1. Introduction

Induction heating has been used in industry over the past three decades. Due to its rapid heating and good reproducibility it is used in heat treatment as well as in applications for special use in mechanical engineering and chemical industry. Induction heating is a non-contact method of heating an electrically conductive material by the process of electromagnetic induction [1]. In a thin layer on the surface of parts alternating electromagnetic field generates currents which are called eddy currents [2]. Eddy currents generate heat due to resistive power losses, which are the main source of heat in the process of induction heating. The induction heating process is dependent on the electrical conductivity of the material, the size of the current in the inductor, the frequency of the applied electromagnetic field and the magnetic properties of the material [3]. Numerical modeling of induction heating allows the optimization of process variables. Using the finite element method, it is possible to calculate the temperature distribution across all components which are exposed to the induction heat. It is thus possible to optimize various process parameters. Induction heating can be addressed as coupling of the electromagnetic and thermal problem [1].

In the following chapters are summarized the fundamental equations and their simplification which are used to describe the physical behavior of materials under electromagnetic and thermal fields.

2. Electromagnetism equations

An electromagnetic field describes a set of equations known as Maxwell’s equations. Equations consist of Faraday’s law and Ampere’s law with Maxwell extension.

The equations contain four different variables, the intensity of the electric field $E$, magnetic induction $B$, the intensity of the magnetic field $H$ and the electric induction $D$ [4].

Maxwell’s equations are expressed as follows [5 and 4]:

$$\nabla \times E = -\frac{\partial B}{\partial t}, \quad (1)$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}, \quad (2)$$

$$\nabla \cdot B = 0, \quad (3)$$

$$\nabla \cdot D = \rho. \quad (4)$$

where $J$ is the current density, $t$ is the time and $\rho$ is the free volume charge density.

Constitutive relations between fields in a homogeneous isotropic environment are expressed as follows [5 and 4]

$$B = \mu H, \quad (5)$$

$$D = \varepsilon E, \quad (6)$$

$$J = \sigma E. \quad (7)$$

where $\mu$ is the permeability, $\varepsilon$ is the permittivity of the dielectric and $\sigma$ is the conductivity of the material.

We assume a harmonous solution to the sinus wave. In addition to these relations there is a vector of magnetic potential $A$ in the following relation to

$$B = \nabla \times A. \quad (8)$$
Maxwell’s equations can then be written by diffusion equation as follows
\[ \frac{\partial A}{\partial t} - \frac{1}{\mu} \nabla^2 A = J_s. \] (9)
where \( \mu \) is the source current density in the inductor.

The diffusion equation (9) can be expressed in the same form for the electric and the magnetic field. In the case of the magnetic vector potential, it can be used in conjunction with any physically noticeable phenomenon of electromagnetic induction, as well as eddy currents induced voltage, inductor impedance, inductor induction. If the current stream as well as eddy currents are considered to be a sinus, and also time-harmonic, electromagnetic fields may be introduced by the following equation [6 and 7]
\[ j\omega\sigma A - \frac{1}{\mu} \nabla^2 A = J_s. \] (10)

Depending on the electrical and magnetic properties, there can be intermittent or continuous electromagnetic fields on each side of a common interface between two different materials.

At the interface between two media, the field must meet the following boundary conditions [4, 8 and 9]:
\[ \hat{n} \times (E_1 - E_2) = 0, \] (11)
\[ \hat{n} \times (D_1 - D_2) = \sigma, \] (12)
\[ \hat{n} \times (H_1 - H_2) = J_s, \] (13)
\[ \hat{n} \times (B_1 - B_2) = 0. \] (14)

The domain border is given by the normal unit vector and is directed from the surface normal of the surface towards the outside and \( q_n \) is a constant heat flux defined by
\[ q_n = - \frac{q}{c} \frac{\partial T}{\partial n}. \] (15)
where \( q \) is the density, \( c \) is the specific heat capacity, thermal conductivity \( k \), and \( q \) is the heat generated in the material per unit of volume and time. Heat transfer equation (15) specifies the layout of the temperature in the environment as a function of time and space [14 and 15]. In the case that the temperature distribution is known, it is possible to calculate the heat flux (15) according to the equation inside the body or on its surface. The solution of the equation of heat conduction requires initial and boundary conditions. The boundary condition may be prescribed by a known temperature at the border or by prescribing heat flux using the flux or radiation [16].

The condition of the heat flux at the border can be defined by the relationship
\[ q_s = - k \frac{\partial T}{\partial n}, \] (16)
where \( n \) is normal to the surface towards the outside and \( q_s \) is a constant heat flux defined by
\[ q_s = h(T_s - T_b) + \sigma \epsilon [T_s^4 - (T_b)^4]. \] (17)
where \( h \) is the convection surface heat transfer coefficient, \( T_s \) is the surface temperature and \( T_b \) is the ambient temperature.

The first part of the boundary condition represents convective heat transfer and the second part of boundary condition (17) represents radiation.

Suppose that the temperature changes on the surface of the material in the sinusoidal mode. Then it can be shown that the amplitude of the changes will be smaller with increasing depth and with a certain time lag, i.e. the phase angle. If the temperature increases, the frequency attenuation and phase angle will increase [6 and 2]. This is the same phenomenon as in the case of electromagnetic waves, but in a different time scale because the electromagnetic field has a much shorter relaxation time.

In spite of the fact that the diffusion equation (10) has the same properties as heat conduction equation (15), their scales are different. Harmonic current in the device for induction heating is usually applied in the frequency range 50 Hz - 70 kHz. For this reason, the use of equation (10) can increase the thermal analysis at each time because the time scale of the electromagnetic problem is much shorter. This assumption is used in the numerical simulation of the solutions strategy [6 and 2].
4. Coupling of electromagnetic and thermal problem

The practical problem associated with the heat transfer is not usually possible to solve analytically, and especially if it has temperature-dependent and non-linear material properties. The current approach to getting a solution for these problems is to use the finite element method. The aim is to calculate temperature field distribution in the body on which it is applied harmoniously through oscillating current in the inductor.

It is possible to calculate magnetic vector potential from the equation (10). Eddy current \( J_e \) generated in the body is then calculated using the following relation [2, 3 and 9]

\[
J_e = -j\omega\sigma A. \tag{18}
\]

The source of the heat in the equation of heat conduction (15) is expressed by [3, 9 and 17]

\[
Q = \frac{1}{2\sigma} |J_e|^2 = \frac{1}{2\sigma} J_e \cdot J_e^*, \tag{19}
\]

where \( J_e^* \) is the complex conjugate of \( J_e \).

At the external border area the magnetic vector potential is chosen so that it is zero at the interface (Dirichlet condition). From the equation (10) we can see that the magnetic vector potential is a long-term solution, a solution in which the characteristics of the material of the time step used the \( t_n \). This is due to the fact that the time scale of the electromagnetic problem is much shorter than the thermal problem.

Figure 1 shows the strategy solutions in the time step. The problem is resolved using the material electromagnetic properties in the time step \( t_n \). The thermal problem is calculated due to eddy currents in a time step \( t_{n+1} \).

5. Two dimensional model of coupled electromagnetic and thermal problem

Let us assume a general planar \( n \)-node element defined in the Cartesian nodal coordinate system \( (x, y) \) [18].

For a finite \( n \)-node element is defined the interpolation of all calculated values in any point inside the element using values from the individual nodal points applying shape functions \( N(x, y) \).

For example, the temperature \( T(x, y) \) in any point of element can be calculated by the following expression from element nodal temperatures \( T_i \) [13, 15 and 19]

\[
T(x, y) = \sum_{i=1}^{n} T_i N_i(x, y). \tag{20}
\]

Energy balance in the modeling area is dedicated to minimizing the energy functional in each node. This can be achieved by setting the first partial derivation of a functional in each node to zero. Instead of minimizing in each node, it is better to perform at the level of element.

The total energy associated to the entire modeling area is then equal to the sum of the energies of all the elements. As a result of the solution of algebraic equations of parallel system with respect to the magnitude of the magnetic vector potential gives an unknown value in each of the nodes. A set of equations can be written in the form

\[
(V_r + jW_r) a_r = g_r, \tag{21}
\]

where \( V_r \) and \( W_r \) are local matrices corresponding to finite element, \( a_r \) is the unknown vector of the magnetic vector potential of the corresponding finite element and \( g_r \) is the vector of the source current density of the corresponding finite element.

The other unknown quantities of the electromagnetic field are obtained using the following expressions [1, 3 and 9].

The intensity of the electric field \( E_i \)

\[
E_i = -j\omega A_i. \tag{22}
\]

Electric induction \( D_i \)

\[
D_i = \varepsilon_0\varepsilon_i E_i = -j\omega\varepsilon_0\varepsilon_i A_i, \tag{23}
\]

where \( \varepsilon \) is the relative permittivity and \( \varepsilon_0 \) is the vacuum permittivity.

Current density \( J_i \)

\[
J_i = \sigma E_i + j\omega D_i = -j\sigma\omega A_i + \omega\varepsilon_0\varepsilon_i A_i. \tag{24}
\]

Joule heat \( Q_{aw} \)

\[
Q_{aw} = \frac{1}{2}\text{real}(J_i \cdot E_i). \tag{25}
\]
6. Computational model

Consider two dimensional problem of electromagnetic heating process of six parallel inductors to steel plate specimen. Martensitic annealed stainless steel is used as material for specimens. The scope of the FE study is to observe how different relative permeability of ferrite core material influences final temperature of heated steel plate specimen. The comparison had been done by the same input parameters shown in Table 1.

Input parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>300 [A]</td>
</tr>
<tr>
<td>Frequency</td>
<td>3500 [Hz]</td>
</tr>
<tr>
<td>Heating time</td>
<td>50 [s]</td>
</tr>
<tr>
<td>Initial temperature</td>
<td>293.15 [K]</td>
</tr>
</tbody>
</table>

The chosen configuration for FE study runs is shown in Figs. 2 and 3.

The model size and component dimensions are shown in Fig. 3.

The distances and diameter values are in millimeters [mm]. The length of the model in "z" direction is 100mm.
Free triangular mesh with finer mesh in the areas with the highest value gradients in observed areas was chosen as the FE mesh shown in Fig. 4. The linear 3-node elements are used for interpolation [21].

Table 2 shows which material constants were used in FE studies. For the simplification, the constant material properties over the time were considered [22]. The relative permeability of ferrite core marked with (*) presents the range of the values used in FE runs.

The inductors were made of copper.

<table>
<thead>
<tr>
<th>Material properties</th>
<th>Table 2</th>
</tr>
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<tbody>
<tr>
<td>Air</td>
<td>Material density 1.293 [kg/m²]</td>
</tr>
<tr>
<td></td>
<td>Specific heat 1.01 \times 10^3 [J/(kg K)]</td>
</tr>
<tr>
<td></td>
<td>Thermal conductivity 26.0 \times 10^3 [W/(m K)]</td>
</tr>
<tr>
<td></td>
<td>Relative permeability 1.000 001 86 1</td>
</tr>
<tr>
<td>Copper</td>
<td>Material density 8930 [kg/m²]</td>
</tr>
<tr>
<td></td>
<td>Specific heat 340 [J/(kg K)]</td>
</tr>
<tr>
<td></td>
<td>Thermal conductivity 384 [W/(m K)]</td>
</tr>
<tr>
<td></td>
<td>Electrical resistance 1.7 \times 10^4 [ohm m]</td>
</tr>
<tr>
<td></td>
<td>Relative permeability 0.999 99 1</td>
</tr>
<tr>
<td>Steel</td>
<td>Material density 7850 [kg/m²]</td>
</tr>
<tr>
<td></td>
<td>Specific heat 460 [J/(kg K)]</td>
</tr>
<tr>
<td></td>
<td>Thermal conductivity 28 [W/(m K)]</td>
</tr>
<tr>
<td></td>
<td>Electrical resistance 4.35 \times 10^1 [ohm m]</td>
</tr>
<tr>
<td></td>
<td>Relative permeability 1000 1</td>
</tr>
<tr>
<td>Ferrite</td>
<td>Material density 6600 [kg/m²]</td>
</tr>
<tr>
<td></td>
<td>Specific heat 700 [J/(kg K)]</td>
</tr>
<tr>
<td></td>
<td>Thermal conductivity 20 [W/(m K)]</td>
</tr>
<tr>
<td></td>
<td>Electrical resistance 50000 [ohm m]</td>
</tr>
<tr>
<td></td>
<td>Relative permeability 1 to 100 (*) 1</td>
</tr>
</tbody>
</table>

7. Results

The scope of the numerical model is to calculate resulting temperature after certain heating time. Fig. 5 shows temperature distribution in [K] at 50s heating time and used ferrite core relative permeability value of 50 (*). Fig. 6 presents impact of ferrite material relative permeability resulting temperature of induction heating process.

The red curve in Fig. 6 shows how relative permeability of ferrite core material influences the resulting temperature of heated material. The green line shows the resulting temperature of heated material without ferrite core. Both induction heating processes use the same process settings mentioned in Table 1.
8. Conclusion

In this article we applied equations of the electromagnetic and thermal field to solve problem of induction heating. The numerical simulation of induction heating process is shown as a coupling of electromagnetic and thermal problem leading to a system of thermal transient analysis by the explicit Euler method used in the time integration solutions.

By the FE solution of the electromagnetic and thermal problem it is possible to observe the effect of the individual variables related to electromagnetic induction as well as the resulting temperature or heat generated by the interaction of the surrounding environments for different time steps. By this numerical approach we can observe how different material properties influence the resulting heat generated during the induction heating process and how the right combination of surrounding components influences the heating efficiency.

The numerical modeling of induction heating process allows optimization of the process parameters, material properties of components or geometry shape optimization.

Acknowledgements

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References