1. Introduction

In modern development process of rail vehicle computer aided simulations are employed. In this way costly experiments and prototypes can be reduced. Production of a rail vehicle is composed of several phases. There is a design phase, a development and optimisation phase, production of a rail vehicle, verification and validation of a rail vehicle and, in the end, commissioning of a rail vehicle. At this time computer software allows to perform complex simulations. Thus, shorter development periods and rising requirements like durability, efficiency or mass reduction demand precise simulations, which intensify the usage of lightweight structures.

2. Principles of rail vehicle multibody system with a flexible body

The need for more accurate models of a rail vehicle to describe the complex behaviour of flexible systems experiencing large motion while undergoing small elastic deformations motivated the development of many powerful analysis techniques. The most popular formulations use time-variant mass matrices to describe the inertia coupling between the rigid body motion and the elastic deformation [1].

To describe the dynamic behaviour of a rail vehicle mechanical system which undergoes large nonlinear working motions the multibody system (MBS) approach is often most useful [2]. A classic MBS of a rail vehicle consists of rigid elements which are connected by ideal joints, coupling elements [3], contact elements [4 and 5] and force elements. The phenomena of the wheel/rail contact [6] significantly influence the rail vehicle properties and wheel/rail contact stress evaluation [7 and 8].

For applications in the field of rail vehicle analysis where the deformation of the bodies cannot be neglected, the method of flexible multibody systems has to be applied. In the flexible multibody system of a rail vehicle the approach is extended by flexible bodies.

The application of flexible bodies into the rail vehicle multibody system with the help of the finite element method introduces a large number of flexible degrees of freedom into the rail vehicle model. The reduction of the linear flexible degrees of freedom is the principal step for an efficient simulation of a flexible multibody system of a rail vehicle [9].

2.1 Foundations of flexible multibody dynamics

For the kinematic description of the motion of flexible bodies that are subjected to large displacements several methods are used. Among these methods are, for example, the floating frame of reference, convected coordinate system, finite segment method and large rotation vector. Large deformation problems in flexible
multibody system can be efficiently solved using the absolute nodal coordinate formulation [10].

In the absolute nodal coordinate formulation, neither infinitesimal nor finite rotations are used as the element coordinates. The locations and deformations of the material points on the finite element are defined in the global coordinate system using the element shape function and nodal coordinates (Fig. 1).

![Fig. 1 Representation of the flexible body kinematics](image)

The floating frame of reference formulation is currently the most widely used method in the computer simulation of flexible multibody systems [10 and 11]. It is used for the systems where the elastic deformation is small compared to the rigid body motion. The basic idea is to separate the motion of the body into a large nonlinear motion of the reference frame and a small linear elastic deformation with respect to the reference frame. The motion of a particular point $P$ on the flexible body $B$ is represented by the vector $r_{p}(t)$ (Fig. 1). A flexible body can be e.g. the bogie frame, the body of wagon or rail etc. Using the floating frame of reference formulation, the position vector $r_{p}(t)$ of the point is separated into a usually non-linear motion $r_{i}(t)$ of the reference frame and a superposed linear elastic deformation $u_{p}(t) = u(R, t)$ with respect to the reference frame [11].

$$r_{p}(t) = r_{i}(t) + R_{p} + u_{p}(t), \quad (1)$$

see Fig. 1. The vector $R_{p}$ corresponds to the position of the point $P$ in the undeformed state.

A similar approach concerning the orientation of the coordinate system at point $P$ leads to the small rotational motion $\Theta_{p}(t) = \Theta_{p}(R_{p}, t)$ by elastic deformation. Once deformation disappeared, this kinematic formulation leads to exact modelling the rigid body dynamics of a rail vehicle. When this formulation is used, the modelling of a flexible multibody system of a rail vehicle can be divided into two parts (Fig. 2) [11].

$$u(R, t) = \Phi(R) \cdot q_{p}(t), \quad \Theta(R, t) = \Psi(R) \cdot q_{p}(t). \quad (2)$$
where \( q_i(t) \) refers to the nodal displacements of a finite element model and \( \Phi(R) \) and \( \Psi(R) \) are the elastic shape functions [9]. This results in the equation of motion of a flexible body

\[
M_r \cdot \ddot{q}_r(t) + K_r \cdot q_r(t) = h_r, \tag{3}
\]

as formulated in [9]. The matrices \( M_r \) and \( K_r \) are the mass and stiffness matrices of the flexible structure and have the following characteristics if the system is constrained sufficiently to avoid rigid body motion

\[
M_r = M_r^T > 0, \quad K_r^T > 0. \tag{4}
\]

The generalised surface and volume forces are summarised in the force vector \( h_r \). To consider the dissipative effects an additional damping matrix \( D_r \) is often introduced and can be approximated, e.g. by viscous damping (Rayleigh damping):

\[
D_r = \alpha M_r + \beta K_r, \tag{5}
\]

with the proportional factors \( 0 \leq \alpha, \beta \in \mathbb{R} \). The need for high precision and complex geometries often leads to a fine spatial discretisation. Mathematically the flexible bodies are described by a large set of linear ordinary differential equations, whose solution increases the computational effort of the simulation. Linear model reduction is a decisive component to efficient simulation. To get a representation used for some model reduction techniques the forces acting on the finite element structure are described by the time dependent excitation \( u_r(t) \) and the input or control matrix \( B_r \in \mathbb{R}^{nr \times or} \). This matrix captures the spatial distribution of the boundary and coupling conditions. Further on, the output or observation matrix \( C_r \in \mathbb{R}^{nr \times or} \) is introduced for the calculation of the interesting displacements \( y(t) \). In this case, equations of motion of a single flexible body can be formulated as a linear time-invariant second order multi input multi output system:

\[
M_r \cdot \ddot{q}_r(t) + D_r \cdot \dot{q}_r(t) + K_r \cdot q_r(t) = B_r \cdot u_r(t). \tag{6}
\]

\[
y(t) = C_r \cdot q_r(t). \tag{7}
\]

Due to increasing demands on the characteristics of technical products and their simulation, the requirements on the calculation accuracy and the calculation time are often extremely high. On the bottom line this trend means that the dimension of the equation of motion rises and the time to run the simulation should be as quick as possible. Such problems particularly require an adequate model order reduction to decrease the number of equations and keep the significant characteristics of the system. Using the floating frame of reference formulation and the linear model order reduction via projection of equation of motion (3), (5), (6) (see [9 and 11]) can be used. Therefore, the large number of degrees of freedom of the flexible coordinates \( q_r \in \mathbb{R}^{nr \times or} \) are approximated in a subspace \( \mathcal{V}_r \) of smaller dimension \( n < N \) by the reduced displacement vector \( \tilde{q}_r \in \mathbb{R}^{nr \times or} \)

\[
q_r \approx V \cdot \tilde{q}_r. \tag{8}
\]

This subspace \( \mathcal{V}_r \) is described by the projection matrix \( V \in \mathbb{R}^{n \times or} \). The use of this relation in FEM equations of motion (5) and (6) leads to an over-determined system and leaves a residuum because the exact solution \( q_r \) is generally not an element of the subspace \( \mathcal{V}_r \). To obtain a unique solution the residual should be orthogonal on a second subspace \( \mathcal{W}_r \) represented by \( W \in \mathbb{R}^{n \times or} \). The orthogonality conditions or Petro-Galerkin conditions result in the reduced FE equations

\[
\tilde{M}_r \cdot \ddot{\tilde{q}}_r + \tilde{D}_r \cdot \dot{\tilde{q}}_r + \tilde{K}_r \cdot \tilde{q}_r = \tilde{B}_r \cdot u_r, \tag{9}
\]

with the reduced matrices \( \tilde{M}_r := W^T \cdot M_r \cdot V, \tilde{D}_r := W^T \cdot D_r \cdot V, \tilde{K}_r := W^T \cdot K_r \cdot V \in \mathbb{R}^{nr \times nr} \) and \( \tilde{B}_r := W^T \cdot B_r \in \mathbb{R}^{nr \times or} \). The projection is called orthogonal if the subspaces are identical \( V = W \) and oblique otherwise. This procedure leads to the reduced equations of motions of one flexible body:

\[
\begin{bmatrix}
M_r & M_r \cdot V \\
W^T \cdot M_r & \tilde{M}_r
\end{bmatrix}
\begin{bmatrix}
\ddot{\tilde{q}}_r \\
\dot{\tilde{q}}_r
\end{bmatrix}
+
\begin{bmatrix}
0 \\
W^T \cdot \tilde{K}_r
\end{bmatrix}
\begin{bmatrix}
\tilde{q}_r \\
\dot{\tilde{q}}_r
\end{bmatrix}
=
\begin{bmatrix}
h_r \\
W^T \cdot h_r
\end{bmatrix}. \tag{10}
\]

A task of different reduction techniques is to find the projection matrices \( V \) and \( W \).

4. Approach for the reduction of rail vehicle parts

For the simulation of the rail vehicle multibody system with flexible bodies some preprocessing steps for obtaining a reduced flexible body are necessary. It is possible to make in FEM software, for example in Ansys [14 and 15]. Ansys software allows engineers to construct computer models or structures, machine components or system, apply operating loads and other design criteria and study physical responses [16 and 17]. This software also allows to reduce flexible bodies for import into MBS software.

The general process to integrate flexible bodies into the rail vehicle multibody system consists of several operations:

- setting up the finite element model,
- integrating the finite element model into the MBS software,
- setting up the MBS model of the rail vehicle.

It is needed to reduce the size (number of freedom) of FEM model before working with the MBS interface. For this it is needed to perform several operations:

- define the interface nodes. The MBS interacts with the FEM superelement on these nodes.
- connect the interface nodes with structure. In Ansys it is recommended to use the following elements types:
Once the FEM model of a part of the rail vehicle is reduced, the input files generation for MBS software is required. The file with the flexible body input data is necessary for including flexible bodies in the MBS software. After loading the file with the FEM input data into the MBS software it is possible to define the interaction between flexible body and MBS system by using joints, constraints or force elements which apply loads to the flexible body. The flexible body deformation is caused by these loads [15].

For the needs of a rail vehicle simulation the FEM model of a bogie frame of a freight wagon was created. This is the most commonly used bogie in the Central and Eastern Europe – the Y25 bogie [19, 20 and 21].

4.1 Reduction of the bogie frame

In this section the procedure of preparation of the flexible model of the bogie frame is described.

The CAD model of the bogie frame was imported into the FEM software. For the preparation of the FEM data the software

- **rigid body element** – interface nodes have independent DOFs, coupling nodes on the FE structure have dependent degrees of freedom, dependent nodes perform rigid body motion only and independent node (interface node) defines this rigid body motion. Element types of rigid body elements for Ansys FEM code are CE, CERIG, MPC184 and RBE 2.

- **force distributing constraints** – interface node has dependent DOFs, coupling nodes on the FE structure have independent DOFs, motion at the interface node is the weighted average of the motion of the coupling nodes, forces and moments at the reference node are distributed either as forces or moments at the coupling nodes. Element types of force distributing constraints for Ansys FEM code are TARGE 170+CONTA173, TARGE 170+CONTA174.

- define the coupling nodes as retained nodes,
- define the retained DOFs. This step is important in the reduction process for yielding accurate superelement matrices [18].
simulation allows better optimization of rail vehicles design as well as prevention of potential problems during their long-term operation.

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Ansys was used. In Ansys the FEM mesh and simulation of flexible body behaviour were performed [14 and 22]. For interface nodes the rigid body element was used. Interface nodes were defined in locations of joints, constraints and force elements (Fig. 3). In Fig. 3 interface nodes on the bogie frame created by using rigid elements are shown.

In Fig. 4 the example of interface node (INode) and relevant constraint equations created on the friction surface of the axle guide are shown. In this interface node friction forces between the bogie frame and the axlebox in the multibody system of the freight wagon are defined.

In the future research this FEM model of a bogie frame of a freight wagon will be used for importing into the MBS model of a freight wagon. After creation of the rail vehicle multibody system with flexible bodies analyses will be performed. These analyses will allow the better assessment of the freight wagon ride properties. It will be needed to consider that the freight wagon most commonly consists of two bogies. The created FEM model of a bogie will require much bigger computer capacity and will extend the computational time significantly.

5. Conclusion

The computer simulation is nowadays an integral part of the development process of rail vehicles. The flexible multibody dynamics is the field that encompasses several subjects such as rigid body dynamics, continuum mechanics, finite element method, numerical and computer method. Multibody simulations with flexible bodies enable more detailed analyses of a rail vehicle behaviour. The inclusion of the flexible body into MBS

References


[18] SIMPACK documentation 2014, user guide (part of the program package).


