vehicles [10].

The model (11)–(15) can also model the problem with crite-
riom (4), in which the number of double covered demands should
be maximized. Nevertheless the constraints (12) must be replaced
by constraints (16):

\[ \sum_{i \in I} a_{ij} y_i \geq 1 + x_j \] \text{ for } j \in J \] (16)

Concerning the solving technique for the problems described
by the above presented models, it can be noted that all of them
belong to the family of integer programming problems, more pre-
cisely zero-one integer programming problems and can be theo-
retically solved by any commercial solver, which contains some
general integer programming algorithm, e.g. the branch and bound
method, the cutting plane method or the branch and cut method.
These general algorithms are able to solve to optimality real-sized
covering problems, but only small instants of the allocation prob-
lems. To solve the problems with the criteria (1) or (2), we can
make use a similarity between the problem (5)–(10) and the unca-
pacitated facility location problem [3]. The problem can be solved
by the approach reported in [4] or [9], where a Lagrangean mul-
tiplier is introduced for the constraint (8) to relax it from the set
of constraints. Then the problem takes a form of the uncapacitated
facility location problem. To solve it, the procedure \( \text{BBDual} \) [9]
was designed and implemented based on the principle presented in
[3], which is the branch and bound method with special methods
for obtaining of the lower bound. The procedure was embedded
into the dichotomy algorithm, which was used to find a fitting
value of the Lagrangean multiplier.

4. Preliminary numerical experiments
and criteria comparison

We performed the numerical experiments with the data origi-
nating at the Slovak road network with 2916 dwelling places, which
represent aggregations of potential patients. In this study, the elec-
tronic road map of Slovak Republic was employed. The numbers
of inhabitants of dwelling places were known together with other
attributes of the nodes. The current proposal of the emergency
medical vehicle location consists of 264 places, but 41 of them
duplicate or triplicate locations at some bigger cities and they have
no influence on the studied accessibility in accordance to criteria
(1), (2) and (3) considering the fact that these towns are repre-
sented by one node each. Based on this reduction, the 223 points
(locations) were taken into consideration as the value \( p \) in the
primary problem. The sum of unallocated ambulances from the
primary problem solution and 41 multiple locations enter as
value \( p \) the secondary problem. These data enable to calculate the
suggested criteria for the given scenarios of the vehicle speeds con-
ected with the individual link classes. The considered speed sce-
enario was \( v = (105, 95, 75, 60, 50) \), which are assumed average
speeds in kilometer per hour on highways, roads of first, second
and third class and on the local roads respectively. The set of can-
didate locations was formed from all towns and villages with more
than 300 inhabitants and present ambulance locations. This way,
a set of 2284 candidate locations was obtained.

We solved all the above formulated problems for \( T^{\text{max}} = 15 \)
minutes, whenever this limit was included into the model. In
accordance to the type of criterion we employed the special algo-
rithm \( \text{BBDual} \) or the general optimisation software \( \text{Xpress-MP} \), if
possible with respect to the size and structure of the associated
model. The associated algorithms were run on a personal computer
equipped with the Intel Core 2 6700 processor with parameters:
2.66 GHz and 3 GB RAM.

The results of numerical experiments are reported in Table 1
where each row corresponds to one instance of the problem, which
is specified by the used criterion and problem type (p-primary or
s-secondary). The row contains the objective function value (Objec-
tive) of the optimal solution, the computation time in seconds
(Time [s]) and the number of located ambulances (Loc). These
figures are placed in the section \( \text{BBDual} \) or \( \text{Xpress-MP} \) in ac-
cordance to the used solution technique.

It turned out that the problem with criterion (2) was insolvable
due to either huge time consumption or model size by both the
approaches. That is why the optimal solution of only problems with criteria (1), (3) and (4) are reported in Table 1.

### Table 1

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Type</th>
<th>BBDual</th>
<th>Xpress-MP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Objective</td>
<td>Time [s]</td>
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<td>13771837</td>
<td>3436</td>
</tr>
<tr>
<td>(1)</td>
<td>s</td>
<td>44203257</td>
<td>3574</td>
</tr>
<tr>
<td>(3)</td>
<td>p</td>
<td>91</td>
<td>0.1</td>
</tr>
<tr>
<td>(3)</td>
<td>s</td>
<td>286180</td>
<td>1.4</td>
</tr>
<tr>
<td>(4)</td>
<td>p</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The comprehensive solutions were obtained from the primary and secondary solutions by simple addition of the zero-one resulting vectors \( y^p \) and \( y^s \). So in the comprehensive solution, 264 ambulances are deployed. The subscript \( i \) of the nonzero allocation variable \( z_{ij} \) for criterion (1) was obtained for each \( j \) so that the equation (17) holds.

\[
t_i(\cdot) = \min \left\{ t_k(\cdot) : k \in I, 1 \leq y_i^k + y_i^t \right\}
\]  

(17)

The value of variable \( x_j \) for criteria (3) and (4) was obtained for each \( j \) in accordance to the equation (18) or (19) respectively.

\[
x_j = \min \left\{ 1, \sum_{i \in I} a_i (y_i^p + y_i^t) \right\}
\]  

(18)

\[
x_j = \max \left\{ 0, \min \left\{ 1, \sum_{i \in I} a_i (y_i^p + y_i^t) - 1 \right\} \right\}
\]  

(19)

This way, comprehensive solutions BBDual(1), Xpress-MP(3) and Xpress-MP(4) were obtained. Then the values of criteria (1)–(4) were computed for the solutions and these results are reported in Table 2.

The row Man-made in Table 2 corresponds to the current distribution of ambulance vehicles over the area of the Slovak Republic.

### Table 2

<table>
<thead>
<tr>
<th>Criterion Type</th>
<th>Criter. (1)</th>
<th>Criter. (2)</th>
<th>Criter. (3)</th>
<th>Criter. (4)</th>
</tr>
</thead>
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<td>Xpress-MP(3)</td>
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<td>188937</td>
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<td>Man-made</td>
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<td>92032</td>
<td>31672</td>
<td>431415</td>
</tr>
</tbody>
</table>

6. Conclusions

We presented four models of the medical emergency system design problem which are based on a different quality criterion. These quality criteria reflect possible approaches to the original problem with a general objective formulated as: to provide the best service to all inhabitants of a considered region. As any sophisticated designing process of real service system needs methods, which are able to provide it with a concrete solution in a sensible time, we tried to assign to these particular problem formulations some solving algorithms and performed preliminary computational experiments to verify suitability of the algorithms. With exception of the second criterion, we found that instances of the particular problem types were solvable in reasonable time. Furthermore, we compared the obtained results with the current structure of the medical emergency system of the Slovak Republic. We proved that the studied approaches could considerably improve the current system in all the considered objectives.

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References