The presented article deals with the methods for the railway wheel profile development on the base of the intended shape of wheelset/track contact geometric characteristics. The article deals with two methods. In short the profiles creation method through arcs radii profile variation is mentioned. The main emphasis is directed to the “Iterative method for a railway wheel profile design”. This method is explained in a more detailed way in the article. The base for the creation of new profiles with the help of the first method is an interactive attitude based on the modification of the original shape which is defined by section of a railway wheel profile, with interconnected and exactly defined arcs radii. It is possible to change them according to shape needs of the final geometrical characteristics. The base of the iterative method, which is of the highest importance in this article, is to develop a new profile under given conditions - a prescribed shape of the contact points distribution and the shape of delta-r function. The newly developed wheel profile in the couple with given rail profile will have requested chosen characteristic shapes.

Key words: railway wheel profile, profile development, iterative method

1. Introduction

Today there is an extremely up to date need for security of vehicles interoperability in its greatest extent in order to use in reality one of the most important advantages of railway transport – economic transport of a huge volume of goods over long distances and comfortable transport of passengers. Track forces are of the highest relevance from the point of view of safe vehicle operation on the track. The wheel/rail geometry is one of the main parameters influencing track forces and vehicles dynamics.

In the article we will deal with methods for the railway wheel profile development on the base of the intended shape of wheelset/track geometric characteristics. In short the profiles creation method through arcs radii profile variation is mentioned, the main emphasis is directed to the “Iterative method for a railway wheel tread profile design”. This method is in detail explained in this article.

2. The wheel profile development on the base of a geometric characteristics shape

Wheelset and rail geometric characteristics help to specify the geometric binding of a wheelset and rail. With their help it is possible to assess relatively quickly some parameters of the binding which help to estimate how a vehicle or, better to say, a wheelset can behave in real operation.

Under geometric characteristics we understand the following: contact points distribution between a wheel and rail at a lateral
movement of wheels profiles of a wheelset over the rail heads profiles, see Fig. 1, Delta r function, Tangent Gamma function, Equivalent conicity function and Radial steering index.

The basic presupposition for the characteristics evaluation is detailed knowledge of a wheel and rail profiles geometric shape.

It is not possible to state a generally “optimum” profile for all types of tracks and railway wheels. The main reason is that in operation there are tracks with various gauges, various rail heads profiles on which vehicles with various railway wheels profiles move. Vehicles move at various speeds, with various wheel forces, they transport various goods and on top of it we require various kinematics behavior. That is why it is very difficult to state unambiguous criteria which could be defined and compared when taking into account all vehicles.

We choose geometric characteristics of a railway wheelset and rail and we will state our intended wheel profile on the base of their specific shape.

Both of the mentioned methods: “Profiles creation method through arcs radii profile variation” and “Iterative method for railway wheel tread profile design” have the same aim, to develop the wheel profile on the base of a requested shape of selected contact geometric characteristics, but each of them use different input parameters, work on different core procedures.

3. Criteria for the search of a requested profile

We stated the following conditions as criteria for optimizing the process:
- fluent distribution of contact points on the wheel and rail surface,
- delta-r function shape without jump change of the course,
- requested equivalent conicity,
- exclusion of two-point-contact

The difference function course shape (delta-r) without jumps indicates a continual increase in lateral forces in the wheel and rail contact point without kick bounces and additional dynamic exciting of the vehicle mechanical system. This phenomenon is shown in the value of safety against derailment.

The size of an instant equivalent conicity value is connected with the size of wavelength of periodic oscillation movement which a wheelset performs. A low conicity value presupposes a higher wavelength value. The sufficient wavelength size is extremely important for a high speed railway operation. The size of equivalent conicity is projected into the stability of vehicle movement and riding comfort of passengers.

The determining value of equivalent conicity is stated at the movement amplitude of 3 mm. The contact points distribution across the profile has a crucial influence on wearing (resistance against wearing) of a wheel and material and, at the same time, the geometric stability of their profiles. From the nature of the way of geometrical characteristic evaluation it is clear that it is not possible to suggest profile geometry directly on the base of a contact points position setting. Various authors have applied various procedures where a continual profile shape modification on the base of a geometric characteristic is the starting point.

4. Profiles creation method through arcs radii profile variation

We used the method of arcs radii profiles variation for the creation of new profiles which meet the given criteria. This method was presented in detail in [1].

The base of the activity is an interactive attitude based on the modification of the original shape defined by parts of the railway wheel profile interlinked with exactly defined arcs radii which it may be changed according to the need to the final geometric characteristics shape.

The basic procedure is as follows:
1. We have the profile of the rail of UIC 60 1:40.
2. We have a requested shape of geometric characteristics: it is low equivalent conicity ≤ of 0.05 in the given case.
3. We choose a “suitable” wheel profile: we will choose the up to now used profile of S1002.
4. We perform numerical division of the whole existing wheel profile in order to obtain simple geometric parts.
5. We create the radii function \( R(y) \) (in dependence on the lateral wheel profile coordinate).

The radii function \( R \) depends on the lateral movement \( y, z \) is a vertical profile coordinate.

\[
R = R(y)
\]

\[
z^* = \frac{1 + (z')^2}{R(y)}
\]  

6. We set the starting conditions in the equation \( y_0 = 0, z_0 = 0 \), \( z_0' = \tan \) profile tangent slope. The initial condition secure that the beginning of the profile coordinate system is placed in a horizontal direction into the plane of a nominal wheel radii circle and the profile crosses the beginning. The first \( z'_0 \) coordinate derivation determines the tangent value of the profile tangent slope in the point \( [y_0, z_0] \). The profile is created by the solution of the differential function (2) continually from the point \( [y_0, z_0] \) to the left side and then from the point \( [y_0, z_0] \) to the right side.

7. In the given example, it is the first derivation \( = \) tangent value in the starting point \( = 0.05 \).
8. We change the selected radius (from the original S1002 profile radii) and compute the equation (2).
9. When the new profile is derived, the geometric characteristics are evaluated and the results are assessed. If the results meet...
our expectations, the profile is accepted, if the results do not fulfill our expectations, the selected radius (radii) is (are) changed and computations have to continue. In this case, there is a need for knowledge and for experience and skills in the radius selection and radius value for the determination of the change.

The Nyström method was used for the solution of the differential function (2) arising from the relationship for curvature. Its application for the solution of the function (2) is in relationship with the starting conditions defined in point Nr. 6.

Results of application of the method of arcs radii profile variation

Wheel profiles (NEWRAD_I) which correspond to our needs thanks to their properties were created on the base of the chosen input parameters at defined optimizing criteria.

5. Iterative method

The other method based on different presuppositions than the first presented method which can be used for the new railway wheel tread profile design is the “Iterative method”. This method can help us reach our aim to have the wheel profile with requested properties according to the given conditions based on the requested shapes of geometric characteristics.

1. We set the functions which describe the rail profiles: \( f_{kp}, f_{kl} \).
2. We set the \( f_x \) function for x-contact points on rail coordinates prescriptions in dependency on the lateral wheelset movement \( y_w \).
3. We set the \( f_{dr} \) function for the requested \( \Delta r \) function characteristics prescription in dependency on the lateral wheelset movement \( y_w \).
4. \( y_i = y_{x_{min}} \cdot i = 0 \) the index \( i \) and a lateral shift of a wheelset initiation.

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Fig. 2 Profiles radii S1002 and UIC60 at contact points

Fig. 3 Profiles radii NEWRAD and UIC60 at contact points

Fig. 4 Contact points at profiles combination NEWRAD_I/UIC60/1:40/1435 with marking of contact points interval at a wheelset lateral movement in a rail with amplitude of 3 mm
5. the contact points on the rail coordinates calculation.

6. the \( d \) parameter calculation.

7. wheel profiles points calculation.

8. wheelset lateral shift increasing \( y_w \) for an increment \( \Delta y_w \) and the index \( i \) increasing.

9. If \( y_w < y_{w_{\text{max}}} \) go to the Nr. 4.

10. Smoothing of the developed wheel profiles with the help of the spline smoothing.

11. The functions courses computation. The functions characterize the rail/wheel contact.

12. The judgment of the \( \Delta r \), tan \( \gamma \), tan \( \gamma_e \), characteristics courses.

13. If the result is acceptable, the wheel profiles are created and whole process is finished.

14. We modify the \( f_x \) function course for contact points on the rail \( x \)-coordinate prescription in dependence on the wheelset lateral shift \( y_w \) and we modify the function \( f_{dr}(y_w) \) for the prescription of the requested \( \Delta r \) characteristic course in dependence on the wheelset lateral shift \( y_w \).

15. Return to point Nr. 3.

Where:

\( y_w \) - is a wheelset lateral movement
\( y_{w_{\text{min}}} \) - starting lateral movement of a wheelset
\( y_{w_{\text{max}}} \) - ending lateral movement of a wheelset
\( f_{rx}(x) \) - right rail profile describing function
\( f_{lx}(x) \) - left rail profile describing function
\( f_{dr}(y_w) \) - the function determining the \( x \)-coordinates of the contact points on the rail
\( f_{dr}(y_{w_{\text{max}}}) \) - prescription function for requested shape of \( \Delta r \) function
\( x_w \) - \( x \)-contact point coordinate at the right rail
\( x_{r_{\text{e}}} \) - \( x \)-contact point coordinate at the left rail
\( y_{r} \) - \( y \)-contact point coordinate at the right rail
\( y_{l} \) - \( y \)-contact point coordinate at the left rail
\( d \) - difference of \( y \)-th coordinates
\( i \) - index of the point of a profile
\( x_w[i] \) - \( x \)-axis coordinate of \( i \)-th point of the right wheel profile

Fig. 5 Equivalent conicity function for the profiles couples NEWARDI/UIC60/I:40/920/1435

Fig. 6 Delta-\( r \) function for the profiles combination NEWARDI/UIC60/I:40/1435

Fig. 7 Delta-\( r \) function for S1002 wheel profile and requested shape of delta-\( r \) for a new developed profile NEWRAD depiction
Fig. 8 Contact points function for S1002 wheel profile and requested shape of contact points placement function (fx) for a new developed profile NEWRAD depiction; left wheel profile/UIC60

Fig. 9 Contact points function for S1002 wheel profile and requested shape of contact points placement function (fx) for a new developed profile NEWRAD depiction; right wheel profile/UIC60

Fig. 10 Contact points at profiles combination NEWRAD/UIC60/1:40/1435

Fig. 11 Course of the difference curve between theoretical wheel profile of S1002 and NEW designed profile NEWRAD with regard to a lateral profile coordinate yw
**Fig. 12** Equivalent conicity function for profiles combination NEWRAD and S1002/UIC60/1:40/1435

**Fig. 13** Tangent gamma function for profiles combination NEWRAD and S1002/UIC60/1:40/1435

**Fig. 14** Contact points function with marked radii for a kinematic movement through a track curve for the rail profile UIC60 and wheel profile NEWRAD

**Tab. 1** Radii for a kinematic movement through a track curve

<table>
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<tr>
<th>Left arc</th>
<th>Radius [m]</th>
<th>yw [mm]</th>
<th>Left arc</th>
<th>Radius [m]</th>
<th>yw [mm]</th>
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<tr>
<td>LEFT arc</td>
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<td>6.5847</td>
<td>S1002</td>
<td>L100</td>
<td>6.615</td>
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<tr>
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**Fig. 15** Graphic expression of the dependency of radii for a kinematic movement through a track curve on the wheelset lateral movement
It is evident from Figs. 14, 15, 16 that a wheel equipped with a NEWRAD wheel profile can move in a kinematic way through the rail arc radii in a fluent continuous way. This happens at the given configuration already from zero to seven mm of lateral displacement. It is not possible when combining the wheel profile S1002 and the profile UIC60. Fig. 10 illustrates the regular distance of contact points. Fig. 18 and Fig. 19 show the stress distribution. NEWRAD shows lower maximum values of the normal stress evaluated by the Hertz method.

6. Conclusion

The presented article deals with the methods for the railway wheel profile development on the base of the intended shape of wheelset/track contact geometric characteristics. In short, the profiles creation method through arcs radii profile variation [1] see Fig. 4 was mentioned. The result wheel profile is named NEW-
RAD_I. The base for the creation of new profiles with the help of the first method is an interactive attitude based on the modification of “the original shape which is defined by a section of a railway wheel profile”, with interconnected and exactly defined arcs radii. It is possible to change them according to shape needs of the final geometrical characteristics in Fig. 5 and Fig. 6. The main emphasis is directed to the “Iterative method for railway wheel tread profile design”. This method is in detail explained in this article. The base of the mainly presented iterative method is to develop a new profile under the given conditions – a prescribed shape of the contact points distribution and shape of delta r function. The new developed wheel profile together with the given rail profile will have requested characteristics. This method can be used after modification for a design of rail profile in accordance with the wheel profile. It is evident from Fig. 14, 15 and 16 that a wheel equipped with the NEWRAD wheel profile can move in a kinematic way through the rail arc radii in a fluent continuous way. This happens at the given configuration from zero to seven mm of a lateral displacement. This is not possible when the wheel profile S1002 and the profile UIC60 are combined. In Fig. 10 there is a regular distance of the contact points. Fig. 18 and Fig. 19 show the stress distribution. NEWRAD shows lower maximum values of the normal stress evaluated by the Hertz method.

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