1. Introduction

A prerequisite for effective functioning of the intermodal system in Europe is a system of shuttle or multiple-section trains regularly operating among intermodal terminals or ports oriented on technologies such as Night Jump (Nachtdprung in German) or Just in Time (with guaranteed term of delivery), as described by Lindner [19] and Panda [22].

An important part of an intermodal train’s travelling time is the reloading time in node terminals. That is why routes of those trains must be coordinated on a track and in a terminal. That is also a dominant theme of this paper. Emphasis on a systematic approach is the key to the solution. This solution will use findings which are described in Caprara [3], Nielsen [20], Sevele [23] or Schmitt [24]. Results of our paper will contribute to better time and space sequence of intermodal trains in node terminals. Our approach will increase the quality of railroad cargo transport.

2. Systematic time tables

Systematic timetabling is one of the possibilities for a timetable construction. Its principle is in a periodically repeated structure of passenger and goods trains so that during the day the same groups of trains are repeated. According to distance among those groups we have a lag timetable or pulse timetable.

In railroad transport we have a household word – lag timetable. Its characteristic is a constant time period among trains. Therefore, we can find one-hour, two-hour, three-hour or eventually four-hour pulses, as described by Bar [1] and Ferchland [7].

On a higher quality level there exists an integrated lag timetable (further only ILT). It systematically coordinates lag timetables among more train paths or other means of transport. More basic information about ILT can be found in Diermeier [6].

Bar [1] and Hesse [12] suggest the most important requirements for ILT:

- Offer of lag transport during every day of the week,
- Offering each train path in a basic pulse (passenger trains one or two hours, intermodal trains – 4, 6, 12 or 24 hours),
- Ensuring train services for passengers from 5am to 10pm, combined with intermodal transport in continuous operation,
- Ensure good connection in node stations among systematic trains and other types of trains,
- Increase standard of quality for passengers and customers in cargo transport,
- Ensure maximum accuracy of trains (following the time table).

Another important aspect of lag timetabling is axis of symmetry. It is an instant of time when trains on the same train route are crossing (on the single track line) or meeting (on double track line). Hesse [12] found that an important requirement for ILT is the same symmetry time for all train routes. If we observe that principle it leads to a mirrored timetable. So that for each train in direction A-B we have a train in direction B-A with the same lag time, similar reloading time and block speed. In standard practice the so-called zero symmetry is often used. It means that one symmetry time is right at the beginning of an hour. For example, if one train arrives to the station in the 54th minute (60 – 6), then the train on the same route in the opposite direction will arrive in 6th minute (60 + 6).
3. Intermodal trains

Intermodal trains are taken as a system of fast train connections of intermodal transport (hereinafter as IMT) which ensures connection among individual IMT terminals in desired quality and sufficient volume of transported goods. Systems of through trains and mixed trains are used for the train connection among IMT terminals. From the railway carrier’s point of view the whole cargo (transport unit) is transported either as an individual set of coaches or as a group of sets of coaches or as the shuttle train. The biggest share of IMT cargos is transported in the shuttle trains which all belong to the “Nex” train category.

Heinrici [11] found that the systems of train connections of IMT transport units can be operated on an individual line or on a specified line network. In the former case we speak of so-called “line system” and in the latter case we speak of the so-called “network system”.

The network system of train connections is characterized as a network service of individual terminals on a specified traffic network. This means that there is a gap in the service of individual IMT terminals using line systems which are included in the traffic network. A system of shuttle trains connecting the individual IMT terminals is usually used in the network system. This network system is in the IMT area represented by the “Hub-and-Spoke” system, as described by Novak [21].

![Fig. 1 Logistic technology „Hub-and-Spoke“](image)

The “Hub-and-Spoke” technology can be combined with the line principle using the technology of multiple-section trains. The schematic representation of this technology is in Fig. 1. Novak [21] and Vrenken [26] found that the network structure based on this technology is used mainly abroad. For example, in Germany (IMT operator Kombiverkehr) this system is represented as the “goods IC-concept”. It is a very similar system to the one used by passenger trains IC, ICE and EC categories, as described by Heinrici [11].

A practical application of this technology has been found not only abroad but also in the Czech Republic. Especially in transportation among seaports (Rotterdam, Bremerhaven, Hamburg) and some inland terminals (Prague-Uhrineves, Melnik, Lovosice) that have the position of node terminals. In these lines the freight traffic volumes are high and the trains are usually set out twice per day (24 hours). In other sequential directions (other terminals in the CR or adjacent countries) the freight traffic volumes are much lower and the train set-out frequency is much lower (2 to 5 trains per week).

4. Train routes model for intermodal trains

The task of any transport problem solved by a mathematical model is finding its attributes, analysing all its states and verifying its scheduled parameters. This is subjected to right definition and dimensioning of system items, efficient regulation and organization of the system.

4.1 General solution

In the node terminal two groups of shipments are assembled for one train. Cerna [4] and Janacek [13] suggest that in the first group are shipments which enter the system in that node. The second group are shipments which came into that node by train. If there is a node \( u \) in time \( t_{ij} \) (arrival time of train \( j \) on line \( i \) ), the shipment will be brought by train \( s_{ij} \) on train path \( L_{ij} \) and will be reloaded for time \( t_{d_{ij}} \) (time of reload from line \( i \) to lane \( r \)) to position on train path \( L_{ir} \) and will wait for the nearest train \( s_{rg} \) of that train path with the depart at time \( t_{rg} \) (departure time of train \( g \) on line \( r \)):

\[
\min \{1\}
\]

Providing that the train \( s_{ij} \) departs from a starting station at time \( t_{ij} \) and that the travelling time to node \( u \) is \( d_{ij} \) then the arrival time to \( t_{ij} \) can be formulated by 2 and 3:

\[
\min \{2\}
\]

and for the train \( s_{rg} \):

\[
\min \{3\}
\]

Blackout time during waiting for reload can be formulated by 4:

\[
\min \{4\}
\]
If we are unable to influence the last three timing logic elements that means that the travelling times of each train are $d^p_{ij}$ and $d^r_{ij}$ and reloading time is $t^r_{ij}$. Then we can lower the blackout time by increasing the value of $r^u_{ij}$ to:

$$r^u_{ij} = t^u_{ij} + x^u_i \quad [\text{min}]$$

(5)

or by lowering $r^u_{ij}$ value to:

$$r^u_{ij} = t^u_{ij} + x^u_i \quad (x^u_i < 0) \quad [\text{min}]$$

(6)

Now we can extend the formula (1) to:

$$t^u_{ij} + x^u_i + d^u_{ij} - t^u_{ij} - x^u_i - d^u_{ij} - l^u_i \geq 0$$

(7)

Besides the boundary conditions like (7) there is also another one which ensures time locations of trains $s_{ij}$. It may vary in fixed bounds $a_i \leq t^u_{ij} + x^u_i \leq b_i$:

$$a_i \leq t^u_{ij} + x^u_i \leq b_i$$

(8)

Similarly for trains $s_{ij}$:

$$a_i \leq t^u_{ij} + x^u_i \leq b_i$$

(9)

Routes $L_i$ and $L_j$ pass through the node $u$ and two lines of those routes $x^u_i$ and $x^u_j$, which fulfill conditions (8) and (9). So we can make a directed graph $G = (S, H)$, where $(v_i, v_j) \in H$ if node $u$ exists we are able to coordinate those lines (S - set of routes, H - coordinated graph of transport network). The edges determine $k \in H$ which pairs of nodes can be coordinated but not all of the pairs from the set $H$ can be coordinated simultaneously by a different value of $x^u_j$. Therefore, a coordinated subgraph $G^r = (S, H^r)$ must be found where for all $i = 1, \ldots, n, j = 1, \ldots, m_i$ exist values $x^u_i$ fulfilling the condition (7).

For the coordinated subgraph $G^r$ there exist usually more possibilities for the $x^u_i$ value, which comply with each pair of nodes $(v_i, v_j) \in H^r$ conditions (7) and (8). The selection of the final value will depend on the consignment volume $f_{ij}$, which will be reloaded in a node from $L_i$ to $L_j$. Here we establish a term optimal solution for coordinated pairs of trains. It is determined by the subgraph $G^r$:

$$X = \{x^u_i\}^i_n \cup \{x^u_j\}^j_m$$

(10)

for $i, j$ which comply with condition (8), for all $(v_i, v_j) \in H^r$ and condition (7) and which minimize the value of the objective function $\hat{u}$:

$$\hat{u} = \sum_{(v_i, v_j) \in H^r} f_{ij}(t^u_{ij} + x^u_i + d^u_{ij} - t^u_{ij} - x^u_i - d^u_{ij} - l^u_i)$$

(11)

If we expect that the values $t^u_{ij}, d^u_{ij}, t^r_{ij}$ are constant according to the optimization task the value of objective function $\hat{u}$ is minimized after fulfilling above mentioned conditions:

$$\hat{u} = \sum_{(x_i, x_j) \in H} f_{ij}(x_i - x_j)$$

(12)

The resultant form of objective function depends on subtraction of values $x^u_i$ and $x^u_j$ which are additional times to arrival of a train on a train route $t^u_i$ or departure of a train on a train route $t^u_j$. So if we want to set up the objective function for the train route model we have to come out from the formula (12) as described by Caprara [3] and Cerna [4].

4.2 Formulation of a mathematical model

Railroad timetable set up is an intensive and long process. The reason is in a limited use of a computer application. It is mainly an interactive process between human experiences and automated generation of timetable tools. Kryze [18] and Nielsen [20] suggest that the whole travelling time for a consignment in intermodal transport contains these components:

- Travelling times between terminals.
- Time spent at stops on the train route.
- Waiting time (result of insufficient capacity of operation equipment).
- Synchronization time (time needed to reload consignments between two trains).

Shortening of first three times (travelling time, time spend and waiting time) is subjected to investments (of infrastructure). On the contrary, synchronization time is determined by timetable so that we are able to shorten the synchronization time only by change of organizational set-up (non-investment character). Therefore it would be better to work on optimization leading to minimization of synchronization time, as described by Dirmeier [6].

In railroad cargo transport there exist two types of synchronization:

1. Waiting of train due to unloading of goods.
   During this first type of synchronization there is an extended travelling time for goods due to unloading, and therefore there is no advantage from synchronized times.

2. Waiting of goods for connecting train
   In the second form is extended travelling time of goods which needs synchronization.

Reloading process (formation of synchronization time principle) of transport units between two trains is in Fig. 2. From that figure it follows that the waiting time has several significant spells. Those spells are due to shipping and traffic reasons. The most important part is synchronization time which depends on a minimal reloading time. Waiting time is a result of insufficient capacity of railroad infrastructure or insufficient locomotives. This time may not apply to each train. The size of that time is variable for different groups of trains. On the other hand, stopping time and time for dispatch of a train must be kept to a minimum for each train. In Fig. 2, we can see a striking difference between the waiting time of "not-reloaded" consignments in the first or second train and the waiting of reloaded consignments between those trains. That can be applied also for more than two trains in a station.
Speaking of transformation into a mathematic model the authors found two different approaches: Krista [16, 17] formulated this problem as a linear programming (for small transport networks) and solves it by simplex algorithm, Kolonko [15] solved it via genetic algorithms (for large transport networks). The linear programming calculation is very slow - it takes several hours. In case of additional conditions, the time of calculation increases. That’s why author used the genetic algorithm, which is useable for large transport networks.

This method is in a couple of aspects different from the “classic” optimization methods. Goldberg [9] found that it uses a random sample and that is why that method is non-deterministic. This means it can give us a different result for each optimization. During the optimization this method holds “population of candidates for best bet”. Only one member of that population is the best bet; the others are sample points where the best bet can be found later. That helps avoiding local optimums. Inspired by natural evolution this method makes periodically random mutations of one or more members from “population of candidates for best bet”. That process gives us new candidates. Best members of the population survive and the weak ones are eliminated. The disadvantage of that method is that we cannot recognize that the solution is optimal; therefore we don’t know a fixed rule for the end of optimization, as described by Debels [5].

Both methods have some common features:

1. The objective function is defined according to optimization criterions. It is derived from the requirement for “as short as possible” travelling time at all stages (minimal network travelling time). Because the only one variable in this value is the waiting time for connecting intermodal train in a terminal; the value can be limited to the sum of waiting times in all terminals. For ensuring favourable sequences for each consignment their volumes must be taken into account because the whole travelling time is affected by volumes of reloaded consignments. The objective function is then defined as a minimum from the sum of products of synchronization time for reloading of consignments to a connecting train Ti synch and volume of reloaded consignments fi.

\[
\min \ u = \min \sum\limits_{i=1}^{n} T_{\text{synch}, i} f_i
\]  

(12)

2. Besides travelling time, stay in terminal or reloading time models can also take into account other constraint condition like impossible occupation of a single line stage by two opposite directed trains or operating intervals.

Within common features there exist some important differences:

1. The solution via a simplex method needs a objective function and constraint conditions in a linear form but the basic problem isn’t linear. A character of lag timetable means that reloading linkage (eventually constraint conditions) may not be set up between two train routes during one period and leads to the use of the nonlinear function which returns a division reminder after integer division. Jentsch [18] found that linearization calls for an integer parameter (pulse multiplier) tm and pulse interval T which enable pulse movement \( T_{\text{p}} \). Then we have:

\[
T_{\text{p}} = t_m \ast T
\]  

(13)

Such multipliers take the value from \(-10\) to \(+10\). For each reloading terminal (constraint condition) a self pulse multiplier must be set up which is an integer parameter.
2. The simplex algorithm finds a definite optimal solution. On the contrary, a genetic algorithm finds a different solution for each calculation. That’s why a genetic algorithm must be used a number of times (the more calculations the higher possibility to find the optimal solution). In the next step we analyze the best values from the genetic algorithm according to other criterions because all the terms of a real system cannot be added into the objective function. Here we can find that the best value from the genetic algorithm will be unsuitable for a real system and the most suitable will be a slightly worse value.

If we want to solve the train route problem we have to define model inputs (sort and range of values). We used the following input parameters:

- Railroad network - graph (vertices and edges) of a real railroad network; vertices are intermodal terminals, edges are tracks (Fig. 3).
- Travelling times between vertices,
- Train routes - present proposed routes of each train according to a train operation technology,
- Stopping time – minimal stopping time of a train in node,
- Sort of pulse – each route has a self pulse between 6 and 24 hours,
- Reloading time – time needed for reload of transport units among trains,
- Terminal outputs – volume of units reloaded in each node.

For a theoretic model proposal a network from Fig. 3 was chosen. Sixteen nodes (intermodal terminals) and 51 reloading links were added. Distances, travelling times between nodes and pulses were proposed. The pulses are 6, 12 or 24 hours. Those values come up to real pulses for intermodal transports. Reloading times were also valued (from 6 to 10 hours). This time is enough for unloading and loading of transport units. In Fig. 3 there are also ratings of each reloading link (importance of each node). This quantity is stipulated as a link rating among each route. The value of rating is a tenth of reloaded transport units in TEU (Twenty-foot Equivalent Unit). It means that if we reload 10 TEU, the rating value is 1.

A very important advantage of this model is its variability. This is important especially during individual computations and can without any problems react to some input changes. This offers us the possibility to acquire more variable solutions, as described by Bienstock [2].

4.3 The solution results

The input data of the model were transformed using the Premium Solver Platform Version 7.0 software, Frontline Systems for Microsoft Excel 2007. Based on this product which uses the method of genetic algorithms, the model solution was run in Microsoft Excel 2007. For the needs of finding the optimal solution this computation was run 200 times. The genetic algorithm finds solutions that cannot be immediately classified as optimal ones, as described by Debels [5] and Goldberg [9]. Gondro [8] suggests that searching for the solution is random and there are usually different results in each computation. The optimization was also carried out several times (the more times the more probability of finding the optimal solution). It also seems to be appropriate to choose a few best results and evaluate them following other criterions. Because we cannot cover all the inputs existing in the real world in our objective function it is possible that the solution with its minimal value can be unsuitable in real life and, on the contrary, the best usable solution can be a solution with a slightly worse value of the objective function. Schmitt [24] and Tuzar [25] found that the finding of several suboptimal solutions can therefore be successfully used.

The solution of this model can be interpreted either in a table or in a graphic form. The results in a graphic form are expressed as a schematic representation of computed results of departures and arrivals of the trains into nodes in a network graph where the vertices are different nodes marked A to P and the edges represent individual train connections. Each node was drawn in all the relevant connections with direction and time positions of all departures and arrivals. These time positions are in hours. A value is
attached to individual tracks shown by the number within each small frame. These values say what pulse was used on that track. A section of this graph can be seen in Fig. 5.

![Fig. 5 Cut of graphic expression of final model solution](image)

Results of 20 best solutions (10% of the total count of found solutions) are shown in Fig. 6. All these solutions have the value of the objective function (sum of product of waiting times during reload and weight of reloading links representing amount of transhipped transport units) from 110 to 195. From the table it is clear that if we choose a simple sum of synchronization times the best solution will be no. 5. On the contrary the solution no. 1 – the best one according to the objective function – will be in 5th spot. It is necessary to say that the weights of the transhipped transport units that significantly influence the results are theoretically estimated. Therefore it is valid to investigate also other solutions.

It has to be said that this designed theoretical model is directed to the phase of the preparation of possible timetables of IMT trains. Therefore, the computations are based on a premise that the timetables are adhered to and there are no reserves for elimination of possible delays.

5. Conclusion

The presented method of optimization of train time positions minimizes the transportation time in a given system of IMT cargo transport through minimization of time spent waiting on the sequential train in the terminal. This method can be used in other traffic systems especially in public passenger traffic, i.e. in regional railway traffic, bus traffic or in the multi-traffic systems.

The presented mathematical model cannot practically solve the set-up of pulse timetable with all constraint conditions because each constraint condition makes the solution more complicated and decreases the probability for an optimal solution. The most important constraint conditions must be incorporated into the model.

A big advantage of the proposed model is the possibility for arbitrary change of inputs. That makes the solution more variable. It is necessary to realize that the change of a route time or change of a line interval (or any other input) can result in a significant change of the total solution for time positions of other lines. Because of this, the main criterion of countries which advocate a working pulse timetable is to spend a great deal of investment in developing a timetable.

The presented results are part of the task MSM 0021627505 “Theory of transport systems”.

### Table

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### References


