RESONANT FREQUENCIES OF SMALL ARTERIAL SEGMENTS AS DETERMINING FACTORS FOR ESTIMATION OF TERMINAL SEGMENTS IN ELECTROMECHANICAL ARTERIAL TREE MODEL

A model of the nonsymmetrical arterial tree was analyzed. The arterial branching model was based on the principle of the electromechanical analogy. The nonsymmetrical arterial tree was divided to 7 arterial generations, starting from larger arteries towards small arteries and arterioles. For each generation the resonant frequency and the transfer function were calculated and there are presented the pressure simulation results for each arterial generation. Terminal segments were determined regarding to calculation of the resonant frequencies and the transfer functions and simulation results.

Keywords: Arterial system, electromechanical analogy, outflow condition, resonant frequency, transfer function.

1. Introduction

The arteries deliver a blood containing oxygen to the tissues of whole human body. Blood delivery starts at heart output connected to aortic arterial segments and continues towards the periphery. On the peripheral side of the arterial system small arteries play more than one role such as blood delivering pipelines. They serve for regulation of systemic vascular resistance by changing their radii which are modulated by innervated smooth muscles [1].

2. Material and methods

1.1 Arterial Branching

In this contribution we adopted the results of works which deal with arterial branching questions [2 - 6]. In most cases, the arterial branching consists of mother vessel which bifurcates into two daughter vessels. We can define the daughter vessel radii by Eq. (1) and Eq. (2):

\[ r_d = \alpha r_p \]  
\[ r_d = \beta r_p \]

where \( r_d \) are daughter vessel radii, \( r_p \) is radius of the mother vessel and \( \alpha, \beta \) are the coefficients modulating the daughter vessel radii [2] and [3].


Based on these assumptions, we used for our analysis and simulation the following adopted values of parameters [3 - 6]:

\[ \gamma = 0.41, \eta = 1.16, \xi = 2.76, \alpha = 0.91 \text{ and } \beta = 0.58. \]

1.2 Model of Arterial System Based on Electromechanical Analogy

The properties of the arterial system can be transformed by using appropriate electromechanical analogies to system consisting of discrete electrical elements such as resistor, inductor or capacitor [7 - 10]. A blood pressure is equivalent to electrical voltage and a blood flow can be modeled as electrical current. These equivalences can be used in the case when we divide the modeled arterial tree to the discrete arterial segments with defined length. We used for our simulation of arterial system behavior the model consisting of interconnection of single vessel segments (see Fig. 1).
are approximately equal to arteriole radii. We attempted to solve this problem in another way.

1.4 Transfer Function of Single Arterial Segments

At first, we made an analysis of transfer function of arterial segments placed in the single generations (see Fig. 2). Pressure or voltage transfer function of the single segments is defined by:

$$H_r(\omega) = \frac{\hat{U}_2}{\hat{U}_1}$$

where $\hat{U}_2$ is output harmonic voltage of the simplified arterial segment (see Fig. 3) and $\hat{U}_1$ is input harmonic voltage of this segment. According to the theory of electromechanical analogy the voltage is interconnected with the pressure in physiological segment described by its mechanical properties.

1.3 Small arteries branching implementation

Before implementation of small artery models to existing model of arterial system involving the large arteries, it is needed to determine a finishing radius of last arterial generation. Radii of arterioles vary from 5 to 30 µm [1]. Therefore the new generation of small arteries should be finished when their radii

$$R_n = 2nR_{ns} = \frac{1}{2^{n-1}}L_{ns}, n = 1, 2, 3, \ldots$$

$$R_0 = \frac{4n}{\pi r_0^2}, L_0 = \frac{\rho}{\pi r_0^2}$$

$$C = \frac{2\pi r_0^2}{k_r E_{\text{stat}}}$$

The meaning of the elements, which are described by (3), (4) and (5), is listed in Table 1 [7] and [8].

![Fig. 1 Vessel segment model](image)

**Elements and parameters of the vessel segments**

<table>
<thead>
<tr>
<th>Element/Parameter</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_n$</td>
<td>Specific resistance of the appropriate vessel segment</td>
<td>[Ω∙m⁻¹]</td>
</tr>
<tr>
<td>$L_n$</td>
<td>Specific inductance of the appropriate vessel segment</td>
<td>[H∙m⁻¹]</td>
</tr>
<tr>
<td>$C$</td>
<td>Specific capacitance of the appropriate vessel segment</td>
<td>[F∙m⁻¹]</td>
</tr>
<tr>
<td>$G$</td>
<td>Specific conductance of the appropriate vessel segment</td>
<td>[S∙m⁻¹]</td>
</tr>
<tr>
<td>$r_n$</td>
<td>Radius of the appropriate vessel segment</td>
<td>[mm]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Blood viscosity</td>
<td>[mPa∙s]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Blood density</td>
<td>[kg∙m⁻³]</td>
</tr>
<tr>
<td>$k_r$</td>
<td>Geometrical factor</td>
<td>[-]</td>
</tr>
<tr>
<td>$E_{\text{stat}}$</td>
<td>Static elastic modulus</td>
<td>[MPa]</td>
</tr>
<tr>
<td>$k_h$</td>
<td>Vessel wall thickness</td>
<td>[mm]</td>
</tr>
</tbody>
</table>

Then we can express the voltage $\hat{U}_2$ by using relation for voltage divider and we get the equation for the voltage transfer function $H_r(\omega)$ in the following form:

$$H_r(\omega) = \frac{1}{1 + Z_L Y_T}$$

where $Z_L$ is longitudinal impedance of arterial segment in Fig. 1 and $Y_T$ is transversal admittance of this segment.

Both parts of this basic arterial segment model can be expressed by (8) and (9):

$$Z_L = R_L + j\omega L_L + \frac{j\omega L_L (R_L + j\omega L_L)}{j\omega L_L + R_L + j\omega L_L}$$
\[ \hat{Y}_t = \frac{j\omega GC}{G + j\omega C} \]  

(9)

The meaning of the elements used in (8), (9) is listed in Table 1.

From the point of view of the electrical circuit theory, we can express impedance of the unloaded segment such as the serial connection of its longitudinal impedance (8) and reciprocal of its transversal admittance (9):

\[ \hat{Z} = Z_L + \frac{1}{\hat{Y}_t} \]  

(10)

The total impedance will be helpful at analysis of transfer characteristics of the single arterial segments placed in different generations. Concretely, there are resonant frequencies which are indicators of the arterial segment properties regarding to inductive and capacitive properties of the equivalent electrical circuit (Figs 1 - 3). The resonant frequency of the corresponding segment (see two-port circuit in Fig. 3) is determined by equaling of the imaginary part of the total impedance to zero:

\[ \text{Im}\{\hat{Z}\} = 0 \]  

(11)

Having expressed the single elements and edited the Eq. (6), we get a biquadratic equation:

\[ \text{Im}\{\hat{Z}\} = \omega L_1 + \frac{\omega^2 (L_1 L_3 + L_2 L_1)}{(\omega L_2 + \omega L_1)^2} + R_2^2 - \frac{1}{\omega C} = 0 \]  

(12)

which has 4 roots but only one of them is positive. Then for resonant frequency, we obtain the following relation:

\[ \omega = \sqrt{\frac{C^2 L_1 R_1^2 + 2C^2 L_1 L_3 R_1 + C^2 L_1 L_2 R_1 + 2C L_1 L_3 L_2 R_1 - 2CL_1 R_1 + 2CL_1 L_1 R_1 L_1 + L_1}{(2CL_1 L_1 + 2CL_1 L_1 + 4CL_1 L_1 + L_1)^2}} \]  

(13)

The values of the resonant frequencies of the arterial segments placed in the different generation are listed in Table 2.

### Table 2

<table>
<thead>
<tr>
<th>Generation</th>
<th>( \omega_r ) [rad·s(^{-1})]</th>
<th>( f_r ) [Hz]</th>
<th>( \omega_r ) [rad·s(^{-1})]</th>
<th>( f_r ) [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mother vessel</td>
<td>97.12</td>
<td>15.46</td>
<td>97.12</td>
<td>15.46</td>
</tr>
<tr>
<td>1.</td>
<td>111.98</td>
<td>17.82</td>
<td>221.94</td>
<td>35.326</td>
</tr>
<tr>
<td>2.</td>
<td>129.15</td>
<td>20.55</td>
<td>508.25</td>
<td>808.90</td>
</tr>
<tr>
<td>3.</td>
<td>149.01</td>
<td>23.72</td>
<td>1147.91</td>
<td>1826.96</td>
</tr>
<tr>
<td>4.</td>
<td>171.98</td>
<td>27.37</td>
<td>2526.53</td>
<td>4021.10</td>
</tr>
<tr>
<td>5.</td>
<td>198.53</td>
<td>31.60</td>
<td>5406.21</td>
<td>8604.25</td>
</tr>
<tr>
<td>6.</td>
<td>229.21</td>
<td>36.48</td>
<td>11319.73</td>
<td>18015.91</td>
</tr>
<tr>
<td>7.</td>
<td>264.64</td>
<td>42.12</td>
<td>23402.74</td>
<td>37246.62</td>
</tr>
</tbody>
</table>

### 3. Results

#### 3.1 Determination of outflow condition

As written in the text above, we used the calculation of the transfer functions for assessment of the properties of the single arterial segments. They can be used for determination of outflow conditions of the segments placed in the last generation of small arteries, concretely in our case in the 7th generation. We can list several approaches how to determine an element which finishes arterial tree. We can start with "windkessel" models and continue with simple resistive elements which represent resistive nature of capillary bed [11] and [12].

In our contribution we focus on determining outflow condition in the following way: in the real world the arterial tree consists of approximately 20 generations. It follows from that there are 2\(^{20}\) elements needed for modeling the last generation [2] and [11]. By using the calculation of the transfer functions we can estimate “cut-off” generation which means that we can determine when we can stop the generating of new elements and by that way it is possible to minimize computing costs of computer simulation. In our work we set the threshold for stopping generating of the new segments by observing the resonant properties of the single segments on the alpha and beta sides of the particular arterial generations.

As defined by (1) and (2) the radius of the new segment descends much slower on the alpha side than on the beta side of the modeled arterial tree. It follows from Fig. 4 which characterizes transfer functions of arterial tree segments (see Fig. 2) on the alpha side. The change in resonant frequency is much evident in single generations on the beta side of arterial bed (Fig. 5).
We performed the simulations of the model of arterial tree consisting of small arteries towards arterioles. This model was implemented to the existing model of arterial system based on electromechanical analogy. Comparing to the previous arterial tree model [7 - 10], where the arterial peripheral segments were ended by terminal segments at the level of large arteries, the arterial branching continues to the seventh arterial generation in our implementation. Input signal of this branching extension corresponds to the mother vessel curve (blue line – see Fig. 6). The radii of the segments placed on the left side of the modeled arterial tree are around 45 µm (see Fig. 7) which is value close to arteriolar tree border (arterioles have resistive character and serve for peripheral vascular resistance regulation [1]). The simulated pressure values (see Fig. 6) in the seventh generation correspond to pressure values measured in arteriolar tree [1].

4. Conclusion

We estimated the criteria for stopping of creation of the new elements in the next generation by evaluating the resonant peaks in single transfer functions. The segment placed on the beta side of modeled arterial tree has resonant frequency approximately equal to $10^4 \text{ rad}\cdot\text{s}^{-1}$. It is negligible from the point of view of possible heart rate frequencies which can exist in arterial system. Therefore, we can assume that the segments placed in the seventh generation have resistive character in the range of the possible heart rate frequencies and with them connected upper harmonic components which form the basic pulse wave propagating through arteries and they can be finished by simple resistive elements – resistors with values comparable to total resistance of the previous segment.

The modeling of the vascular system by using the electromechanical analogies can lead to better understanding of processes which occur in arteries. Also it could be possible to evaluate the degree of pathological changes in arterial system by using appropriate measurement methods (e.g. photoplethysmography) [13] and by reverse comparing of measured and simulated data. In this way the malformation such as arterial stenosis or aneurysm could be deeply studied.
References


