1. Introduction

Let us consider an emergency situation when population of a given set of towns and villages is endangered by some threat. The casualties can be avoided to some extent by evacuation of the endangered population to some safe places, which have been pre-destinated for each evacuated place in advance. To perform the evacuation, some available vehicles are disposable at several places located in the neighborhood of the endangered dwelling places. It is necessary to determine a route for each used vehicle so that the population is evacuated from its original places to the predetermined refuges. The evacuation should be performed so that the time of evacuation is as short as possible. The end of evacuation is given by the time when the last inhabitant reaches his/her pre-destinated shelter.

If any vehicle is used to provide an evacuated community with this service, the route of vehicle may take a prescribed form. The route starts at the original vehicle location, continues to the served village or town, picks up a portion of the evacuated inhabitants and takes them to the predetermined refuge [4]. If necessary, the vehicle may return back to the evacuated place and save another portion of its population by taking them to the refuge. This cycle can be repeated several times.

Even under this simplifying assumption about a form of route, time optimal assignment of the vehicles to the evacuated places represents a hard combinatorial problem, whose solution must be usually found in a short time of several minutes. In this paper, we present different approaches to this problem. Each of these approaches enables to employ a commercial IP-solver, to obtain a final concrete set of decisions on the vehicle assignment. All these approaches consist of a linear programming model formulation and solving process performed by commercial software with usage of its particular characteristics. The presented approaches differ in models and following quality of obtained solutions. These properties were studied by numerical experiments and their results are presented in the concluding part of this paper.

To formulate the following mathematical models for the individual approaches, we shall use a common denotation, where symbol $I$ denotes the set of all considered homogenous fleets of vehicles. Each homogenous fleet $i \in I$ is characterized by a number $N_i$ of vehicles and by vehicle capacity $K_i$. We shall assume that the fleet $i$ is located at a node $u(i)$ of a road network covering the serviced area. The endangered dwelling places form a set $J$ and each dwelling place $j \in J$ is described by a number $b_j$ of its population and by a road network node $v(j)$, where the village or town $j$ is located. We assumed for the purpose of evacuation that a destination place $w(j)$ is assigned to each dwelling place $j \in J$. Furthermore, let $t_{ij}$ denote the time, which is necessary for a vehicle from the fleet $i$ to traverse the distance between the nodes $u(i)$ and $v(j)$. In addition, let $s_j$ denote the time necessary for traversing the distance between the nodes $v(j)$ and $w(j)$. Using these denotations, we can introduce two approaches to the evacuation plan design problem. The first approach assumes that each considered homogenous fleet is indivisible, i.e. all the vehicles of one fleet perform simultaneously the same activities like a convoy or vehicle train. The second approach is based on the assumption that each fleet can be split into arbitrary integer parts and only vehicles of one part have to act as a convoy.

The first approach leads to a simpler but larger model, whereas the second one pays for its smaller size by non-linearity of the associated model. Advantages and disadvantages of the both approaches are studied in the next two sections and some results of numerical experiments are presented in the concluding part of this paper to demonstrate efficiency of the both approaches, in
the case when a commercial software tool is used for obtaining final decisions on the evacuation plan.

2. Evacuation problem with indivisible fleets

2.1 Problem formulation and model building

Let us consider that a set $J$ represents evacuated dwelling places where each element $j \in J$ is characterized by a location $v(j)$, a number of population $b_j$ and a location $w(j)$, to which all population must be transported. For this purpose a set $I$ of indivisible fleets is disposible where each fleet $i \in I$ is characterized by a location $u(i)$, a number $N_i$ of identical vehicles and a vehicle capacity $K_i$. Let $t_{ij}$ and $s_j$ be travelling times between the locations $u(i)$ and $v(j)$ and between $v(j)$ and $w(j)$ respectively. The objective is to determine a route of each indivisible fleet so that all population is evacuated and the longest fleet route is minimal [3].

Coming out of the assumption on the possible form of a fleet route, we introduce the variable $z_{ij} \in \{0,1\}$ defined for each pair of the fleet $i$ and dwelling place $j$. This variable takes the value of 1 if and only if the fleet $i$ is assigned to the dwelling place $j$.

![Fig. 1 A route with two visits at v(j)](image)

Taking into account that the fleet $i$ can visit an evacuated place several times, we denote the number of journeys of fleet $i$ from $v(j)$ to $w(j)$ by variable $x_{ij} \in Z^{+}$. Now, when the fleet $i$ is assigned to the dwelling place $j$, its travelling time (see Fig. 1) is equal to:

$$t_i + s_j + 2s_j(x_j - 1)$$

(1)

The last introduced variable is $T \geq 0$, which denotes an upper bound of all route times.

We can take into account that a maximal sensible time $T^{\text{max}}$ of the evacuation can be given. In this case the maximal possible number of visits of fleet $i$ at the place $j$ can be evaluated to comply with this limit. The inequality (2) must hold for the $x_{ij}$

$$t_i + s_j + 2s_j(x_j - 1) \leq T^{\text{max}}$$

(2)

The inequality (2) can be rewritten as:

$$x_j \leq \frac{T^{\text{max}} - t_i + s_j}{2s_j}$$

(3)

Denote $P_i(T^{\text{max}}) = \frac{T^{\text{max}} - t_i + s_j}{2s_j}$.

If $P_i(T^{\text{max}}) \leq 0$, then the fleet $i$ is not able to service the place $j$. To minimize the size of a built model, we introduce a set $I(i)$ of all the places from $J$, for which the inequality $P_i(T^{\text{max}}) \leq 0$ holds. Similarly, we define a set $J(i)$ of all the fleets from $I$, for which $P_i(T^{\text{max}}) > 0$ holds.

The linear model of an evacuation plan design can be completed now as follows:

Minimize $T$

Subject to

$$(t_i - s_j)z_{ij} + 2s_jx_{ij} \leq T \text{ for } i \in I, j \in J(i)$$

(5)

$$\sum_{j \in J(i)} z_{ij} = 1 \text{ for } i \in I$$

(6)

$$x_{ij} \leq P_i(T^{\text{max}})z_{ij} \text{ for } i \in I, j \in J(i)$$

(7)

$$\sum_{i \in I} N_iK_i x_{ij} \geq b_j \text{ for } j \in J$$

(8)

$$z_{ij} \in \{0,1\} \text{ for } i \in I, j \in J(i)$$

(9)

$$T \geq 0$$

(10)

The constraints (5) assure that the travelling time of each fleet is less or equal to the upper bound $T$. The constraints (6) enable for the fleet $i$ to be assigned at most to one evacuated dwelling place. Constraints (7) cause that if the variable $z_{ij}$ is equal to zero, then the variable $x_{ij}$ is also zero, which means that if the fleet $i$ is not assigned to the place $j$, then the fleet cannot visit this place at all. The constraints (8) ensure that each dwelling place $j$ is provided with a sufficient capacity, which enables to evacuate all the population of the size $b_j$.

2.2 Numerical experiments

To verify the suggested use of the IP-solver, we formulated ten different instances of the problem. One of the instances denoted as “Hradza” comes out of the possible emergency situation, which can occur if the dam Liptovska Mara breaks. Then, under given assumptions, 26 communities would have to be evacuated to 26 predestined places. For this evacuation, 411 vehicles of different capacities located at three bigger towns of the area are available. The other instances were formulated in nine areas of the Slovak Republic in a similar way. These instances are denoted by names of the biggest towns of the areas. The numbers and capacities of available vehicles were generated similarly to the first instance for the vehicles to be able to satisfy the demand on evacuation. These benchmarks were used to verify the suggested method, which consists in a particular model building and employing the general IP-solver for obtaining of a good solution of the problem. To be able to perform the computation in a given time, we used the general
optimisation software environment XPRESS-IVE for our study \[5\], \[6\]. This software system includes the branch-and-cut method and it also enables exploitation of the premature stopping rules. The software is equipped with the programming language Mosel, which can be used for both the input of model and writing of input and output procedures. Furthermore, the language has its own tools for the stopping rules adjustment.

The main disadvantage is born-in to the branch-and-bound method, which is the basic solving method of any IP-solver. This is the possibility that the list of determined but unfathomed solution subsets may grow exponentially instead of being reduced to an empty set. When a special IP-solver is designed, all specific properties of the problem can be used to improve quality of the upper and lower bounds, whereas only a general problem relaxation can be employed in the case of this general solver. We used the possibility of the solver, which enables a premature termination of the searching process whenever a fixed time limit is exceeded. The experiments were performed on a personal computer equipped with Intel Core 2 6700 with parameters: 2.66 GHz and 3 GB RAM. The first series of experiments corresponds with the case, when each vehicle is considered as one individual fleet. The best results obtained in the computational time of 20 minutes are presented in Table 1, where “No. of F” denotes the considered number of fleets, “No. of EP” denotes the number of evacuated dwelling places, Rows denotes the number of structural constraints of the model and Columns denotes the number of used variables including the auxiliary ones, which are automatically introduced by the solver. In the row denoted as “Tmax”, there are reported the predetermined values of $T_{max}$ in minutes. The symbol “Tbest” denotes the row, where the best-found time of evacuation is plotted.

As the preliminary experiments showed that decreasing the parameter $T_{max}$ to diminish the numbers of constraints and var-
ables of the model had no effect on obtaining a better solution in the given time limit, we decided to explore another way of a model size diminishing. We formed models with a smaller number of fleets so that we grouped the vehicles by two, four and eight if possible to fleets of several vehicles with the same capacity. The derived problems were solved and the results are presented in tables 2, 3 and 4 respectively.

It can be observed that the process of forming the bigger fleets ended at eight-vehicle fleets for some instances, when the solving process fails in finding a solution due to infeasibility. These cases are denoted by asterisk in the row “Tbest”. Furthermore, the found resulting evacuation times turned worse for instances, where a feasible solution was found. This result evoked an idea to formulate the evacuation problem for divisible fleets, which is the topic of the next section.

3. Evacuation problem with divisible fleets

3.1 Problem formulation and model building

Similarly to the previous section, we consider that the set J represents evacuated dwelling places, where each element j ∈ J is characterized by a number of the elements 2

\[ t_i - x_i \leq T \text{ for } i \in I, j \in J(i) \]

\[ x_i \leq P_i (T^{max}) \text{ for } i \in I, j \in J(i) \]

\[ q_i \leq N_i z_{ij} \text{ for } i \in I, j \in J(i) \]

\[ \sum_{j \in J(i)} q_i \leq N_i, \text{ for } i \in I \]

\[ \sum_{j \in J(i)} K_i q_i x_i \geq b_j, \text{ for } j \in J \]

\[ z_{ij} \in \{0,1\}, \text{ for } i \in I, j \in J(i) \]

\[ T \geq 0 \]

\[ q_i \in Z^+, \text{ for } i \in I, j \in J(i) \]

The constraints (13) assure that the travelling time of each part of the fleet is less or equal to the upper bound T. The constraints (14) represent binding constraints between the variables x_i and z_{ij}. These constraints cause that if the variable x_i is equal to zero, then the variable z_{ij} is also zero, which means that if the fleet i is not assigned to the place j, then no vehicle of the fleet i can visit this place. The constraints (15) represent binding constraints between the variables q_i and z_{ij}. These constraints cause that if the variable z_{ij} is equal to zero, then the variable q_i is also zero, which means that if the fleet i is not assigned to the place j, then no vehicle of the fleet i can visit this place. The constraints (16) assure that the total number of designed vehicles of the fleet i does not exceed the number N_i.

The constraints (17) ensure that each dwelling place j is provided with a sufficient capacity, which enables to evacuate all the population of size b_j. Unfortunately, these constraints are non-linear and thus this model cannot be input to the IP-solver. To be able to solve this much simpler problem, it must be rearranged to a linear form. An approach to the model linearization is shown in the next section.

Results of numerical experiments for indivisible fleets of eight vehicles

<table>
<thead>
<tr>
<th>Instances:</th>
<th>Bratislava</th>
<th>Dubnica nad Vahom</th>
<th>Hradza</th>
<th>Kosice</th>
<th>Liptovsky Mikulas</th>
<th>Leopoldov</th>
<th>Michalovce</th>
<th>Nitra</th>
<th>Nove Zamky</th>
<th>Puchov</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of F.</td>
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<td>28</td>
<td>71</td>
<td>71</td>
<td>23</td>
<td>48</td>
<td>24</td>
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<td>1687</td>
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<td>1623</td>
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<td>278</td>
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<td>140</td>
<td>1420</td>
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<td>93</td>
<td>202</td>
<td>184</td>
<td>140</td>
<td>140</td>
<td>1420</td>
<td>423</td>
<td>1497</td>
</tr>
<tr>
<td>Tbest [min]</td>
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<td>2353</td>
<td>1623</td>
<td>2847</td>
<td>140</td>
<td>1497</td>
<td>1273</td>
<td>278</td>
<td>278</td>
<td>278</td>
</tr>
</tbody>
</table>

Minimize T

Subject to:

[equations as shown in the document]
3.2 Problem reformulation to a linear model

To rewrite the model into a linear form, we make use of the fact that the variable \( x_{ij} \) may take only one of several few values from the range of 0, 1, ... , \( P_i \). We introduce auxiliary variables \( m_{ij} \), which serve as a lower bound of the number of visits at the dwelling place \( j \), which are performed by the vehicles of fleet \( i \). Then the following constraints must hold:

\[
q_i x_{ij} \geq m_{ij} \quad \text{for } i \in I, j \in J(i) \tag{22}
\]

Now the series (17) of constraints can be replaced by the constraints (23).

\[
\sum_{i \in J} K m_{ij} \geq b_j \quad \text{for } j \in J \tag{23}
\]

Now the constraint \( q_i x_{ij} \geq m_{ij} \) can be replaced by the following system of logical constraints:

- If \( x_{ij} = 0 \) then \( 0 \geq m_{ij} \),
- If \( x_{ij} = 1 \) then \( q_i \geq m_{ij} \),
- If \( x_{ij} = 2 \) then \( 2q_i \geq m_{ij} \),
- ...
- If \( x_{ij} = k \) then \( kq_i \geq m_{ij} \),
- ...
- If \( x_{ij} = P_i - 1 \) then \( (P_i - 1)q_i \geq m_{ij} \).

It holds here that if some of the constraints is fulfilled for \( k = p \), then they are fulfilled for each \( k > p \) and, vice versa, if a constraint is not fulfilled for \( k = p \), then they cannot be fulfilled for any \( k < p \).

Now we introduce variables \( y_{pq}^k \in [0, 1] \) for \( i \in I \) and \( j \in J(i) \) and \( p = 0, 1, ..., P_i - 1 \) and the system of logical constraints can be replaced by the series (24) of constraints.

\[
NP_i y_{pq}^k + pq_i \geq m_{ij} \\
\text{for } i \in I, j \in J(i), p = 0, 1, ..., P_i - 1 \tag{24}
\]

If the system (24) holds for given values of variables \( q_i \) and \( k \) is the minimal value of subscript \( p \) for which \( y_{pq}^k = 0 \), then the system must also hold for such a setting of variables \( y_{pq}^k \) where \( y_{pq}^k = 1 \) for \( p = 0, 1, ..., k - 1 \) and \( y_{pq}^k = 0 \) for \( p = k, ..., P_i - 1 \), i.e. the setting of variables \( y_{pq}^k \) fulfills the equation (25).

\[
\sum_{p=0}^{k-1} y_{pq}^k = k \quad \text{for } i \in I, j \in J(i) \tag{25}
\]

Then the following linear model describes the evacuation plan design problem with divisible fleets:

Minimize \( T \) \tag{26}

Subject to \( (t_i - s)z_i + 2y_{ij} \sum_{x=0}^{k_{pq}-1} y_{pq}^x \leq T \)

\[\text{for } i \in I, j \in J(i) \tag{27}\]

The constraints (27) have the same meaning as the constraints (13) in the non-linear model (12) - (21), i.e. they assure that the travelling time of each part of the fleet is less than the upper bound \( T \). These constraints were derived from the constraints (13) by substitution of the left-hand-side of the equality (25) for \( x_{ij} \). The constraints (28) are the binding constraints, which assure relations between the variables \( z_{ij} \) and the sum of \( y_{pq}^k \), which corresponds with the number \( x_{ij} \) of visits of the part of the fleet \( i \) to the community \( j \). The constraints (29) were equivalent to the former constraints (15), which assure that if no part of the fleet \( i \) is designated to the place \( j \) (\( z_{ij} = 0 \)), then the number \( q_{ij} \) of vehicles is equal to zero. The constraints (30) assure that the total number of designed vehicles of the fleet \( i \) does not exceed the number \( N_i \).

As \( m_{ij} \) represents a lower bound of the product \( x_{ij} q_{ij} \), which is the number of visits of vehicles from the fleet \( i \) at the place \( j \), then the constraints (31) ensure that population of the place \( j \) can be evacuated. The constraints (32) ensure that the estimation \( m_{ij} \) does not exceed the upper bound \( N_i P_i(T_{\text{max}}) \) of visits and the constraints (33) ensure that the sum of \( y_{pq}^k \) over \( p \) corresponds with the number of necessary visits of a group of \( q_{ij} \) vehicles of the fleet \( i \) at \( j \).

3.3. Numerical experiments

To find characteristics of the second approach involving the linear model of evacuation by divisible fleets, we solved the same instances, which are described in section 2.3. We used the same setting of \( T_{\text{max}} \) and we focused on the evacuation times, which were obtained in the computational time of 20 minutes. The results associated with these experiments are reported in Table 5.
To analyze the sensitivity of the best-found solution on the permitted computational time, we performed these numerical experiments once more for the time limit of 40 minutes. The results are shown in Table 6 and it can be easily found that they were improved only in two instances (Liptovsky Mikulas and Nove Zamky).

### 4. Comparison and conclusions

We suggested two approaches to the evacuation plan design problem. The first approach was based on the concept of indivisible fleets, which enables a direct formulation of the linear model and easy use of the IP-solver. Nevertheless, the linear model was too large and that is why we suggested a process of its shrinking by clustering evacuation vehicles into bigger fleets.

This process reduced the size of the associated model, but it brought an additional condition in the model and it caused that the resulting evacuation times turned worse. The overview in Table 7 shows that the best results were achieved for the biggest models. This result evoked an idea to formulate the evacuation problem for divisible fleets. This task was successfully solved by a complicated reformulation of the originally non-linear model to the linear one. Even if the new model is much more complicated than the first approach model, it has a much smaller size and enables to reach better results in the same computational time. This fact is demonstrated in Table 8, where the row “Divisible” reports on the best evacuation times obtained by the first approach and the row “Indivisible” reports on the times obtained by the second approach.

As we have the possibility to estimate the optimal solution using an iterative approach, we are able to compare the obtained results of both suggested approaches to the best-known results, which are reported in the row “Best Known” of Table 8. The row “Gap” of this table, where differences between the divisible fleet and the iterative approaches in percentage are given, shows that the suggested approaches constitute a promising way to the evacuation plan design.
Table 7

<table>
<thead>
<tr>
<th>Instances:</th>
<th>Bratislava</th>
<th>Dubnica nad Vahom</th>
<th>Hradza</th>
<th>Kosice</th>
<th>Liptovsky Mikulas</th>
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<tr>
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<td>11</td>
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<td>3.6</td>
<td>0.6</td>
<td>12.1</td>
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References


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