INFLUENCE OF THE TECHNICAL CONDITION OF THE RUNNING SYSTEM OF ARTICULATED BUSES ON STABILITY OF THEIR STRAIGHT-LINE MOTION

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Resume

Among the most important operational properties of articulated buses, ensuring their safety, it is necessary to note the course stability of traffic (CST), as often the loss of CST of a vehicle is accompanied by an accident. The parameters of the CST of a road train are closely related to the design of the running gear, in particular to the skew of the bus and trailer axles. The critical speed of rectilinear traffic was determined for different states of the articulated buses running system, which, in the absence of a bridge skew, was 32.1 m/s (116.6 km/h). The movement of the SSA is asymptotically stable. Under the action of perturbation, the stabilization time for lateral velocity is about 1 s and it increases to 2 s for the folding angle, but the amplitude of the backscatter of this angle is 10 times less than the initial perturbation, which indicates a stable asymptotic law of its change at small amplitude.

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1 Introduction

The modern development of public and freight transport leads to an increase in demand for large vehicles and city buses. This is justified by the arguments of economic savings, energy consumption conservation and pollution reduction due to the limitation of the number of vehicles and drivers needed to move a large number of goods and people. As a result, manufacturers of trucks and city buses are currently designing high-capacity structures in the form of articulated and multi-member vehicles. The use of hinged structures makes long vehicles flexible in use and allows mobility even in a chaotic (urban) environment [1]. We should also not forget about the corrosive effect of different aggressive chemical compounds on elements of vehicle metal structures, [2-6]. Improving the efficiency of modern freight and passenger trains involves increasing their movement speed and bringing the load level to the maximum.

Therefore, aims are to create the safe traffic conditions for these vehicles and improve their performance. In order to improve the situation in the field of passenger transportation, which has recently become very tense due to the increase in number of vehicles on the roads of megacities, there is an urgent need to improve it, namely - the unloading of city streets.

The appearance of new wheeled vehicles moving on specially designated lanes in large cities will be able to establish a system of passenger movement in the city district, which will primarily reduce the time of their movement from sleeping areas to the city center. This type of transport is the most promising at the moment because it carries passengers without delays during the peak and inter-peak periods. The basis of such a transport consists of two- and three-link articulated buses, the stability of which is affected by a large number of factors. Those are the design features of the layout scheme (load distribution on the axles of the bus and the towing link, the height of the center of mass, the control system, the type of supporting and coupling device and the construction of the running system). At the same time, the influence of each of the factors on the stability of the movement is ambiguous, since their operation is inextricably linked to the change in the characteristics of their structural elements, which cannot but affect the kinematic and stiffness properties of the chassis, in particular, the bus and trailer due to the skewing of their bridges, which can cause a deterioration in the traffic safety of public transport vehicles. However, the operation of articulated buses is inextricably linked with changes in the characteristics of their structural elements, which inevitably affects the kinematic and rigid properties of the chassis, in particular the bus and trailer and changes in distribution of reactions in the contact area of tires articulated buses with the road due to skew bridges, in particular. A large number of buses and articulated buses in operation have different technical conditions and, accordingly, different properties. In this case, there is a question of traffic safety of articulated buses with different technical conditions of the chassis of the bus and trailer.

It is obvious that even at the same technical condition of a running gear of the bus and the trailer at their manufacturing, after some period of operation it is possible to find out the various degree of wear of tires, elements of a suspension bracket of bridges of the bus and the trailer. It is known that the intensity of their wear is influenced by: the angles of the axle, the load on the wheel, lateral forces, tangential forces (traction and braking) and air pressure in the tires. Quantitatively, those factors are not identical for each of the axles of articulated buses. So, if there are different angles of installation of axles and various wear of a protector, it is possible to speak about the change of resistance to the lateral assignment of bridges and, as a result, parameters of maneuverability and stability of articulated buses, as a whole. The costs of maintenance and repair of the chassis are a significant part of the total cost of articulated buses, replacement of the entire set, in the case of extreme wear of one of the elements, is economically unreasonable. Therefore, a large number of articulated buses are operated with a chassis that has different technical conditions and, accordingly, different properties. In this case, there is a question about the stability and, as a consequence, the safety of the road train with the different technical conditions of the running gear.

However, the operation of articulated buses is inextricably linked with changes in the characteristics of their structural elements, which cannot but affect the kinematic and rigid properties of the chassis, including bus and trailer due to skew of their axles, which can lead to deterioration of articulated buses.

Given the fact that in Ukraine during the peak hours there is a significant overload of rolling stock, during its operation there may be deviations in the geometry of the chassis, which negatively affects the dynamics of articulated buses in different modes. Therefore, the question of taking into account the geometry of the axes of the links of articulated buses in the study of their stability is relevant.

2 Analysis of recent research

Among the most important operational properties of the road trains, including articulated buses, which ensure their safety, it is necessary to note the stability of traffic, as often the loss of stability of a vehicle is accompanied by an accident.

Maneuverability and stability of vehicles are in the focus of many researchers. Thus, in [1] a general method for modeling the kinematics of articulated vehicles, with different locations of the drive axle and different steering capabilities, is proposed. The main advantages of the method are its scalability (relative to a number of vehicle segments) and modularity, which allows direct (even automated) derivation of compact models that can be used in real time by relatively low energy computing devices.

In [6], a unified model is proposed, which includes the dynamics of rotation of any axis or articulation for any articulated bus. Comparative results by simulation prove that the approach applicable to any rotation scheme provides fairly high accuracy. In [7], is shown that the relationships between axles and between links of vehicles with several trailers can give rise to specific oscillating behavior of trailers during the vehicle maneuvers. The article shows that such oscillations are a direct consequence of properties of the vehicle kinematics, associated with the design of the traction coupling device. On the example of one pair of vehicle components, the patterns of its rotation are recognized, which then apply to a road train with any number of trailers. Numerical results, obtained for the kinematics of three uniaxial trailers, confirm the theoretical considerations that give a certain quantitative view of the problem.

In [8], a single-track dynamic model of articulated buses with a number of assumptions is considered, in particular, that the transverse accelerated front of the body and the angles of rotation of the steered wheels are insignificant. The kinematics and dynamics of articulated buses are considered in detail. The simulation results are functions of kinematic and dynamic parameters that allow determining the parameters of maneuverability and stability. In [9], a multivariate extension of the D2-IBC (Data-Driven - Inversion Based Control) method for determining the traffic parameters was considered and its application to control the stability of road trains was discussed in detail.

The equations of vertical and lateral dynamics of a road vehicle with 6 degrees of freedom are considered in [10], reduced to a matrix form. The motion of such a tool in the vertical and lateral planes is studied. It is shown that the developed method can be applied to analysis of the stability of the movement, in particular passenger trains. In [11], a simplified analysis of the maneuverability and stability of combinations of vehicles, such as a tractor in combination with one or two semi-trailers or a truck and a complete trailer. Car combinations are considered as linear dynamic systems with two degrees of freedom for each unit. The equations of motion are derived taking into account the effect of braking and acceleration and the characteristic equation for motion with constant speed is obtained. To improve stability, a new design of the saddle device for stabilization of the tractor-semi-trailer combinations is presented.

In [12], three-dimensional dynamic models of a car and a trailer were developed, based on which a dynamic model of a train was built. Based on the first-order approximation theory of ordinary differential equations and Hopf's bifurcation theory, the linear and nonlinear stability of each element and the road train as a whole, in rectilinear motion are studied. Numerical results show that for the nonlinear and linear models, the critical velocities differ little. At the same time, the costs of purchasing bridges and suspensions for articulated buses manufacturers are the highest, due to the fact that they mostly use air suspension with integrated longitudinal jet rods and a bridge beam.

From the above analysis of the approach to determining the parameters of maneuverability and stability of articulated buses, it follows that in each case it is necessary to develop a model that would meet the objectives of the study.

The technical condition of the trailer composition of road trains is influenced by a large number of factors, the degree of influence of which on different systems and components is not the same. The selection of the most significant factors, from the total number of defects and failures of components and parts, can be obtained using the mathematical and statistical method of expert evaluation (a priori ranking) [13]. According to this method, the least reliable components and systems are platform, drawbar, swivel and main frames and the brake system. These systems have the highest number of failures and damage to parts.

It is established that the suspension of the road train has the highest probability of failure. The skew of the bridges leads to uneven wear of the tire tread and in cases of difficult road conditions - a violation of controllability, stability of road trains. The main indicator of stability of movement is the critical speed V_{cr}.

In [13-15] a complex method of choosing the parameters of a road train was developed, based on mathematical models of its rectilinear and controlled motion, taking into account the angles of installation of the axles of a tractor and a semi-trailer. It is shown that the movement of a road train, consisting of a two-axle tractor and a three-axle semi-trailer, without the skew axles, is asymptotically stable. Increasing the skew of any axle of the semi-trailer and the second axle of the tractor in one direction or another leads to a decrease in the critical speed of the road train. The skew of the rear axle of the semi-trailer has the most significant effect. In this case, any combination of skew bridges causes a deterioration in the stability of rectilinear motion, due to fluctuations in the trailer link.

This technique can be applied to analysis of the stability of articulated buses.

The aim of the work is to study the influence of articulated bridges of articulated buses on their stability.

3 The results of the study

In the theory of the controlled movement of a road train, the following basic assumptions are considered to be quite justified in modeling [13]:

- the road train moves on a flat horizontal surface;
- unsprang mass is considered to be non-rolling;
- the controlling influence on the traffic parameters of the road train is carried out through the driven wheels of the traction vehicle, therefore the steering dynamics are not taken into account;
- the presence of gaps in the traction coupling device is not taken into account;
- the longitudinal speed of the road train is constant;
- the distance between the links of the road train does not change due to the smallness of the assembly angles;
- the constituent elements of the road train are absolutely solid bodies;
- the load on the road train is located so that the centers of mass of the towing vehicle and the trailer, as well as the traction-coupling device connecting them, are located in the vertical plane of symmetry of the link;
- the trajectory of the center of mass of the tractor is taken as the main trajectory;
- the interaction of the wheels with the supporting surface is expressed through the reaction of the road surface.

Since the stability of the road train in rectilinear motion is being considered, where the change in the normal reactions of the road bed to the wheels of one axis can be neglected, the plane-parallel movement of its links can be spread out when composing the cocks of the road train.

In [14], a system of equations describing the plane-parallel motion of an articulated bus was obtained. This system is written as:

- for the bus

$$m_1(\dot{v}_1 - \omega_1 v_1) = -X_1\cos\theta_1 - X_{1r}\cos\theta_{1r} - Y_1\sin\theta_1 - Y_{1r}\sin\theta_{1r} + (X_2 + X_{2r})\cos\psi_2 + (Y_2 + Y_{2r})\sin\psi_2 + X_S;$$
$$m_1(\dot{u}_1 - \omega_1 v_1) = -X_1\sin\theta_1 - X_{1r}\sin\theta_{1r} + Y_1\cos\theta_1 + Y_{1r}\cos\theta_{1r} - (X_2 + X_{2r})\sin\psi_2 + (Y_2 + Y_{2r})\cos\psi_2 + Y_S;$$
$$I_1\dot{\omega}_1 = (-X_1\sin\theta_1 + Y_1\cos\theta_1)(a_1 - \varepsilon\cos\theta_1) - (X_{1r}\cos\theta_{1r} - Y_{1r}\sin\theta_{1r})(a_1 + \varepsilon\cos\theta_1) + \quad (1)$$
$$+(X_1\cos\theta_1 + Y_1\sin\theta_1)(h_1 + \varepsilon\cos\theta_1) - (X_{1r}\cos\theta_{1r} + Y_{1r}\sin\theta_{1r})(h_1 + \varepsilon\cos\theta_{1r}) -$$
$$-X_2(h_2 - b_1\sin\psi_2) + X_{2r}(h_2 + b_1\sin\psi_2) - (Y_2 + Y_{2r})b_1\cos\psi_2 + Y_S(b_1 - c_1)a.$$

- for the trailer

$$m_2(\dot{v}_2 - \omega_2 u_2) = -(X_3 + X_{3r})\cos\psi_3 + (Y_3 + Y_{3r})\sin\psi_3 - X_S\cos\phi_1 - Y_S\sin\phi_1;$$
$$m_2(\dot{u}_2 + \omega_2 v_2) = (X_3 + X_{3r})\sin\psi_3 + (Y_3 + Y_{3r})\cos\psi_3 - X_S\sin\phi_1 - Y_S\cos\phi_1; \quad (2)$$
$$I_2\dot{\omega}_2 = (-X_S\sin\phi_1 + Y_S\cos\phi_1)a_2 + X_3(h_3 - b_2\sin\psi_3) - X_{3r}(h_3 + b_2\sin\psi_3) - (Y_3 + Y_{3r})b_2\cos\psi_3.$$

$$Y = \frac{k\cdot\delta}{\sqrt{1 + \left(\frac{k\cdot\delta}{\varphi\cdot G_\kappa}\right)}}, \quad (3)$$

where:
k - the coefficient of lateral deviation due to the tangent of the angle of inclination of the linear part of the curves;
φ - coefficient of adhesion between the tire and the support surface;
G_κ -the normal load on the wheel, N.

In the systems of Equations (1) and (2) the following notations are accepted: m_1, I_1; m_2, I_2 - respectively, the mass and central moments of inertia of individual modules are articulated buses; v_1, u_1 - longitudinal and lateral projection of the velocity of the center of mass of the i-th link on the axis of the movable coordinate system, invariably connected to the modules of articulated buses; ω_1 - angular velocity of the link; φ_1 - the angle of assembly of the road train; ψ_1 - skew angle i - axles articulated buses; X_i, X_{ir}, - longitudinal reactions of the road surface on the wheels i - their axles are defined as rolling resistance forces; Y_i, Y_{ir} - lateral reactions of the road surface on the wheels i - their axes are determined according to Rokar's axilmatics; i = 1,.., 3 - indices that belong to each of the axles of the road train; r - index indicating the starboard side of the links of the road train; a_1, b_1 - the distance from the center of mass of the bus to the front and rear axles, respectively; a_2, b_2 - the distance from the center of mass of the trailer to the point of coupling and the axis of the trailer, respectively; h_1 - the distance from the longitudinal axis of the bus to the axis of the pin; h_2 - the distance from the longitudinal axis of the bus to the axis of the rear wheel; h_3 - the distance from the longitudinal axis of the trailer to the axis of the wheel; ε - the distance from the axis of the pin to the axis of the front wheel; c_1 - distance from the center of mass of the bus to the coupling point with the trailer.

To solve the problem of stability of rectilinear motion of a road train it is necessary to make a system of equations of its perturbed motion. This system allows to determine the reactions of the links of the road train in a single disturbance (a sharp turn of the steering wheel of the tractor), as well as the critical speed of the road train. The theory of stability of motion of wheeled machines is based on the mathematical apparatus of research of differential equations developed by A.M. Lyapunov [13]. Steady motion, according to Lyapunov, is realized in a previously unknown region of initial perturbations, which is called the region of attraction of undisturbed motion. There is a problem of defining the boundaries of this area. The critical speed (CS) V_{cr} will be the speed at which at least one of the links of the road train loses stability. Stability means the ability of the road train to keep within the specified limits, regardless of speed and external forces, direction and orientation of the longitudinal and vertical axes in the absence of control effects from the driver [13, 15].

The system of equations of motion of a road train allows solutions $u_1 = 0$, $\omega_1 = 0$, $\omega_1 = 0$, ($\theta_1 = \theta_{1r} = 0$), which on the plane of the road corresponds to the movement of all points of the road train with speed v along the line θ = *const*.

The stability of the stationary solution $v_1^*, u_1^*, \omega_1^*, \varphi_1^*$ (in the case of rectilinear undisturbed motion, all these values, except for v, are equal to zero) is investigated next. At a constant speed of movement (v = const) and the linearity of the lateral removal forces, which are determined by the dependence of I. Rokar [4] Equation (3).

Let us investigate the stability of a stationary solution $v_1^*, u_1^*, \omega_1^*, \varphi_1^*$ (in the case of rectilinear undisturbed motion, all these values, except v, are zero). At a constant speed (v = *const)* and the linearity of the lateral deflection forces, determined by the dependence Rokar's $Y_i = k_i\delta_i$ [13], taking Equation (3) into account, one obtains:

$$\begin{aligned}&(m_1+m_2)\dot{u}+\dot{\omega}_1 m_2(b_1-c_1)+m_2a_2[(\dot{\omega}_1+\ddot{\phi}_1)\dot{\omega}_0-v_1\omega_1)=k_1\delta_1+k_1\delta_{1r}-m_1v_1\omega_1-\\&-(X_2+X_{2r})\sin\psi_2+(k_2\delta_2+k_2\delta_{2r})\cos\psi_2+(X_3+X_{3r})\sin\psi_3+(k_3\delta_3+k_3\delta_{3r})\cos\psi_3;\\&m_2[(\dot{u}-\dot{\omega}_1)(b_1-c_1)-a_2(\dot{\omega}_1-\ddot{\phi}_1)+v_1\omega_1]=I_1\dot{\omega}_1-(k_1\delta_1+k_1\delta_{1r})a_1-(X_1+X_{1r})(h_1+\varepsilon)+\\&+X_2(h_2-b_1\sin\psi_2)-X_{2r}(h_2+b_1\sin\psi_2)+(k_2\delta_2+k_2\delta_{2r})b_1\cos\psi_2+X_3(h_3-b_3\sin\psi_3)+\\&+X_{3r}(h_3+b_3\sin\psi_3)\cos\psi_3+(k_3\delta_3+k_3\delta_{3r})\sin\psi_3;\\&I_2(\dot{\omega}_1+\ddot{\phi}_1)-m_2a_2[\dot{u}+\dot{\omega}_1(b_1-c_1)]-a_2(\dot{\omega}_1+\ddot{\phi}_1)+v_1\omega_1]=-(X_3+X_{3r})b_3\sin\psi_3X_3-\\&-(k_3\delta_3+k_3\delta_{3r})b_3\cos\psi_3+X_3(h_3-b_3\sin\psi_3)-X_{3r}\sin\psi_3(h_3+b_3\sin\psi_3).\end{aligned}\tag{4}$$

After solving these equations concerning the higher orderderivatives, one obtains:

$$\dot{u}_1=-\frac{m_2B0-m_2(b_1-c)+A0m_2C^2I_1+\\+m_2I_2a_2^2A0+A0+A0I_1I_2}{m_2C^2m_1I_1+m_2I_2B^2m+m_2I_1I_2+m_1I_1I_2},\tag{5}$$

$$\dot{\omega}_1=-\frac{m_1m_2(b_1-c)B0+m_2I_2B0+\\+m_2I_2A0+I_1m_1B0}{m_2C^2m_1I_1+m_2I_2B^2m_1+m_2I_1I_2+m_1I_1I_2},\tag{6}$$

$$\ddot{\varphi}_1=-\frac{m_2(b_1-c)m_1B0+m_2B^2m_1C0+\\+m_2C^2m_1B0+I_2m_2B0}{m_2C^2m_1I_1+m_2I_2B^2m_1+m_2I_1I_2+m_1I_1I_2},\tag{7}$$

where:

$$\begin{aligned}&A0=2\Big(\frac{A2}{v_1}+(m_1+m_2)v_1\Big)\omega_1+\frac{A1u}{v_1}+A3\varphi_1-\\&-\frac{A4\dot{\varphi}_1}{v_1};\\&B0=2\Big(\frac{B2}{v_1}+(B+2C)v_1\Big)\omega_1+\frac{B1u_1}{v_1}+B3\varphi_1+\\&+\frac{B4\dot{\varphi}_1}{v_1};\\&C0\Big(m_2v_1C+\frac{C2}{v_1}\Big)\omega_1+\frac{C1u_1}{v_1}+C1\varphi_1+\frac{C3\dot{\varphi}_1}{v_1};\\&A1=2(k_1+k_2+k_3);\\&A2=2(k_1a_1-k_2b_1-k_3(B+C-c);\\&A3=2k_3;A4=2k_3a_2;\\&B1=2(k_1a_1-k_2b_1-k_3(a_2+B);\\&B2=2(k_1a_1^2-k_2b_1^2+k_3(B+C-c)(B+a_2);\\&B3=-2k_3(a_2+B);B4=-2k_3a_2(a_2+B);\\&C1=2k_3a_2;C2=2[k_3a_2(-B-C+c);\\&C3=2k_3a_2^2\cdot B=b+c;C=c+a_2\end{aligned}$$

The system of Equations (4) is reduced to the vector-matrix form:

$$\|a_{ij}\|_1^3\cdot\begin{vmatrix}\dot{u}_1\\\dot{\omega}_1\\\ddot{\varphi}_1\end{vmatrix}+\|b_{ij}\|_{3,4}\cdot\begin{vmatrix}u_1\\\omega_1\\\varphi_1\\\dot{\varphi}_1\end{vmatrix}=0.\tag{8}$$

Then, for the partial solution of the system reduced to the vector-matrix form is sought, if and only if λ is the root of the characteristic equation

$$D(\lambda)=A_0\lambda^4+A_1\lambda^3+A_2\lambda^2+A_3\lambda+A_4=0.\tag{9}$$

Matrix of a characteristic equation, in the general form, is:

$$\begin{vmatrix}a_{11}\lambda+b_{11}&a_{12}\lambda+b_{12}&a_{13}\lambda^2+b_{13}\lambda+b_{14}\\a_{21}\lambda+b_{21}&a_{22}\lambda+b_{22}&a_{23}\lambda^2+b_{23}\lambda+b_{24}\\a_{31}\lambda+b_{31}&a_{32}\lambda+b_{32}&a_{33}\lambda^2+b_{33}\lambda+b_{34}\end{vmatrix}=\\=\sum_{i=0}^{n=4}A_i\lambda^{n-i}=0,\tag{10}$$

where and b_{ij} - the corresponding coefficients, which depend on the geometric parameters of the road train, are obtained analytically in the program Maple 14.

According to the Rauss-Hurwitz stability criterion, a necessary but insufficient condition for stability is that all coefficients Ai be positive. The system will be stable if the determinant and its minors are positive. Analysis of the roots of the characteristic equation can characterize the state of the system.

In the general case, the following values of the roots of the characteristic equation are possible: λ is a real and a positive value - the system is unstable, the motion will be unstable; λ - real and negative value - the system eventually returns to a stable position. If the coefficient λ is a complex number, then its positive real part indicates the presence of increasing oscillations and the negative real part indicates the presence of attenuating oscillations.

Determinants of the Hurwitz characteristic Equation (9): first Δ_1 - is responsible for the presence of positive valid roots and the third Δ_3 - in the presence of a positive real part of the imaginary complex connected roots. From Equation (9) one obtains the factors on which the critical velocity depends:

$$v_{kp}=f(m_1,m_2,a_1,c,k_1,k_2,k_3,\psi_1,\psi_2,\psi_3,\varphi).\tag{11}$$

The uniform rectilinear motion can be analyzed by use of Equation (11), namely, to determine the value of the critical velocity of rectilinear motion of the road train and to identify the nature of the influence of factors, including the angles of the bus and trailer bridges [13].

The structure of the characteristic determinant in Equation (9) of the system in Equation (10), as well as the expressions of its coefficients a_{ij}, b_{ij} so cumbersome in an analytical form that in the following derivations, the general expressions of coefficients A_0, A_1, A_2, A_3, A_4, and the determinant Δ_3 of the characteristic equation are used Therefore, the required coefficients were calculated numerically using the computer simulation in Maple 14.

In the general case, the values of operational and design parameters of the road train, at which the determinant of the system A_4 = 0, are called critical and when $\Delta_3 = 0$ - fluttering. When considering the dynamics of the road train, taking into account the angles of the bridges, the main factor determining the stability of rectilinear traffic ν is the course speed of the vehicle.

Typical situations are:

$$A_4 = 0 \Rightarrow \nu = \nu_{cr}; A_4 > 0 \Rightarrow \nu < \nu_{cr}; \\ A_4 < 0 \Rightarrow \nu > \nu_{cr}, \quad (12)$$

where ν - speed of the road train; ν_{cr} - critical speed of the road train;

$$\Delta_3 = 0 \Rightarrow \nu = \nu_0; \Delta_3 < 0 \Rightarrow \nu > \nu_0; \\ \Delta_3 > 0 \Rightarrow \nu < \nu_0, \quad (13)$$

where ν_0 - maximum speed of oscillating instability of the road train.

Since ν_{cr} and ν_0 are functions of the parameters of the road train, then in the space of these parameters of the equation $\nu = \nu_{cr}$ and $\nu = \nu_0$ determine the hypersurfaces on which the characteristic equation has one zero and a pair of complex roots.

The first equations of Equations (12) and (13) can be written as functions of the speed of the road train:

$$A_4 = f(\nu_{cr}, other\ factor) \\ \Delta_3 = f(\nu_0, other\ factor), \quad (14)$$

Thus, there are two characteristic values of the speed of the road train $\nu = \nu_{cr}$ i $\nu = \nu_0$, which can be obtained from Equations (14). However, their reduction to an explicit form due to the large size and number of input parameters is generally an unsolvable problem and does not allow the use of purely analytical research methods.

Those dependencies can be obtained using numerical methods of computer simulation. Since there are no explicit expressions for the solutions of Equations (14), it is necessary to find the dependences $\nu_{cr} = f(A_4)$ i $\nu_0 = f(\Delta_3)$ use the interval method [13]. This method allows to calculate any implicit dependencies. In the case of ν_{cr} and ν_0 for $A_4 = 0$ and $\Delta_3 = 0$ accordingly one has:

$$A_4 = (\nu, X_i) = 0; \\ \Delta_3 = (\nu, X_i) = 0; \\ i = 1 \ldots n, \quad (15)$$

where ν - current value of road train speed; X_i - road train parameters; n - number of parameters.

To find the critical velocity and the limiting velocity of the oscillating instability of the road train movement, one proceeds as follows, [13]. The initial velocity, at which condition in Equation (15) is satisfied, is chosen. Increasing the current speed of the road train ν from ν_{min} to ν_{max} by magnitude $\nu = (\nu_{max} - \nu_{min})/n$, in the range of ν_{min} to ν_{max} the conditions are checked at each step, $A_4 = 0$ or $\Delta_3 = 0$. If any of these conditions are not met, the current value ν is assigned to the corresponding of the extreme speeds ν_{cr} or ν_0. Thus, it is possible to obtain velocity dependences ν_{cr} and ν_0 from any of the parameters of the road train.

According to the selected layout of the road train, the critical speed is 29.8 m/s or 107.3 km/h.

As an example, Table 1 shows the values of the roots of the characteristic equation, which can determine the type of stability or instability of the road train.

As follows from Table 1, the first positive root appeared at a speed of 28 m/s, which can be considered the rate of oscillation instability and articulated buses. This speed is 6.28% less than the critical speed of rectilinear movement of the road train (v_{cs} = 29.8 m/s) and in the stability calculations, it is necessary to take this speed since when driving a vehicle in specially allocated traffic lanes (BRT system) movement at a speed of 25-30 m/s is allowed.

Figure 1 shows the effect of skew of the axle of trailer 1 and bus 2 articulated buses on the critical speed of rectilinear motion, provided that when determining the effect of skew of one of the axles, the skew of the other is absent.

Increasing the skew of any bridge in any direction reduces the critical speed of the road train. The skew of the trailer axle has the most significant effect. Thus, the skew of this bridge by 1 º reduces the critical speed of the road train by 4.6% and by 30 - by 15.2%, Further

***Table 1** The roots of the characteristic equation*

v_a, m/s	λ_1	λ_2	λ_3	λ_4
25.6	-26.15539534	-9.405998162	-1.250289683-3.017103424*I	-1.250289683 +3.017103424*I)
25.7	-23.53052177	-8.536488196	-0.6186081350-3.304419783*I	-0.6186081350 +3.304419783*I)
25.8	-21.51410109	-7.890250409	0.070258412e-1-3.454617665*I	0.070258412e-1 +3.454617665*I)
25.9	-19.91789858	-7.396702500	0.0356100428-3.531547024*I	0.0356100428 +3.531547024*I)
26.0	-18.62333016	-7.011559127	0.7068169155-3.565900596*I	0.7068169155 +3.565900596*I)

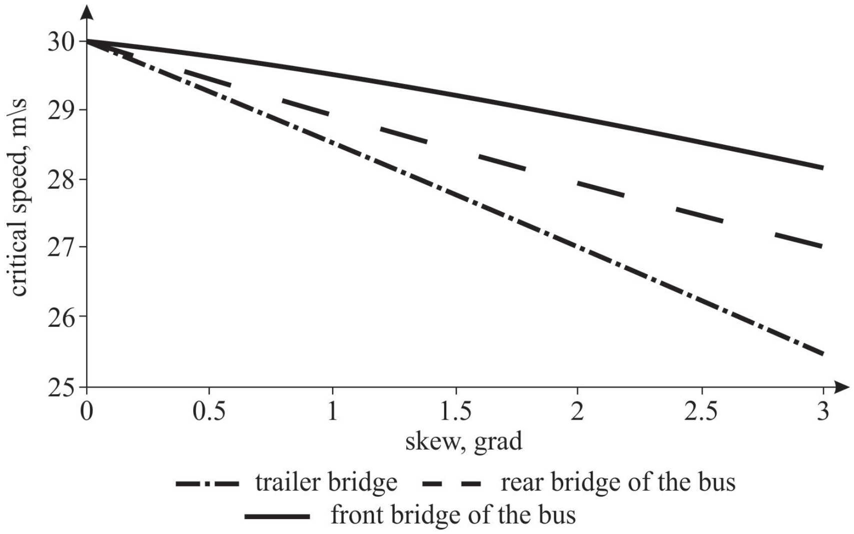


Figure 1 *Variation the critical speed of rectilinear articulated buses in terms of their bridges skew*

increase in the skew of more than 3 º leads to oscillating instability of the road train.

The combination of skew of different bridges articulated buses is a function of three variables, i.e. graphically $\nu_{cr} = f(\psi_1, \psi_2, \psi_3)$, impossible to represent.

The change in the critical speed of rectilinear motion, as a result of the simultaneous skew of different bridges, is considered only for typical schemes of bridge installation articulated buses, separate for articulated buses without skew axles, with an one-sided skew of all the axles and multi-sided skew of the bus and trailer drive axle, using the phase portrait method. For these bridge skew variants, the critical velocity was:

- in the absence of skew of the axles - 32.12 m \ s;
- for the one-sided skew of the bridges -25.61 m/s;
- for the one-sided skew of the rear axle of the bus and trailer - 26.95 m/s.

The method of phase portraits has long been used in mathematics and mechanics to analyze the behavior of systems described by systems of differential equations that have no analytical solution. Phase portrait is a graphical representation of ratios of the system state parameters that vary with time. Each point of the phase portrait (point of phase space) characterizes the state of the system at a particular time and the movement of the point along the line of the graph (phase trajectory) characterizes the change in the state of the system over time.

For analysis of the articulated buses movement stability, taking into account the skew of the bridges, phase portraits of lateral speed from the time are of interest $[t, u_1(t)]$, angular velocity from time to time $[t, \omega_1(t)]$, folding angle from time to time $[t, \varphi_1(t)]$, folding angle speed $[\varphi_1(t), \dot{\varphi}_1(t)]$, coordinates x and y of the center of mass of the bus from time to time $[x(t), y(t)]$.

Among the considered portraits the most informative are the portraits of the lateral velocity of the folding angle.

In the absence of skew of the road train bridges, it is established that the nature of the flow of both phase variable parameters of the road train is asymptotic. Figure 2 shows the dependences of phase variables on time at a speed of 32 m/s

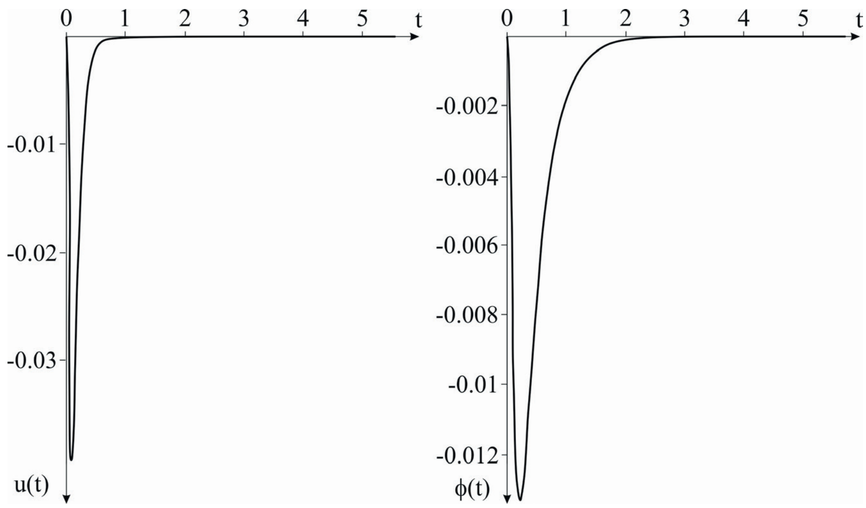
The results of the research have shown that the movement of articulated buses: without skew bridges is asymptotically stable. For the lateral velocity, the stabilization time is about 1 s, however, for the folding angle increases to 2 s, but the amplitude of the feedback is 10 times smaller than the initial perturbation (Figure 2b). The lateral displacement is initially about 5 cm per 12.5 m of the distance traveled, which is then eliminated.

Consider the three main options for skew bridges of road trains. The results for each option are presented in Figures 3 - 4. The sequence of drawings corresponds to the variant of skew of axles of articulated buses:

a) skew of the leading bridge of the bus,
b) unilateral skew of the leading bridge of the bus and the bridge of the trailer,
c) versatile skew axles of the semi-trailer.

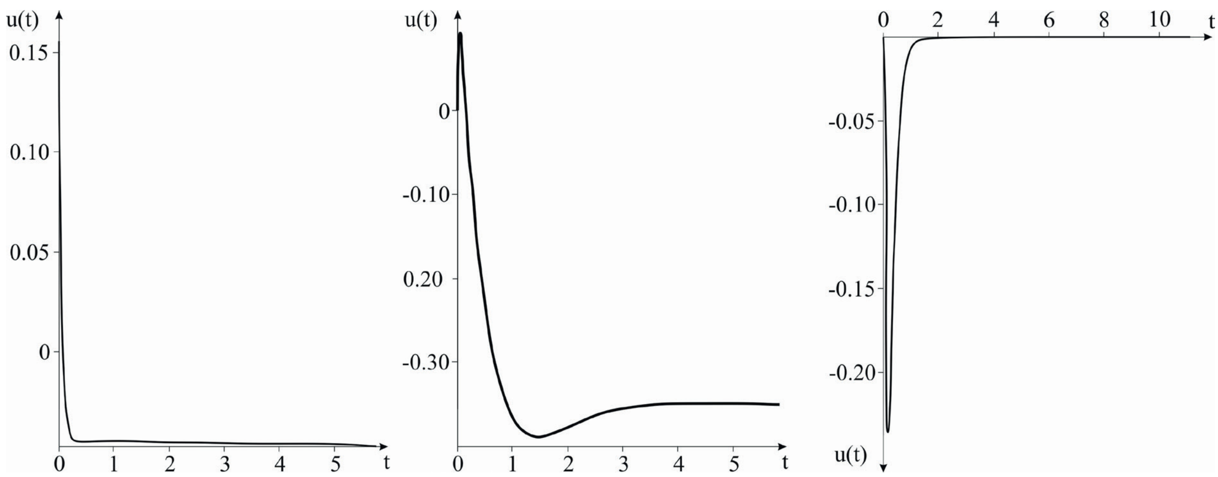
The motion parameters of articulated buses largely depend on their speed. If there is a skew of the leading axis of the bus, the critical speed of rectilinear motion is reduced to 30.2 m/s (without skew - 32.12 m/s). In addition, even at the correct angles of the bridges articulated buses there is a small lateral speed of the center of mass of the bus (Figure 3a). As a result, the driver must take corrective action by turning the steering wheel to the side by the amount of skew. Depending on the speed of movement, appearance of oscillating instability of the bus and, as a consequence, the occurrence of oscillations of the trailer link.

The results of the articulated buses, with a one-sided skew of the bridges, stability study are shown in Figure 4b. The critical speed for this mode is 25.61 m/s. There are modes of steady motion and oscillatory steady with a stabilization time of 2 s and unstable. When moving to critical speed, with a one-sided skew of axles



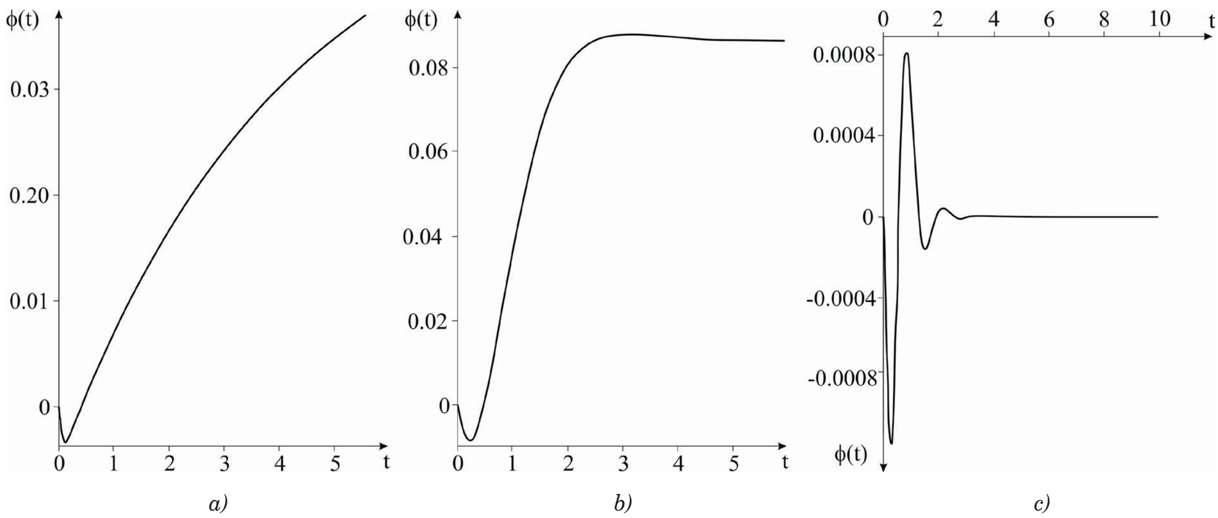
a) b)

Figure 2 *Variation of the lateral velocity a) and folding angle b) during the transition process, ν =32 m/s*



a) b) c)

Figure 3 *Variation of the lateral velocity during the transition process: a) ν =30.2 m/s, b) υ=25.61 m/s, c) ν =26.95 m/s*



a) b) c)

Figure 4 *Variation of the angle of addition during the transition process:*
a) ν =30,2 m/s, b) ν =25.61 m/s, c) ν =26.95 m/s

articulated buses there is a lateral speed of the center of mass of the bus, which exceeds the lateral speed for 4 times, in the case of a skew of the driving axle of the bus only (phase portrait of the folding angle in Figure 4b). When the maximum speed of the movement is = 25.61 m/s, the articulated buses become unstable. The trajectory of the center of mass of the bus with increasing speed is first slightly distorted, then there is movement at a significant angle to the initial direction and with further increase in speed, there is a loss of stability.

As in the case of skew of the bus drive axle, this type of skew can be overcome if the driver makes corrective actions with the steering wheel, but given the length of articulated buses, this will significantly increase its overall lane. Due to the divergence of the direction of movement of the axles, articulated buses on the plane of rolling of the wheels will have the lateral forces, which will lead to increased wear of the suspension elements and tire tread.

Results of the articulated buses, with a versatile skew of the bridges, stability study, are shown in Figure 4c. The critical speed for this case is 26.95 m/s. The zone of oscillating steady motion expands, the stabilization time increases to 3.5 to 4 s, which is quite long. In this case, the corrective actions of the driver may be to blame for the appearance of undamped oscillations, due to the long stabilization time of the system. The amplitude of oscillations exceeds the initial perturbation on the system by 1.7 to 2.5 times. The frequency of oscillations of phase variables increases to 1.5 to 3 Hz. (Figure 4 c). Phase portraits and the trajectory of the center of mass indicate an oscillating steady motion with damped oscillations, but with a large transition period.

Due to the misalignment of the axles of the semi-trailer, there is a force that forms the moment of rotation of the semi-trailer relative to its center of mass. As a result, the rolling resistance of the semi-trailer wheels increases, the wear of the tires and the wear of the suspension elements increase. All these indicate the need for periodic control of the angles of installation of articulated buses.

4 Conclusions

The critical speed of the rectilinear articulated bus has been determined, which in the absence of skew of its bridges was 32.1 m/s (116.6 km/h). In the case of skew of the leading axis of the bus, the critical speed of rectilinear movement decreases to 29.95 m/s. Increasing the skew of any bridge articulated buses, in one or the other direction, reduces the critical speed of the road train. The skew of the trailer axle has the most significant effect. Thus, increasing the skew of the trailer bridge by 1º reduces articulated buses; road trains by 4.6% and by 30 - by 15.2%. Further increase in skew over 3º leads to oscillating instability of articulated buses. It is shown that the movement of articulated buses; without skew bridges is asymptotic stable. Under the action of perturbation, the stabilization time for the lateral velocity is about 1 s and increases to 2 s for the folding angle, but the amplitude of the backscatter of this angle is 10 times smaller than the initial perturbation, which indicates a stable asymptotic law of its change at a small amplitude. The lateral displacement of the trailer is about 5 cm per 12.5 m of the distance traveled (part of a degree), which is then eliminated. This indicates the need to regularly check the angles of installation of bridges articulated buses.

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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