COMMUNICATIONS

Dear reader,

Owing to its fifty five years long history, the University of Zilina occupies an outstanding place within the Slovakian educational system not only with the number of students, offer of accredited study programs but, particularly with its significant research and international activities based on extensive cooperation with companies an institutions both in Slovakia and abroad. The University currently houses seven faculties and seven institutes and employs 650 university teachers. The total number of employees is 1450. There are currently about 12 thousand students enrolled in all forms of studies. About 45 thousands graduates have graduated from the University of Zilina, 1600 of them were from abroad.

In our quickly transforming world higher education actually plays an ever more central role by creating tools and technologies driving economic development and meeting societal challenges. We have opportunities to educate future generation who could connect ideas in unexpected ways, integrate technological innovations into daily life, and apply new knowledge to make a positive impact in the world. At the same time, we provide students with a deeper understanding of the role of science and technology in today's world. Numerous student exchange programs and research collaborations with foreign universities are every day part of the life at the University of Zilina. We are particularly interested in utilizing the new areas of autonomy opened up in order to comparatively strengthen the image and capabilities in research and education at home and abroad. The University is willing and determined to meet challenges posed by the rapidly changing social and economic environment.

In addition to the quantity and quality of learning opportunities, research plays also a key role in determining a university's reputation and acceptance both at home and abroad. As a research institution, the University must make its contribution to scientific and intellectual progress and educate the next generation of scientists. The University of Zilina can justifiably claim that not only in this country but also throughout Europe, it holds a highly respected position in several disciplines. This is also showed by the fact that the faculties are very successful in obtaining research grants. The results of the Framework Programs of the European Union present a similarly good picture. The University received project-related grants and created unusual structural units.

While noteworthy scientific results are usually not the only goal, interest is placed on promoting close cooperation between business, public institutions and national as well as international partners in order to transfer new knowledge and technologies in industry.

The research projects mentioned here as well as the papers published in this issue of the journal provide only a brief look of the work done in the individual departments over the past years.

Jan Bujnak
Rector

Jaroslav Janacek – Ludmila Janosikova *

COMPUTABILITY OF THE EMERGENCY SERVICE SYSTEM DESIGN PROBLEM

The paper deals with emergency medical system design using methods of mathematical programming. The problem consists in optimal location of stations, where ambulance vehicles should be placed. Several possible objective functions are discussed and the relevant mathematical programming models are presented. The comparison of the solutions is reported based on the computational experiments in the conditions of the Slovak Republic.

1. Introduction

The medical emergency system design is a crucial task for each responsible designer due to the interaction of two opposite demands on the system performance. On the one hand the designer is forced not to exceed a given number of located facilities – ambulance vehicles and to solve a large facility location problem. On the other hand, he must ensure the accessibility of the service for potential patients. This accessibility is usually given by a fixed time limit, in which some ambulance vehicle should reach an arbitrary located potential patient [2], [12], [13].

This last demand is hard to meet, because of random travel time on a real road network. In addition, when an accident occurs, an ambulance starts its trip to the accident location to provide the service, which consists of first aid to causalities and their transportation to a hospital. Within this service, the facility (ambulance vehicle) cannot perform any service of other demands. It means that if some other accident occurs simultaneously in the area of this vehicle, then some other ambulance must serve it, or service of the later accident must be considerably postponed. This way, the service system works like a queuing system [11]. Under these circumstances, the access condition cannot be fulfilled absolutely, but only with some probability. Due to the impossibility to include means of the queuing theory into analytical models of the location problem, there are used various surrogate criteria such as an average or total travel time from the ambulance location to potential patients, which belong to the ambulance servicing area [6], [7], [8]. Another type of criterion (covering criterion) is that a maximal travel time from the nearest ambulance location to a customer should not exceed a given value. Designers face the above-mentioned ambulance occupation by using so called double coverage criterion, which is formulated so that the number of potential patients, which lie within a given time radius of two or more facilities should be maximal [2].

Usage of each of these criteria leads to a particular model of mathematical programming [5]. An exact method applied to the particular model has its specific demands for computational time and memory. In the next sections, we present an overview of these criteria, report about preliminary computational experiments and perform a comparison of them.

2. The emergency service system design problem and quality criteria formalization

Within the scope of this paper we confine ourselves to the problem, in which a medical emergency service system is designed. In contrast to the private service systems, the objective of this sort of public service system should stress equity of a "customer" in access to the provided service.

The emergency service system design belongs to the family of location problems [1], in which it must be decided on centre locations, where ambulance vehicles should be placed, because an effective satisfaction of the potential patient demands is possible only if the corresponding service provider concentrates its sources at several places of the served area and provides the service from these places only. The served area consists of dwelling places placed in nodes of a road network. These dwelling places form a finite set J. The number of inhabitants of dwelling place $j \in J$ will be denoted as b_j. The emergency service system design problem can be formulated as a decision about location of at most p emergency centers at some places from a larger set I of possible center locations so that the value of chosen criterion is minimal. The question, which must be answered first, is: "How to estimate the time of access to a customer?" Let j be customer's location and i be a centre of the service provider. Both the locations are nodes of a road network, which consists of links and nodes. Based on the link quality, each link belongs to a class from a finite classification

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system. In accordance to this system, an average speed is assigned to each link. This way, an estimation of the necessary traversing time for each link can be obtained from the link length and the average speed corresponding to the link class. Using this time instead of the link length, the accessibility time t_{ij} can be enumerated as the time length of the shortest path in the network connecting i and j. Time $t_{ij}(v)$ is a function of vector $v = \langle v_1, v_2, ..., v_r \rangle$ of the speeds, which corresponds to the particular link classes. Nevertheless, the average speeds are not constant, but they depend on weather, traffic volume and other dynamically changing conditions. Considering this condition variability, no system design ensures full satisfaction of the estimated time and each further developed criterion is enumerated in accordance to a given speed scenario v. Let $i(v, j)$ represent the located centre, which is the time-nearest one to j considering the link speeds given by v. Further, let $I_1 \subseteq I$ denotes the set of places, in which an ambulance vehicle is located.

After these preliminaries, we formulate the particular criteria. The first family of "allocation criteria" is represented by the total travel time from the ambulance location to potential patients. This criterion can be described by the following expression:

$$\sum_{j \in J} b_j t_{i(v, j), j}(v) \tag{1}$$

This criterion doesn't reflect equity of a "customer" in access to the provided services at all. The original requirement of the concerned public is that each inhabited place must be reachable within time T^{max} from at least one service centre from set I_1. The next criterion, which also belongs to the family of "allocation criteria" takes into account the size of affliction of potential patients, which are out of the time limit:

$$\sum_{\substack{j \in J \\ t_{i(v, j), j}(v) > T^{max}}} b_j \left(t_{i(v, j), j}(v) - T^{max} \right) \tag{2}$$

The second family of "covering criteria" [2] is represented by the criterion, which simply counts the potential patients, which are out of the time limit:

$$\sum_{\substack{j \in J \\ t_{i(v, j), j}(v) > T^{max}}} b_j \tag{3}$$

The third family of "double coverage criteria" [2] is represented by the criterion, which counts the potential patients not covered at least from two ambulance locations. It is said that a patient is covered if the distance to the nearest ambulance station is less than a given limit T^{max}. Let $s(v, j)$ represent the second time-nearest station to j considering the link speeds given by v; $s(v, j)$ belongs to the set $I_1 \subseteq I$ of places, in which an ambulance vehicle is located. The formulation of the last criterion can be as follows:

$$\sum_{\substack{j \in J \\ t_{s(v, j), j}(v) > T^{max}}} b_j \tag{4}$$

The expressions (1)-(4) are to be minimized subject to the constraint that the number of located facilities must not exceed the given number p.

The next generalization of these criteria may issue from observation of possible scenarios of the vehicle speeds. The family of the scenarios constitutes finite set V of possible speed vectors v_q, $q = 1, ..., m$ and each scenario may be weighted by coefficient h_q. The weights can be set proportionally to the empirical frequencies or arbitrary else to reflect the necessity to keep the accessibility condition at a sensible level. The further generalization can be obtained by optimising a linear combination of criteria, where particular criteria are weighted according to their importance.

3. Models and solving techniques for the emergency service system design problem

The mathematical programming approach to the emergency system design comes out from the assumption that the ambulance vehicles are allowed to be located only at some places from the finite set I of possible locations. The decision on placing or not placing an ambulance vehicle must be done for each candidate location $i \in I$. This decision can be modelled by the variable y_i, which takes the value 1 if a vehicle is placed at location i and it takes the value 0 otherwise. The case, in which it is possible to place more than one vehicle at one location, can be rearranged to the considered zero-one decision problem by duplication or triplication of the relevant locations.

The emergency system design problem with the criterion (1) cannot be described only by the location variables y_i, due to the fact that the individual contribution to the objective function value depends on the distance between the customer and the nearest located ambulance. To be able to describe this sort of relations, we introduce zero-one variables z_{ij} for each pair $\langle i, j \rangle$ of a possible location and a customer. Using these variables, the assignment of each customer to some ambulance location can be easily described. If we denote $c_{ij} = b_j t_{ij}(v)$, then the following model describes the emergency system design problem with the criterion (1).

$$\text{Minimize} \quad \sum_{i \in I} \sum_{j \in J} c_{ij} z_{ij} \tag{5}$$

$$\text{Subject to} \quad \sum_{i \in I} z_{ij} = 1 \qquad \text{for } j \in J \tag{6}$$

$$z_{ij} \leq y_i \qquad \text{for } i \in I, j \in J \tag{7}$$

$$\sum_{i \in I} y_i \leq p \tag{8}$$

$$y_i \in \{0,1\} \qquad \text{for } i \in I \tag{9}$$

$$z_{ij} \in \{0,1\} \qquad \text{for } i \in I, j \in J \tag{10}$$

The expression (5) corresponds to the sum of the real access times multiplied by numbers of afflicted inhabitants. The constraints (6) ensure that each dwelling place (customer) is assigned to the

exactly one of possible locations. The constraints (7) are so called binding constraints, which force the variable y_i take the value 1, whenever a customer is assigned to location i. The constraint (8) puts the limit p on the number of located vehicles.

The model (5)-(10) describes also the emergency system design problem with the criterion (2). It is sufficient to denote $c_{ij} = b_j(t_{ij}(v) - T^{max})$, if $t_{ij}(v) > T^{max}$ and $c_{ij} = 0$ otherwise.

The problems connected with criterion (3) can be modelled using a set of the auxiliary zero-one variables x_j, which express by the values 1 or 0, whether the demand of customer $j \in J$ is or is not satisfied. To be able to recognize, whether customer j is or is not accessible from location i, we introduce zero-one constant a_{ij} for each pair $\langle i, j \rangle \in I \times J$. The constant a_{ij} is equal to 1 if and only if customer j can be reached from location i in the access time T^{max}, i.e. $t_{ij}(v) \leq T^{max}$. Otherwise, the constant a_{ij} is equal to 0. Then we can formulate the problem as:

$$\text{Minimize} \quad \sum_{j \in J} b_j(1 - x_j) \tag{11}$$

$$\text{Subject to} \quad \sum_{i \in I} a_{ij} y_i \geq x_j \qquad \text{for } j \in J \tag{12}$$

$$\sum_{i \in I} y_i \leq p \tag{13}$$

$$y_i \in \{0,1\} \qquad \text{for } i \in I \tag{14}$$

$$x_j \in \{0,1\} \qquad \text{for } j \in J \tag{15}$$

The objective function (11) gives the volume of uncovered demands. The constraints (12) ensure that the variables x_j are allowed to take the value 1, if and only if there is at least one ambulance vehicle located in the access time T^{max} from the customer location j. The constraint (13) puts the limit p on the number of located vehicles [10].

The model (11)-(15) can also model the problem with criterion (4), in which the number of double covered demands should be maximized. Nevertheless the constraints (12) must be replaced by constraints (16):

$$\sum_{i \in I} a_{ij} y_i \geq 1 + x_j \quad \text{for } j \in J \tag{16}$$

Concerning the solving technique for the problems described by the above presented models, it can be noted that all of them belong to the family of integer programming problems, more precisely zero-one integer programming problems and can be theoretically solved by any commercial solver, which contains some general integer programming algorithm, e.g. the branch and bound method, the cutting plane method or the branch and cut method. These general algorithms are able to solve to optimality real-sized covering problems, but only small instants of the allocation problems. To solve the problems with the criteria (1) or (2), we can make use a similarity between the problem (5)-(10) and the uncapacitated facility location problem [3]. The problem can be solved

by the approach reported in [4] or [9], where a Lagrangean multiplier is introduced for the constraint (8) to relax it from the set of constraints. Then the problem takes a form of the uncapacitated facility location problem. To solve it, the procedure *BBDual* [9] was designed and implemented based on the principle presented in [3], which is the branch and bound method with special methods for obtaining of the lower bound. The procedure was embedded into the dichotomy algorithm, which was used to find a fitting value of the Lagrangean multiplier.

4. Preliminary numerical experiments and criteria comparison

We performed the numerical experiments with the data originating at the Slovak road network with 2916 dwelling places, which represent aggregations of potential patients. In this study, the electronic road map of Slovak Republic was employed. The numbers of inhabitants of dwelling places were known together with other attributes of the nodes. The current proposal of the emergency medical vehicle location consists of 264 places, but 41 of them duplicate or triplicate locations at some bigger cities and they have no influence on the studied accessibility in accordance to criteria (1), (2) and (3) considering the fact that these towns are represented by one node each. Based on this reduction, the 223 points (locations) were taken into consideration as the value p in the primary problem. The sum of unallocated ambulances from the primary problem solution and the 41 multiple locations enter as value p the secondary problem. These data enable to calculate the suggested criteria for the given scenarios of the vehicle speeds connected with the individual link classes. The considered speed scenario was $v = \langle 105, 95, 75, 60, 50 \rangle$, which are assumed average speeds in kilometer per hour on highways, roads of first, second and third class and on the local roads respectively. The set of candidate locations was formed from all towns and villages with more than 300 inhabitants and present ambulance locations. This way, a set of 2284 candidate locations was obtained.

We solved all the above formulated problems for $T^{max} = 15$ minutes, whenever this limit was included into the model. In accordance to the type of criterion we employed the special algorithm BBDual or the general optimisation software *Xpress-MP*, if possible with respect to the size and structure of the associated model. The associated algorithms were run on a personal computer equipped with the Intel Core 2 6700 processor with parameters: 2.66 GHz and 3 GB RAM.

The results of numerical experiments are reported in Table 1 where each row corresponds to one instance of the problem, which is specified by the used criterion and problem type (p-primary or s-secondary). The row contains the objective function value (Objective) of the optimal solution, the computation time in seconds (Time [s]) and the number of located ambulances (Loc). These figures are placed in the section *BBdual* or *Xpress-MP* in accordance to the used solution technique.

It turned out that the problem with criterion (2) was insolvable due to either huge time consumption or model size by both the

approaches. That is why the optimal solution of only problems with criteria (1), (3) and (4) are reported in Table 1.

Table 1

Criterion	Type	BBDual			Xpress-MP		
		Objective	Time [s]	Loc	Objective	Time [s]	Loc
(1)	p	13771837	3436	222	–	–	–
(1)	s	44203257	3574	42	–	–	–
(3)	p				91	0.1	198
(3)	s				286180	1.4	66
(4)	p	–	–	–	11684	0.3	264

The comprehensive solutions were obtained from the primary and secondary solutions by simple addition of the zero-one resulting vectors y^p and y^s. So in the comprehensive solution, 264 ambulances are deployed. The subscript i of the nonzero allocation variable z_{ij} for criterion (1) was obtained for each j so that the equation (17) holds.

$$t_{ij}(v) = min\{t_{kj}(v) : k \in I, 1 \le y_k^p + y_k^s\} \qquad (17)$$

The value of variable x_j for criteria (3) and (4) was obtained for each j in accordance to the equation (18) or (19) respectively.

$$x_j = min\left\{1, \sum_{i \in I} a_{ij}(y_i^p + y_i^s)\right\} \qquad (18)$$

$$x_j = max\left\{0, min\left\{1, \sum_{i \in I} a_{ij}(y_i^p + y_i^s) - 1\right\}\right\} \qquad (19)$$

This way, comprehensive solutions *BBDual(1)*, *Xpress-MP(3)* and *Xpress-MP(4)* were obtained. Then the values of criteria (1)–(4) were computed for the solutions and these results are reported in Table 2.

Table 2

	Criter. (1)	Criter. (2)	Criter. (3)	Criter. (4)
BBDual(1)	13677537	35109	15613	285324
Xpress-MP(3)	23613635	182	91	188937
Xpress-MP(4)	25597594	182	91	11684
Man-made	16378985	92032	31672	431415

The row Man-made in Table 2 corresponds to the current distribution of ambulance vehicles over the area of the Slovak Republic.

6. Conclusions

We presented four models of the medical emergency system design problem which are based on a different quality criterion. These quality criteria reflect possible approaches to the original problem with a general objective formulated as: to provide the best service to all inhabitants of a considered region. As any sophisticated designing process of real service system needs methods, which are able to provide it with a concrete solution in a sensible time, we tried to assign to these particular problem formulations some solving algorithms and performed preliminary computational experiments to verify suitability of the algorithms. With exception of the second criterion, we found that instances of the particular problem types were solvable in reasonable time. Furthermore, we compared the obtained results with the current structure of the medical emergency system of the Slovak Republic. We proved that the studied approaches could considerably improve the current system in all the considered objectives.

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COMMUNICATIONS

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VILLON – A TOOL FOR SIMULATION OF OPERATION OF TRANSPORTATION TERMINALS

The paper deals with the modelling of transportation terminals operation (e.g. marshalling yards, factories, train care centres, airports, etc.) utilizing generic simulation tool Villon, which was developed using agent based simulation architecture ABAsim. Villon is complete simulation tool providing support for creation, configuration, experimentation and results evaluation tasks connected with simulation modelling of transportation terminals. Villon's unique features like its flexibility, detailed microscopic modelling of operation of various types of terminals, flowchart driven definition of operational procedures, user interaction, selection of decision strategies, 3D animation output and model creation process are explained. Project references and problems that can be addressed using presented simulation tool are also mentioned.

Key words: computer simulation, simulation tool, transportation terminal

1 Introduction

The most important and evidently the most costly part of a logistic chain is transportation. The transportation itself is composed of a movement along the transportation route and of necessary manipulations (service processes) applied to transportation means and to transported commodities at specialised locations called transportation terminals (e.g. marshalling yards, factories, train care centres, airports, etc.). Terminals belong to the most complex service systems involving sophisticated technological processes and are equipped with quite complicated and costly technical devices.

Simulation methods currently represent widespread techniques supporting optimisations and planning related to transportation logistic terminals. Presented simulation tool *Villon* supports tactical (middle-term) and strategic (long-term) planning related usually to infrastructural or operational proposals, which are supposed to guarantee optimal (or at least effective) behaviour of modelled terminal. Villon is a generic simulation model which supports microscopic modelling of various types of transportation logistic terminals containing railway and road infrastructures (e.g. marshalling yards, railway passenger stations, factories, train care centres, depots, airports, etc.). Using Villon, users are able to create detailed simulation models of terminal operation, define simulation scenarios, make experiments with the model and evaluate results of simulation runs in one integrated user-friendly environment.

2. VILLON Simulation Tool

The simulation tool Villon allows users (professionals in the field of transportation) to create simulation models of transportation terminals, to run prepared scenarios as well as to evaluate results of simulation runs, all without the need to write a single line of program code – utilizing only Villon's user-friendly interface. The creation of complex simulation models of logistic systems, of course, requires a certain level of experience and knowledge. However, using the Villon simulation tool, even a less experienced user is able to create simulation models of simple logistic systems within a short period of time (few days).

Even though the development of this tool was motivated by the ambition to create a complex simulation model of a railway marshalling yard, nowadays Villon is able, thanks also to the architecture flexibility and its other valuable properties, to support modelling of various types of logistic transportation terminals (e.g. railway passenger stations, train depots, factories including road transportation, ground handling within airports etc.).

Villon is a complete generic simulation system; it provides the user with comfortable user-friendly editors to edit all needed data to run a simulation model, supports customisation of many aspects of simulation runs, offers animated output of modelled activities in 2D or 3D view as well as extensive set of post-run evaluation tools (including statistics, graphical protocols and others).

In comparison with other commercially available simulation tools used for modelling of railway operation (among many, we can mention RASIM [2], RailSys [1] or OpenTrack [4], Villon offers its users more precise modelling and visualisation of infrastructure (no schematic approach), detailed modelling of resource (personnel, engines) activities, as well as the possibility to model railway and road traffic in a single model to examine mutual interference. On the other hand, due to its focus on modelling of operation of terminals (nodes), Villon lacks support for modelling of large-scale railway networks or timetable creation abilities.

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In the following text some features of Villon simulation tool and the process of simulation model creation will be presented.

3. Creation of simulation model

The creation process of a simulation model of complex logistic system cannot be successfully accomplished without proper data related to various aspects of a modelled system. Following the internal structure of the simulation system, we can divide needed data into three main categories – resources, customers (orders) and operation (services executed). The creation process of a simulation model will be explained on the example of Bratislava-východ marshalling yard [3].

3.1 Resources

Since resources represent the vital element in operation of any logistic terminal, special attention has to be paid to their modelling. The infrastructure model is created by direct transformation of a map (plan), which is available either on paper, or in electronic form – an accurate, not only simplified (schematic) model of infrastructure is used in the simulation and therefore we can evade distortions caused by rough approximations. In case of Bratislava východ marshalling yard both forms were used (two thirds of a marshalling yard map were available as MicroStation files, one third as a scanned paper map). It was necessary to join both maps and to use AutoCAD software to prepare digital file (DXF format) which can be used by Villon. After the marshalling yard layout was "translated" for Villon, so called physical level of the infrastructure model was created. On this physical level a definition of track's profession is not possible. Profession is understood as an information about the purpose of each element, e.g. in a marshalling yard model the track will be used as a reception siding, hump track or departure siding, etc. Information about track profession is essential to further development of a simulation model, especially during the editing of service technologies and technological activities. For example, in technological activity "Shunting locomotive movement to train" it is necessary to define the profession which will be used for the locomotive movement. Professions are assigned to tracks by a simulation model creator, based on the knowledge of the real infrastructure. The result of professions assignment is a logical level of an infrastructure model.

Creation of a physical level, track professions list and logical level of infrastructure is followed by the definition of routes, which are used by locomotives and trains for movement from one track to another. The definition of routes is based on the knowledge of the real station.

Mobile resources (e.g. personnel, engines) are modelled individually, respecting their working hours, profession and other properties. For example, in the model of Bratislava východ marshalling yard we modelled shunting locomotives that are used for hump processes and for displacement of wagons from sorting tracks to departure tracks. It was not necessary to model all the personnel

of the marshalling yard – only those personnel professions were modelled that work on trains in reception or departure yards.

3.2 Customers

Data about customers (from a simulation theory point of view - e.g. trains, cars) are provided in the form of arrival timetables with the possibility of probabilistic modification of their properties (arrival time, loading, composition, ordering of groups, etc.). Timetables could be imported from XLS files (many terminal disposition systems are able to export to this format), which greatly reduces the time required to input the data.

For Bratislava-východ marshalling yard the data from "PIS" (Service information system) were used. Further data about trains and wagon groups were used from "Train-formation plan" of the ZSR Company. Before usage, all the data were checked by employees of the ZSR Company. Thanks to these data, approximately 70 incoming trains were imported and about 50 outgoing trains defined.

3.3 Operation

The main operational procedure inside logistic terminals (similarly as in any other service systems) is serving customers entering the system. Since Villon is a generic model and has to support the modelling of various logistic terminal types, it does not contain any hard-coded service procedures – simply said, Villon itself is not able to perform any task without proper "program", which is entered in a form of a flowchart by the user during creation of the model. Flowcharts are composed of activities which represent single tasks executed during serving customers (e.g. loading, resource assignment, brake testing, etc.).

Flowcharts define succession and mutual dependence of activities in a service process. The defined parameterized activities are reusable and can be used in more than one flowchart. Flowcharts are created in a comfortable graphical editor (Fig. 1) with support for automatic validation of entered flowcharts (guarding required succession of some activities and appropriate resource handling). The ready flowchart is then assigned to a customer (e.g. a train or truck). Once defined the flowchart can be reused – the same flowchart can be applied to different customers with the same attendance procedure.

In the model of Bratislava-východ marshalling yard it was necessary to define nearly twenty flowcharts for incoming trains and more than ten flowcharts for outgoing trains. One flowchart has approximately 35 nodes and nearly 50 edges. Because all the trains incoming from same input line are served in the reception yard in the same manner, only one defined flowchart can be used for all of them.

Other operational procedures that were, thanks Villon capabilities, possible to simulate were, for example, the exchange of
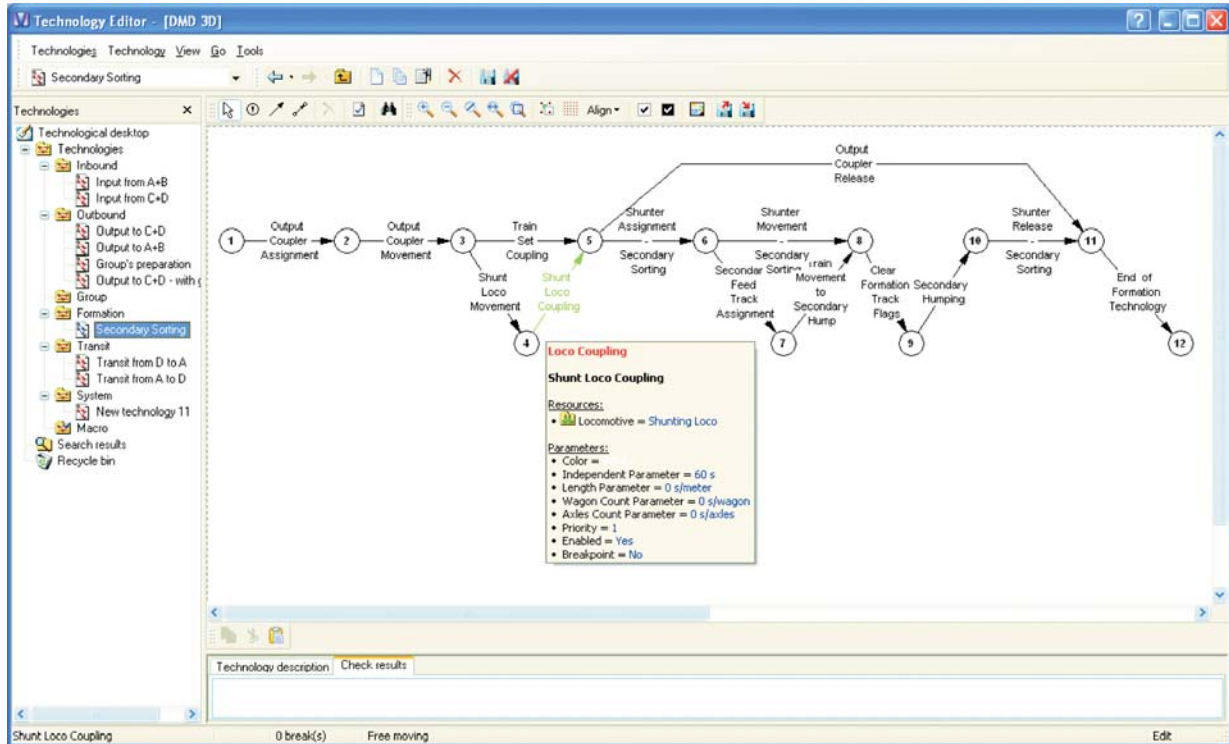
Fig. 1 Example of a flowchart defining technological procedure of serving a customer

wagon groups in trains, secondary humping and sorting, the handling of wagons with special service requirements (e.g. wagons loaded by gasoline that can't be sorted on hump).

4 Simulation model data structure

In Villon each model consists of *Model Data, Run Properties* and scenarios (called *Configurations*).

Model data are loaded respecting the hierarchy – high level data are loaded first, which enables to perform pre-run validation of entered data and also allows comfortable editing. For example, Villon checks if the incoming track for a train, which is defined at lower level (when defining timetable), is present in the infrastructure data and, on the other hand, Villon's timetable editor allows a user to select only such an incoming track that is defined in the infrastructure data as a track suitable for arrival of trains.

Run Properties portion of model define attributes of simulation run – duration, animation settings, simulation protocol options and cooperation options.

A user has the chance to create many variants (represented by distinct data files) of every data type. A simulation scenario is then simply created by selection of a single variant of every data type (e.g. the user selects one variant of infrastructure, one variant of personnel and so on Fig. 2). The scenario created such a way

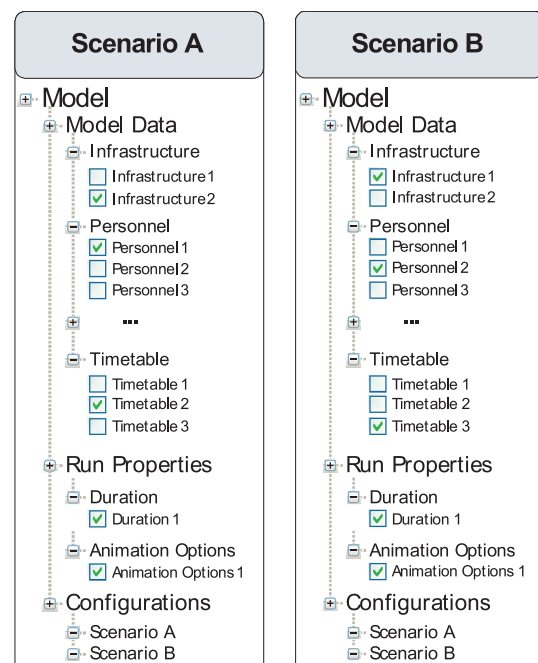is associated with a specific name and can be stored in the database of scenarios (configurations).

Fig. 2 Villon model data structure and scenarios

For the model of Bratislava-vychod marshalling yard we defined three scenarios (configurations). The first scenario reflects current operation. The second scenario reflects operation after the modified schedule for secondary sorting application. The third scenario shows the operation after changes in train-formation rules in Bratislava region.

5. Simulation run

Once all the needed data are collected and the scenario is chosen, the simulation run can be started. During the simulation run animation of all movements of vehicles (train sets, cars, airplanes, locomotives, etc.) is presented to the user. The user can choose between two or three dimensional view of the scene (Fig. 3).
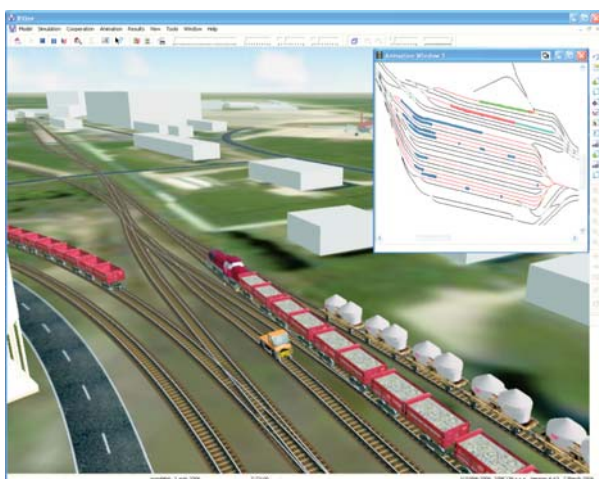

Fig. 3 Example of 2D and 3D animation in Villon

6. Results evaluation

As the simulation proceeds the user can follow a "live" development (change of values) of selected characteristics in a graphical presentation (e.g. status of personnel or utilization rate of a selected locomotive). Another type of output information is the overall information retrievable after termination of the simulation run. To this purpose, a detailed protocol on simulation is generated during the simulation run. These protocols can be processed separately and a sought for information can be retrieved from them. Various statistics are obtainable from the palette of pre-defined evaluations. The palette is open for adding new items according to the user's wish. Besides the graphical presentation of simulation results using time dependent reports on utilization of resources, waiting times, etc. (Fig. 4), statistical evaluations are also provided, in the form of tables, graphs and charts.

Villon also offers the chance to export all the collected information to the XLS file for further processing using a spreadsheet editor.

An important feature of Villon is the possibility of deploying a special Viewer version executable of the simulation tool to the customer. This Viewer version enables customers (e.g. the owner of examined terminal) to execute simulation runs prepared by model designers and also exploit complete range of run-time and post-run evaluation options Villon offers. Of course, the Viewer does not allow the customer to change any model data, which could have unpredictable consequences.
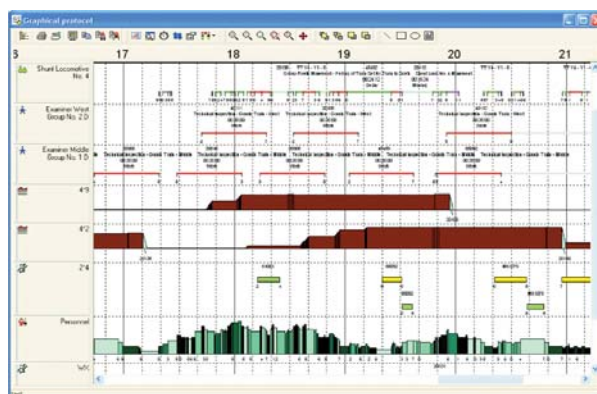

Fig. 4 Graphical time dependent reports example

7 Conclusion

The tool Villon was applied in commercial environment within simulation studies of marshalling yards in Austria (Vienna, Linz), in Germany (Hamburg Alte Süderelbe, Oberhausen-Osterfeld), in Switzerland (Lausanne, Basel), and in China (Mudanjiang, Harbin). Other applications of the discussed tool concentrated, for example, on investigation of a new proposed design of the railway depot in Ulm (Germany); verification of operation of the Prague Masaryk passenger station (Czech Republic), or proof of the internal railway-road traffic concepts inside the car production company Volkswagen Bratislava (Slovakia). Among other applications in industry (mostly connected with production increase in modelled facilities) following can be mentioned – the factory sidings of the chemical plant BASF Ludwigshafen (Germany), the internal railway traffic of the paper producing company SCA Laakirchen (Austria) and steel production companies Voest Alpine Linz (Austria) and Teesside Cast Products Middlesborough (United Kingdom).

The substantial decisions in the field of transport, respecting the contemporary level of information technologies and simulation methodologies, should not be adopted without modelling their consequences. The software tool Villon supports microscopic simulations of the most complex and large-scale transportation terminals and enables not only to investigate the consequences of adopted decisions but also (by means of the reasonable sequence of experiments) to choose the best solution and to save the financial resources.

The generic character of the Villon simulation tool brings one important consequence: once built, verified and validated the model of one specific node can be used repeatedly in seeking for solutions of a different type. For example, the same simulation model can be utilized for investigation related to optimal reconstruction of a terminal (long-term planning); on the other hand, it can be also useful during the real reconstruction in order to study operation within different reconstruction stages (medium-term planning).

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Jozef Hrbcek – Ales Janota *

IMPROVEMENT OF ROAD TUNNEL VENTILATION THROUGH PREDICTIVE CONTROL

Design of an effective ventilation system for the road tunnel is usually a challenging task whose solution is less or more tending to optimum. Predictive control seems to be a promising approach that can help to improve properties of existing ventilation systems applied in road tunnels. Advantages of predictive control result mainly from its ability to solve both SISO and MIMO tasks, to have regard for dynamics of process changes in a broad extent, to compensate effect of measurable and non-measurable failures and to formulate the task as an "optimization control task" considering limiting conditions of control actions, changes of control actions and output variables. Data characterising the existing ventilation system can be used to analyse and identify the system and create its stochastic model. Thereafter the predictive control of ventilation can be designed enabling to predict concentrations of pollutants and optimize system operation.

1. Introduction

Predictive control is basically based on discrete or sampled models of processes. Therefore relevant relations and derivations are presented mainly in discrete area. The term "predictive control" denotes a class of control methods having a set of common properties: a mathematical model of the control system that is used for prediction of the future controlled output, known future trajectory of the required quantity, calculation of sequence of future control actions involving minimization of a proper cost function (usually quadratic) together with future trajectories of control increments and control deviation. Only the first proposed control action is performed and the whole minimization procedure is repeated in the next sampling period again. Usability of predictive control algorithms is quite wide and quality of control is usually higher than in the case of PID-controllers. They are applicable to unstable, multidimensional processes or processes with transport delay and compensate effects of measurable and non-measurable failures [1]. The road tunnel is also such a system since a proper algorithm must be used to keep concentrations of harmful pollutants under the certain level. To make a design of the predictive controller possible the existing tunnel system must be identified first using methods for system identification. Since each tunnel is unique, the design must be realised for the particular road tunnel – in this case for the Prague's urban tunnel Mrázovka. Data measured in the control centre of the tunnel are used to create basic types of stochastic parametric models and to realize them in the programme environment Matlab. Effects of ventilator operation on concentration of pollutants in the tunnel tube are also subject to identification.

2. System identification

For the purpose of identification it is interesting to describe the sought process using input-output relations [3]. The general procedure for estimation of the process model consists of several steps: determination of the model structure, estimation of parameters and verification of the model. Several stochastic models were considered as discussed below. In all of them existence of the stochastic component $\xi(t)$ was assumed. Most often we considered ξ to be a white noise; however more complex cases are possible too. The stochastic models mentioned below are in discrete area all [3].

The model **ARX** (**A**uto-**R**egressive with e**X**ogenous variable) assumes the error appearing as a white noise ξ in the equation of the system:

$$Ay = Bu + \xi. \qquad (2\ 1)$$

The model **ARMAX** (**A**uto-**R**egressive **M**oving **A**verage with e**X**ogenous Input) assumes the error appearing as the MA model, i.e.

$$Ay = Bu + C\xi. \qquad (2\ 2)$$

In the case of the model **OE** (**O**utput **E**rror) we suppose that the stochastic component appears as a white noise additive to the output quantity (measurement noise).

$$y = \frac{B}{A}u + \xi\ ; \qquad (2\ 3)$$

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where:
$w(k) = -a1w(k - 1) - ... - a_{na}w(k - na) + b1\underline{u}(k - 1) + ... + b_{nb}u(k - nb); y(k) = w(k) + \xi(k).$

The structure **Box-Jenkins** gives a complete model with separately formed properties of failure from system dynamics. This model is suitable for those systems where failures are delayed in the process [3]:

$$y = \frac{B}{A}u + \frac{C}{D}\xi .$$ (2 4)

2.1. Prediction of the output

Estimation of the output $y_P(k)$ using a predictor is shown in Fig.1. Prediction of the output is usually a non-linear function $y_p(k) = f[y_{k-1}, u_{k-1}]$, where y_{k-1} is a set of measured outputs, u_{k-1} is a set of measured inputs.
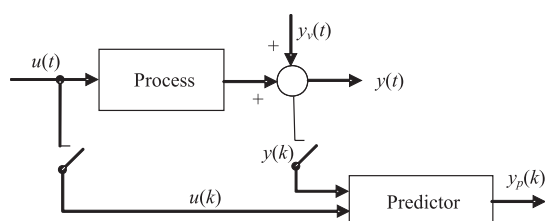

Fig. 1 Estimation of the output using a predictor

3. Requirements for tunnel ventilation

Tunnel ventilation is expected to fulfil the following requirements at least [5]:

- Concentration of emissions in the tunnel kept within the acceptable limits for the monitored harmful pollutants, in consideration of time spent by persons inside the tunnel;
- Good visibility for through passage of vehicles under polluted air inside the tunnel;
- Reduction of effects of smoke and heat on persons in the case of vehicle fire;
- Regulation of dispersion of pollutants in the air caused by petrol fumes from vehicles into the surround environment of the tunnel.

3.1. Ventilation system of the Mrázovka road tunnel

The ventilation system represents one function unit designed as longitudinal ventilation with a central efferent shaft and protection system avoiding spread of harmful pollutants into the tunnel surround area. Ventilation is longitudinal facing in direction of traffic with air suction at the south opening of the eastern tube (ETT) and at the branch B, with air being transferred at the north opening to the western tunnel tube (WTT) [7].

The task is to design a control system of ventilation based on traffic parameters, i.e. to find relationship between traffic intensity, speed of traffic and concentration of pollutants inside the tunnel. To do that the eastern tunnel tube (ETT) was chosen as a model example due to principle of mixing polluted air from ETT to WTT (measured concentrations of pollutants in the WTT are also influenced by traffic intensity in the ETT). To get the required description, the following data were taken from the tunnel: traffic intensity of trucks and cars, their speed, concentrations of CO, NOx and visibility from the ETT. These concentrations are measured by sensors installed inside the tunnel (at five different places of the ETT). Traffic parameters are measured at three places, air flow at three places and concentrations of NOx in the north portal. Primary purpose of these measurements is to use data for ventilation control
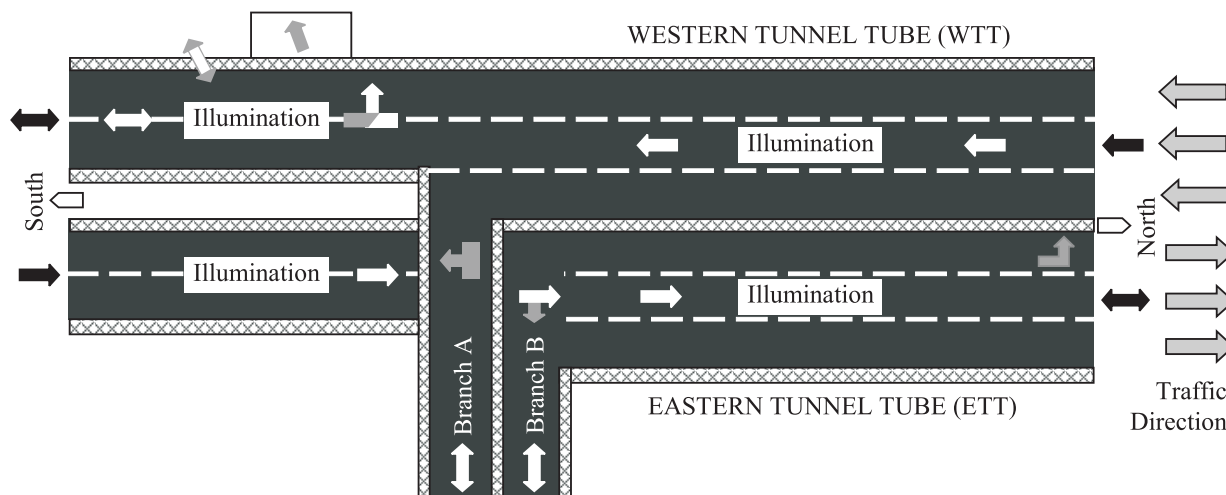

Fig. 2 Configuration of the road tunnel Mrázovka

(switching ventilators) based on sensed level of emissions in the tunnel.

4. Model Predictive Control (MPC)

Model Predictive Control (MPC) comes from the late seventies when it became significantly developed [1] and several methods were defined. In this work we have applied the Dynamic Matrix Control (DMC) method which is one of the most spread approaches and creates the base of many commercially available MPC products. It is based on the model obtained from the transition or impulse characteristics:

$$y(k) = \sum_{i=1}^{N} h_i u(k - i),$$ (4.1)

Where h_i are FIR (Finite Impulse Response) coefficients of the model of the controlled system. Predicted values may be expressed:

$$\hat{y}(n + k \mid n) = \sum h_i \Delta u(n + k - i) + \hat{d}(n + k \mid n) =$$
$$= \sum h_i \Delta u(n + k - i) + \sum h_i \Delta u(n + k - i) +$$
$$+ \hat{d}(n + k \mid n) \qquad .$$

We assume that the additive failure is constant during the prediction horizon:

$$\hat{d}(n + k \mid n) = \hat{d}(n \mid n) = y_m(n) - \hat{y}(n \mid n) .$$ (4.2)

Response can be decomposed to the component depending on future values of control and to the component determined by the system state in time n:

$$\hat{y}(n + k \mid n) = \sum h_i \Delta u(n + k - i) + f(n + k)$$,(4.3)

Where $f(n+k)$ is that component which does not depend on future values of action quantity:

$$f(n + k) = y_n(n) - \sum (h_{k+i} - h_i) \Delta u(n - i) .$$ (4.4)

Predicted values within the prediction horizon p (usually $p \gg N$) can be arranged to the relation (4-5):

$$\hat{y}(n + 1 \mid n) = h_1 \Delta u(n) + f(n + 1);$$

$$\hat{y}(n + 2 \mid n) = h_2 \Delta u(n) + h_1 \Delta u(n + 1) + f(n + 2);$$

$$\dots$$

$$\hat{y}(n + p \mid n) = \sum_{i=p-m+}^{p} h_i \Delta u(n + p - i) + f(n + p).$$ (4 5)

Where the prediction horizon is $k = 1 \dots p$, with respect to m control actions. Regulation circuit is stable if the prediction horizon is long enough. The values may be arranged to the dynamic matrix G:

$$G = \begin{vmatrix} h_1 & 0 & \dots & 0 \\ h_2 & h_1 & \dots & 0 \\ \vdots & \vdots & & \vdots \\ h_p & h_{p-1} & \dots & h_{p-m+1} \end{vmatrix},$$ (4.6)

And expression used for prediction can be written in the matrix form:

$$\hat{y} = Gu + f ,$$ (4.7)

Where $\hat{y}$ is a vector of contributions of action quantity and f are free responses. Test function is a quadratic criterion which can be expressed in the case of SISO (Single-Input Single-Output) system:

$$J = \sum_{j=1}^{p} [\hat{y}(n + j \mid n) - w(n + j)]^2 +$$
$$+ \lambda \sum_{j=1}^{m} [\Delta u(k + j - 1)]^2 = ee^T + \lambda uu^T$$ (4.8)

Where e is a vector of future errors within the prediction horizon and u is a vector of increments of the action quantity during the control horizon. The variant of DMC denoted as the Quadratic Dynamic Matrix Control (QDMC) is an optimization task solved numerically as a task of quadratic programming with limitations in the form of the system of linear inequalities:

$$\sum_{i=1}^{p} C_{yi}^{j} \hat{y}(n + i \mid n) + C_{ui}^{j} u(n + i - 1) + c^{j} \le 0$$
$$j = 1, ..., N_c$$ (4.9)

Since future output values are connected to control values through the dynamic matrix G, the limiting condition can be expressed in the form:

$$Ru \le c.$$ (4.10)

5. Simulations of predictive control in Matlab

Fig. 3 shows a block diagram of the predictive closed-loop controller. Models of the tunnel and ventilator were obtained through identification of real equipment. Higher traffic intensity causes increase of pollutant concentrations in the tunnel. This intensity is expressed as a vector containing really measured data. The Matlab environment is used to simulate behaviour of the system according to Fig. 3. It is a closed-loop control (regulation) of the system with limitations imposed to control quantity and outputs. It uses MPC mode format and solves optimization problem with the use of quadratic programming.
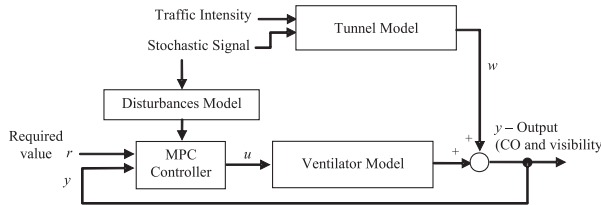
Fig. 3 Closed-loop control of ventilation using a predictive controler

5.1. Components of the control circuit

The "Ventilator Model" block is a model in the MPC step format that characterizes effect of ventilator on CO concentration and visibility (opacity). It is a system with 1 input (u) and 2 outputs (dilution of CO concentration and opacity OP). One of the main advantages of predictive control is incorporation of limiting conditions directly to the control algorithm. In this case the control quantity u is limited to the range 0–10V. The ventilator model has the form:

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} G_1(k) \\ G_2(k) \end{bmatrix} \cdot u(k)$$

The "Tunnel Model" block is also a model in the MPC step format that is used to estimate a controller state. It enters the process as a vector of values calculated on the base of traffic intensity in a certain prediction horizon. Generally this model may also be expressed in a different form [8]. The "Disturbances Model" block has a MPC step format. Disturbing quantity in this simulation is a white noise. For each present and in future obtained setting of control actions $Du(k)$, $Du(k+1)$, ..., $Du(k+m+1)$, future behaviour of outputs of the process $y(k+1|k)$, $y(k+2|k)$, ..., $y(k+p|k)$ can be predicted in the time horizon p. Number of present and future control actions m (while $m \leq p$) is calculated in such a way that the following quadratic criterion is minimized:

$$min_{\Delta u(k)...\Delta u(k+m-1)} \sum_{l=1}^{p} \left\| \Gamma_l^y \left([y(k+l\,|\,k) - r(l+l)] \right) \right\|^2 +$$

$$+ \sum_{l=1}^{m} \left\| \Gamma_l^u [\Delta u(k+l-1)] \right\|^2$$

Where Γ_1^y and Γ_l^u are weighing matrices for evaluation of individual elements y or u in a certain time interval to future. $r(k+1)$ is a vector of future reference values (required value). Despite m control actions was calculated (entire trajectory), only the first one $Du(k)$ is really used. In the next sampling period the whole procedure is repeated again. This principle is known as a strategy of moving horizon [3]. Predicted outputs of the process $\hat{y}(k+1|k)$, ..., $(k+p|k)$ depend on actually measured quantity $y(k)$ and calculations will be performed with respect to non-measurable failures and measurable failures influencing outputs.
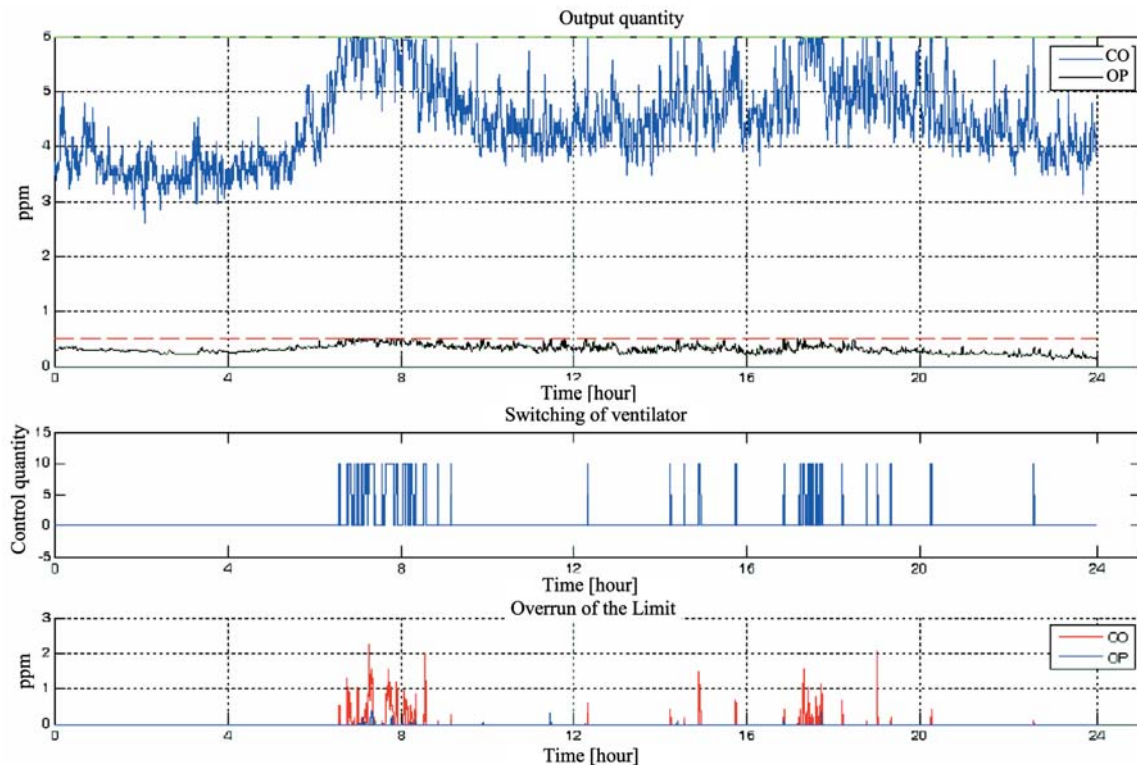

Fig. 4 Characteristics of MPC controller

5.2. Results of simulation

The presented simulation results are obtained for the following concentration limits: 6 ppm for CO concentrations and 0.5 ppm for visibility concentrations. These values are below really defined maximum limits. According to the curve of the output quantity (Fig. 4) it is apparent that no emission value extended the defined limit. However, the value of under-set maximum limit may be extended since one ventilator need not be able to dilute CO concentration sufficiently. In the next simulations we used a possibility to set weighing matrices (*uwt*) for tuning the controller. Correct setting of weighing matrices can avoid frequent switching of ventilators. In real operation the control quantity u must be adapted to the input of ventilator control unit.

6. Conclusions

The paper presents a methodology that was used for design of predictive control of road tunnel ventilation. Such a design needs identification of ventilator properties based on data obtained from the real ventilation system. Model of a two-dimensional system was created and simulated in Matlab environment using the method DMC. Presented results confirm higher effectiveness of predictive control approach where no monitored value extended the maximum set limits.

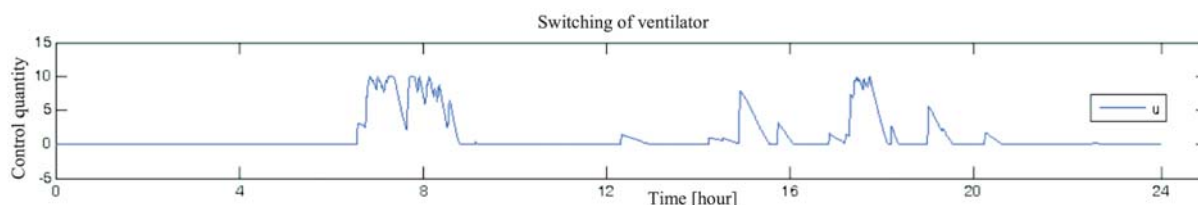The paper was elaborated with support of the project SK-123/CZ-108: ARICET.

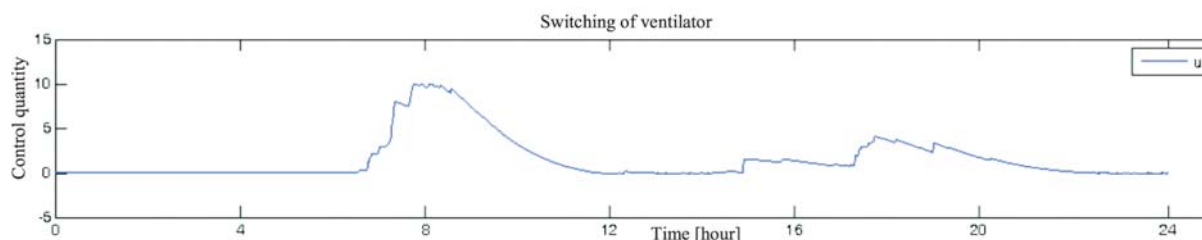*Fig. 5 The course of control quantity of the controller for uwt = 0.05*

Fig. 6 The course of control quantity of the controller for uwt = 0.3 and ywt = [0.8 0.8]

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COMMUNICATIONS

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NUMERICAL INVESTIGATION OF OPTICAL BURST SWITCHING

The simulation analysis of a physical layer of the "all optical burst switching" using a VPI Photonics™ simulation program is presented in this article. The optical burst networks adopt dedicated control channel to transmit headers of burst, at the same time the payload is transmitted on the other optical data channels after the offset time. We have proposed OBS network with four data channels and one control channel consisting of the following main optical elements: lasers with external modulation, a wavelength converter using Kerr effect cross phase modulation (XPM) in optical semiconductor amplifiers (SOA), a wavelength switch 4x4 and photodetector. We present the main connection of the OBS network, data and control message flow, processing in the core node and sending to next node. The results are visualized in graphs.

1. Introduction

Despite of the fact, that the optical burst switching (OBS) concept has been known since 1980s[1], it has not had a great success in electrical domain. The main reason for it was great complexity and requirements to run burst switching which are comparable with packet switching and the resulting flexibility is lower. However, deploying high capacity WDM links have led to new possibilities for utilisation of this principle and OBS is one of promising solutions for WDM networks [1, 2, 4].

The main characteristic of OBS is [5]:
- User data (IP packets) are accumulated in a constant or variable length of the optical burst, which would have up to one hundred kB.
- Separation between the headers (control information) and the payloads is in space and time. The heads are sent to the other OBS nodes in a single wavelength channel and are in all OBS nodes processed in electrical domain. The headers set OBS node for receiving and routing inbound payload burst to the next node.
- The burst data are switched in the OBS nodes asynchronously whereby remain in optical domain while required target is reached.
- The sources are allocated with using the one-way reservation, i.e. the burst is not sent while confirmation about successful set end to end way is not received

2. The simulation scheme of the OBS node

The simulation scheme in VPI Photonics TM is shown in Fig. 1. Four data wavelengths and one control channel use external modulated lasers with on/off modulation. The optical channels are setup on 1 mW power and the type of code is non return to zero. The channels' wavelengths were set with respect to the ITU recommendation for DWDM channels in C band with 0.8 nm (100 GHz) separation. Four wavelengths (1555.2 nm, 1554.4 nm, 1553.6 nm, and 1555.8 nm) were used for data channels and wavelength 1556 nm was set up for control channel. After multiplexing in DWDM multiplexer (insertion loss 2 dB), data were sent into a single mode fibre with 50km length, with attenuation of 0.2 dB.km^{-1} and dispersion of 16.6 ps.km^{-1}nm^{-1}. The dispersion compensation was realised by the dispersion compensation fibre with the length of 10 km and strong negative dispersion. Signals then pass through the optical amplifier with 12 dB amplification of an optical signal. Followed by the optical signal demultiplexing (power loss 2 dB) in the data and control channels, each channel was filtered by a bandpass filter with a Gaussian transfer function. The control signal was detected with the PIN photodetector. After the reconstruction of signal timing and elimination of d.c. component the signal went through a level comparator and at the control input of 4x4 wavelength switch (Fig. 2). According to the bit combination at the control input data burst was switched to the specific output [3, 6, 7]

Whereas the control signal was processed electronically, the optical signals are after sideband filtration brought into a wavelength converter. The wavelength converter, which uses nonlinear Kerr effects as XPM in SOA, was simulated in Mach-Zehnder interferometric structure. The advantage of this scheme is in utilisation of XPM as a main effect. In a such case not as high power of optical signals is needed as in the case of XGM. Brillouin and Raman scattering was avoided. Due to the other aspects, the amplification was needed to keep high SNR [6].

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[2]University College Dublin, Dept. of Electrical, Electronical and Mechanical Engineering, Ireland, dsabol@centrum.sk
[1] OBS was designed after 1990s and it was focused on new reservation protocol for accumulating data in bursts, prototypes and architectures [5].
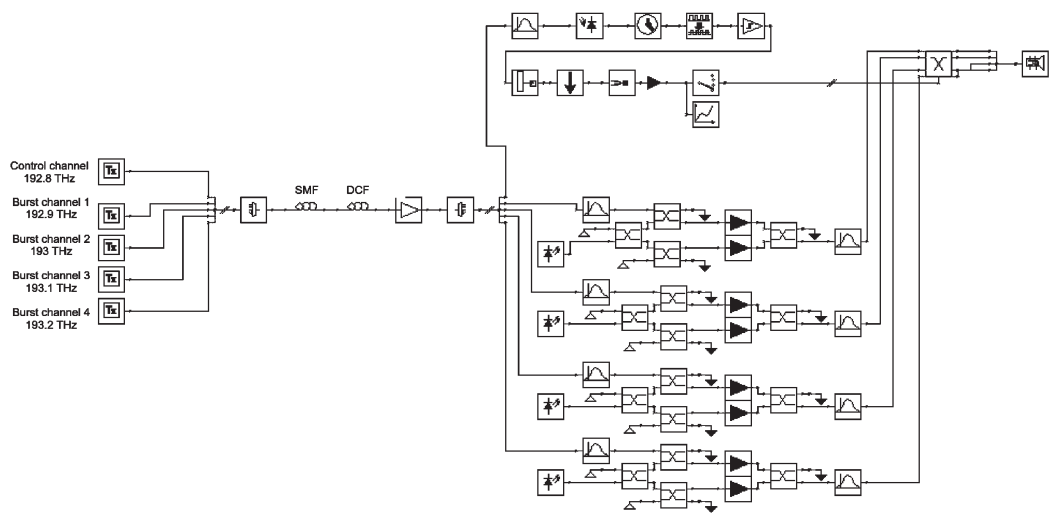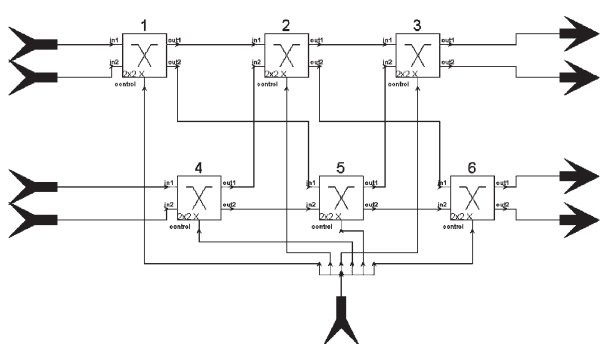
Fig. 1 Simulation scheme of OBS node

Fig. 2 Wavelength switch 4x4

3. Simulation

The OBS node sends data burst in time offset in accordance with the header. The delay of the time offset is possible to perform directly by transmission either from the edge nodes of network or from FDL lines in backbone nodes. In our case the delay is performed from the edge nodes in network [5, 7, 9].

In Fig. 3 there is a combination of 6 bits and 4 control headers, which were created in transmitters of control messages for each data channel. Table 1 shows relevant switch-over in dependence on these bits.

Switching in dependence on combination   Table 1
of bits in the burst heads

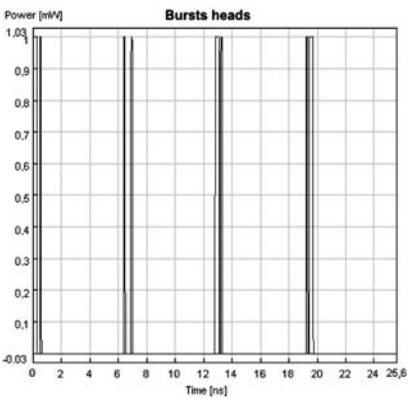|	Burst head	Input 1 in	Input 2 in	Input 3 in	Input 4 in
1.	1 1 1 0 0 1	1			
2.	1 1 0 0 1 1		4		
3.	1 1 1 0 1 0			2	
4.	1 0 1 1 1 0				3

Fig. 3 Burst heads for four data channels

In case of simulation 10 Gbit.s^{-1} bit rate the time length of one bit is 0.1 ns. This means that all the bursts are delayed behind
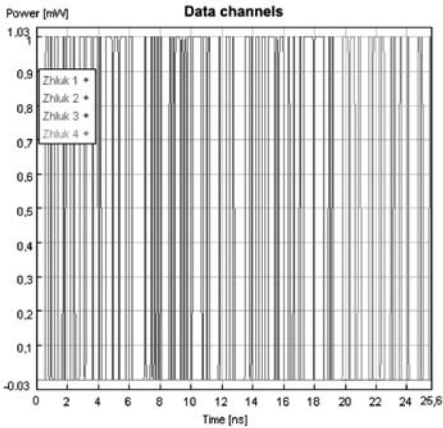

Fig. 4 Bursts with delay 0.6 ns after head

the headers in sources by 0.6 ns (see Fig. 4). The bit length of a sequence is constant and was set up on 58 bits.

After overcoming the optical way with a length of 60 km, the optical signal is amplified (12 dB). As can be seen in Fig. 5, SNR is sufficient for the next signal processing.
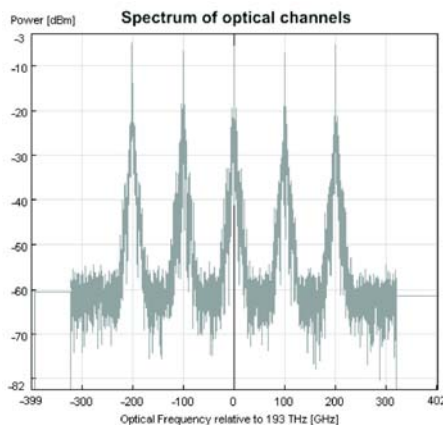

Fig. 5 Spectrum of optical channels after amplification

4. Header processing

After demultiplexing the channels, the control channel was linked to the optical band-pass filter and into the blocks for opto-electrical conversion and processing. A PIN photodiode is used as a photodetector. A very good eye diagram opening can be clearly seen in Fig. 6
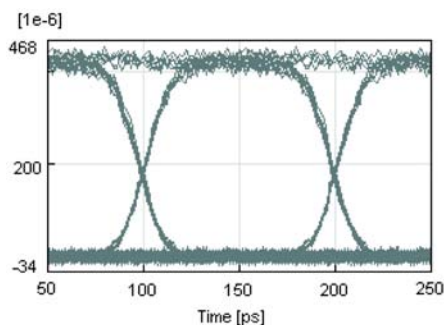

Fig. 6 Eye diagram of header after photodetector

The eye opening represents an influence (interference) between the channels. Crosstalk between the channels is low because an impulse spread is compensated by a dispersion fibre and the channel spacing of 100 GHz was sufficient. In the lower and upper parts of the eye diagram a superposed noise can be seen, which is caused by characteristic attributes of the PIN diode and fibre nonlinearity. In Fig. 7 a received control message of first burst is depicted.
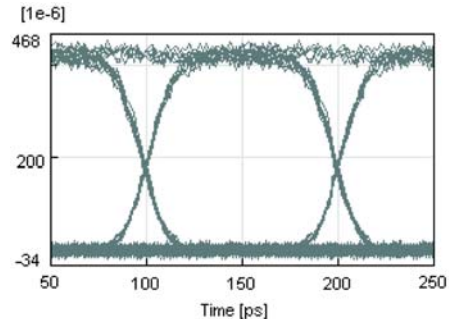

Fig. 7 Control message of first burst

5. The wavelength conversion of the optical data chanels

When the optical signal passes along optical fibre and demultiplexer and before switching to output, the signal goes through SOAs connected in Mach-Zehnder interferometric structure. An optical filter is connected for original wavelength filtering on each output of converter.

The best results for the conversion were achieved when an input power of the optical channel was set up on the half of power of the converted wavelength. The final values for SOA settings are in table 2.

Settings for the SOA   Table 2

OA values	
Length [μm]	500
Heigth [μm]	3
Width [nm]	80
Input power [μW]	400

The next effect which can be observed by conversion is an amplification by SOA of the input signal.

The input and output power was measured with a visualiser and a gain value of SOA was adjusted on 28 dB. The time graph, eye diagram input and the converted channel can be seen in Figs. 8 and 9.

6. A wavelength switch of the optical channels

The last block of the designed OBS node is a wavelength switch 4x4 controlled by the control bits. One input to one output can be switched at the same time only. The MEMS was chosen from more potential switches for the OBS networks. An advantage of MEMS is a low insertion loss (about 0.2 dB) and low crosstalk (less than -60 dB). After receiving electronical bits on the control input (see table 1) the inputs of an optical switch are switched to the desirable direction (Fig. 9).
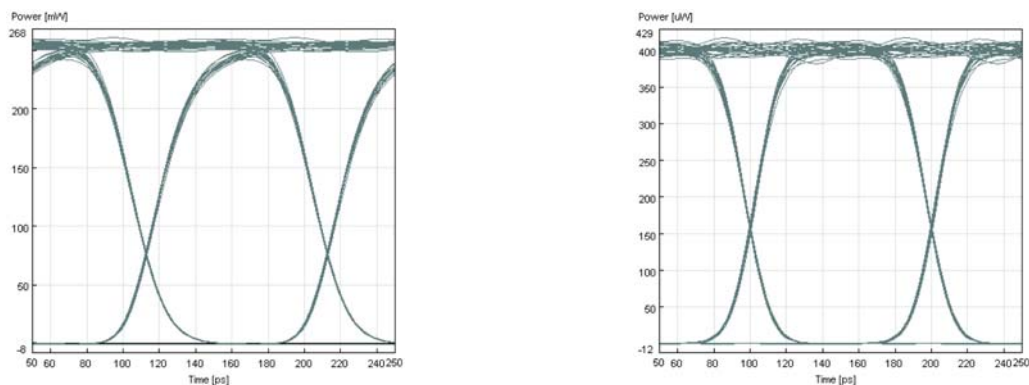
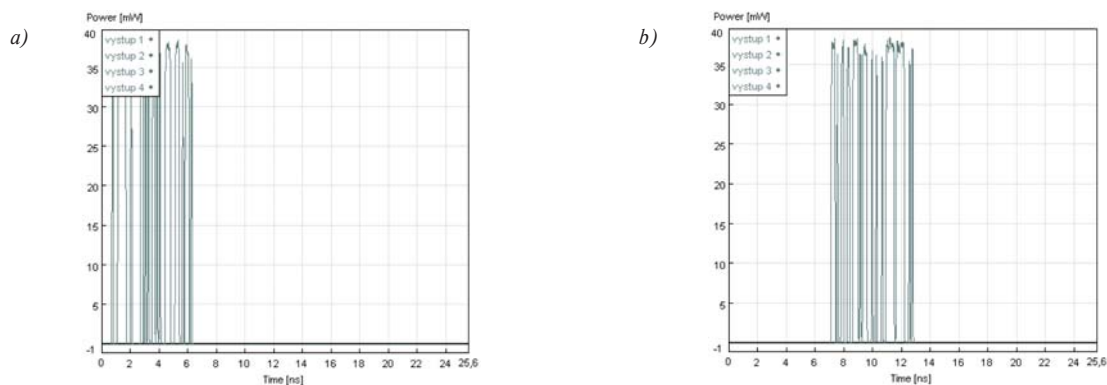*Fig. 8 Eye diagram before and after wavelength conversion*

a)

b)

Fig. 9 Outputs of optical switch, a) 1. burst, b) 2. burst,

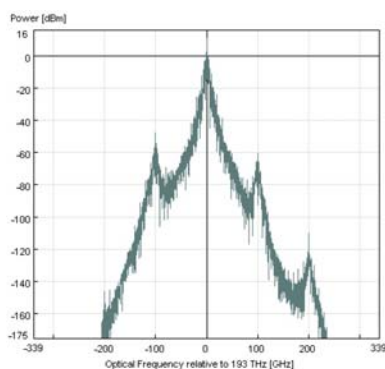The channel spectral diagram at the output port of switch is shown in Fig. 10.

Fig. 10 Crosstalk from another channels

7. Conclusion

Optical burst networks are promising technology of the near future. The main advantage is a possibility of full transparent optical transmission between the communication nodes and the results are higher bit rates between these nodes. Another benefit of OBS usage is possibility to utilize an older infrastructure with upgrading existing WDM networks [5, 8].

The OBS node was created in a simulation environment (wavelength conversion with SOA, header processing, wavelength switch controlled by control signal, etc.) as mathematical models [7]. The main goal was to achieve results which would be usable for prediction of systems parameters in a real situation. Due to this fact all the parameters of each block used in the presented simulation model were set as parameters of real components.

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PHOTONIC CRYSTALS – OPTICAL STRUCTURES FOR ADVANCED TECHNOLOGY

This paper presents experimental results in the field of photonic crystals investigation. There are presented our results of experimental investigation of one-dimensional photonic crystals appearing in nature, two-dimensional photonic crystals we formed in LiNbO3 and thin photoresist layers and results of our investigation of chromatic dispersion of photonic crystal fibers as well as the influence of fiber bending on their transmittance.

1. Introduction

Submicron structures known as photonic crystals (PCs) attract great attention because of their unique optical properties. First attempts of their implementation in technology show improvement of optical and electrical properties of optoelectronic devices. Photonic crystals are formed by periodic inhomogeneities of the refractive index with periodicity near the wavelength of an interaction light. Such PC's show then typical effect of wavelength selection known as photonic band gap. The first Yablonovitch established this term as an analogy to the electronic band gap arrangement in semiconductor materials [1].

It has long been known that metallic structural coloration of birds and insects was due to the wavelength selection in PC's. Photonic crystal of one (1D), two (2D) and three dimensional (3D) arrays are found in natural world and became the topic of the scientific studies [2, 3] (Fig. 1).
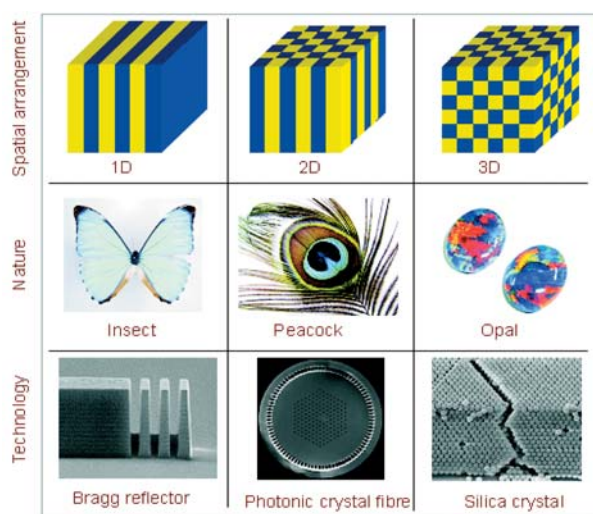

Fig. 1 1D, 2D and 3D photonic crystals in nature and technology

Inspiration in natural world started the progress in technology. Formation of 1D PC was successfully implemented 30 years ago as Bragg reflectors. 2D PC are topic of interest in optical fiber technology, where in last decade fabrication of photonic crystal fibers developed, while the 3D PC fabrication is only starting nowadays [4].

This paper presents our experimental results in the field of PC fabrication and investigation. We demonstrate experimental investigations on the 1D PCs in nature, planar 2D PCs and 2D PC formed in photonic crystal fibers.

2. Photonic crystals in nature

The surface of several beetles and other arthropods show structural colors originated from the light interference in the multilayer of alternative high and low refractive index materials [2, 3]. We investigated the structural colors, which arise in the 1D PCs in the cuticle of some species of class *Insecta*. Optical properties of studied insect were analyzed from angular spectral dependencies of the reflected light. The measured reflected spectra of samples for different incident angles are shown in Fig. 2a. In measured dependencies the individual maxima of measured spectra are in the region of visible light.

The evident blue shift of reflected spectra was observed with the increasing angle of incidence. The reflected spectrum from the sample of cuticle of dead insect was investigated under different conditions. In experiments we found that structural colors of the studied insect can be changed by the heating of samples. The blue shift of the main maximum of the reflected spectrum was observed with the increasing temperature for all investigated samples (Fig. 2b). According to a theoretical analysis the reflected spectrum from the multilayer structures (Fig. 3a) shows the blue shift as the layers thickness decreases [5].

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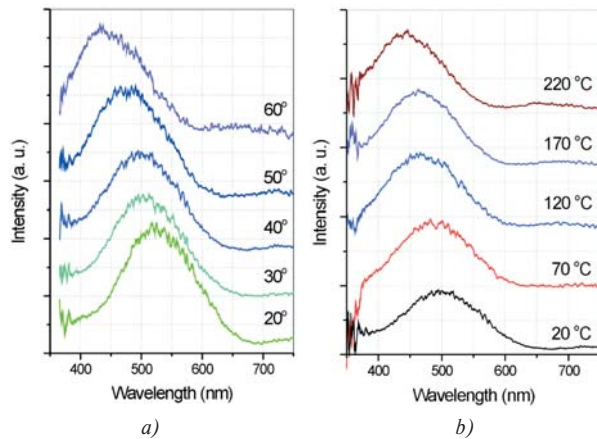
a)   *b)*

Fig. 2 a) The angular dependence of the reflected spectra measured from the samples of Lucilia sericata. b) The temperature dependencies of reflected spectrum of cuticle of Lucilia sericata measured for 40° angle of incidence
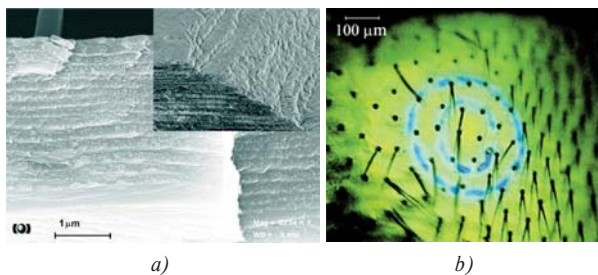

a)   *b)*

Fig. 3 a) SEM image of the cleaved edge of the multilayer structure of Lucilia sericata cuticle. b) The local structural color changes on the back of Lucilia sericata in the shape of sign "©" prepared by the focused HeNe laser beam

Using such experimental arrangement the sign "©"on the cuticle of *Lucilia sericata* (Fig. 3b) was created. The figure documents the local changes of the multilayer photonic structures caused by a local heating of the cuticle with the focused laser beam, what finally results in the blue shift of the reflected spectrum.

3. Fabrication of photonic crystals

Because of their unique optical properties which are advantageous for various device applications [4], the fabrication of photonic crystals has attracted a great attention. The essential for the photonics application is to have a material with properly modulated physical characteristics, e.g. refractive index. Various techniques have been used to fabricate such periodic structures [e.g. 6] but those of techniques that employ the refractive index changes induced due to illumination by light seem to be more interesting for applications.

We recognize various materials for fabrication of photonic structures such as lithium niobate – $LiNbO_3$, barium tantalate – $BaTiO_3$, photoresists and liquid crystals like polyvinylkarbazol – PVK, polymetylmetaakrylat – PMMA and so on. We focused our effort on the efficient design of simple test structures prepared in a $LiNbO_3$:Fe crystals and a positive photoresist AZ 4562 (Clariant Corp.) based on novolac resin.

In the case of $LiNbO_3$ crystals we utilize our knowledge of the photorefractive effect, i.e. the effect observed in materials that respond to light by modifying their refractive index [7, 8]. It is well-known that the non-homogeneous illumination of a crystal results in the spatial redistribution of charge-carrying electrons, including those captured in traps or donor centers. This induces a space charge field in the crystal which contributes to the refractive index changes of the crystal via electro-optic effect or via dependence of the refractive index on population of traps. The result is a spatially-varying refractive index region that occurs throughout the crystal's volume. The simplest (1D) photonic structure can be formed by a two-beam interference method [9]. The basic setup consists of an argon ion laser operating at 488 nm, a beam splitter and a sample holder (Fig. 4). Due to the construction of the splitter beams intersect each other in the place of the crystal sample seating. Laser beams form the optical field with harmonic dependence on coordinate with spatial period depending on the angle between incident beams

$$I(z) = I_0 \cdot \left(1 + m \cdot cos(K \cdot z)\right), \tag{1}$$

where I_0 is the average intensity of light, $K = 2\pi/\Lambda$ is the spatial frequency (Λ is the spatial period), z is the coordinate along the c-axis (optical axis of the crystal) and $m = 2 \cdot \sqrt{I_1 \cdot I_2} / (I_1 + I_2)$ is the modulation index dependent on the intensities I_1 and I_2 of the interfering beams.
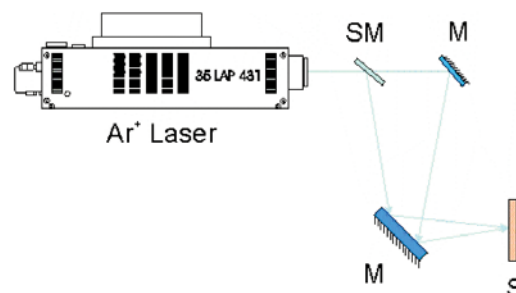
Fig. 4 Set up for two-beam interference recording. SM – semitransparent mirror, M – mirror, S – sample

This optical field creates a region with spatially modulated refractive index, which acts as the phase diffraction grating. In principle, the grating can work in both, the transmission and reflection regimes [9]. Investigation of the transmission grating formation provides the useful information such as recording time and amplitude of refractive index modulation that can be used, employ-

ing the coupled mode theory, for preparation of grating working in reflection regime. Besides the 1D structure we formed also more complex structures (Fig. 5).

a)   *b)*

Fig. 5 Diffraction patterns used for recording

a)   *b)*

Fig. 6 Interferograms of recorded optical fields shown in Fig. 5

However, there is the disadvantage of crystalline materials – their anisotropy. It means that one has to choose the proper orientation of the cryst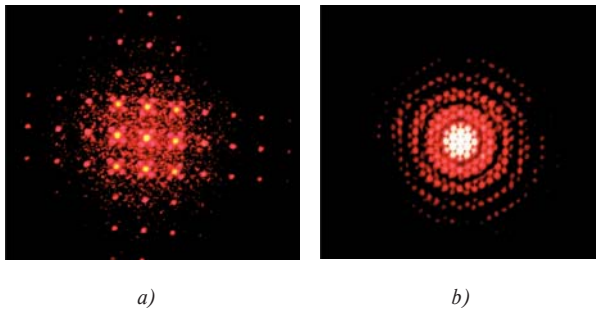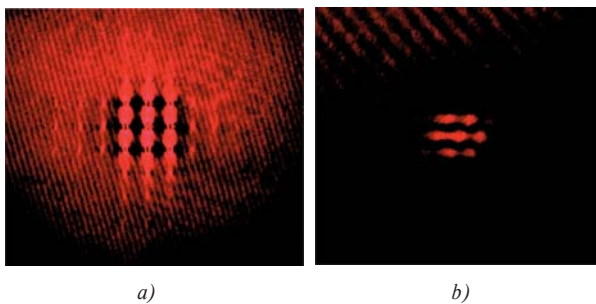al with respect to the gradient of illumination during recording or choose the special form of the field according to the anisotropy in order to produce a record. The optical fields used for recording were made using one beam of Ar laser which
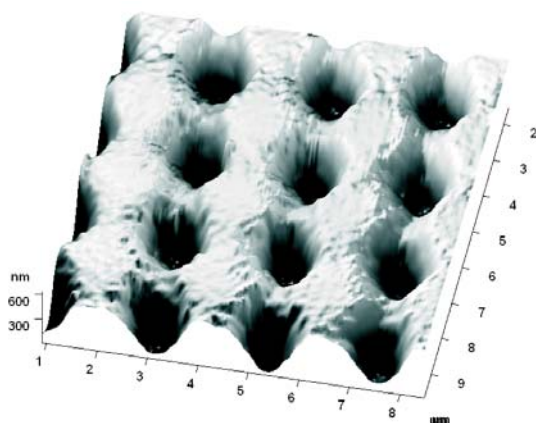
irradiated the suitable mask. The originated diffraction pattern was then projected by a lens onto the crystal. The process of recording took from 1 minute to 2.5 minutes. The average intensity of the recording beam was 4.2 mWmm^{-2}. Recorded optical fields (Fig. 6) can be well-observed using Mach-Zehnder interferometer [10].

The troubles according anisotropy can be overcome by using another type of material which is isotropice.g. a layer of a photoresist. The experimental set up for recording can be the same although the mechanism of the record formation is different. In this material light with the proper wavelength will start the sequence of chemical reactions leading to formation of regions where original monomeric material changes to polymeric one. The change of the refractive index itself or change of the optical thickness is due to process of polymerization. In addition to 1D structure we prepared also 2D structures using combination of two-beam interference method and multiple exposure technique (Fig. 7) [11].

4. Photonic crystal fibers

The unique phenomenon of photonic crystals can be used for construction of optical waveguides – called *photonic crystal fibers* (PCFs). PCFs are fibers based on Bragg reflection on the photonic crystal surrounding the "core" of the fiber, while the guiding mechanism of conventional optical fibers is based on the total internal reflection of the light in boundary of the core and its cladding. The difference between the guiding mechanisms leads to different properties of PCFs and conventional fibers. Because of PCFs ability to confine light with confinement characteristics not possible in conventional optical fibers, PCF gives new applications in fiber-optic communications, nonlinear devices, highly sensitive gas sensors and many other areas [12].

It is essential, for better utilization of PCFs, to know their geometric parameters and transmission properties. Because PCFs have a more complicated cross section area than conventional optical fibers, sometimes it is necessary to modify the methods used for measuring their parameters as chromatic dispersion, transmittance and so on.
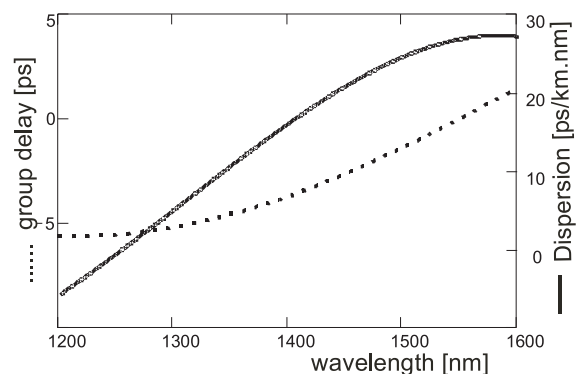

Fig. 7 AFM image of 2D structure prepared by double exposure with angle of incident laser beams $\alpha = 90°$

Fig. 8 Group delay $\Delta\tau_g$ and chromatic dispersion D_λ of measured photonic crystal fibre sample

Chromatic dispersion is not only an important parameter determining performance of optical fiber networks but it is also a key parameter for many other optical fiber applications, e.g. nonlinear applications using photonic crystal fibers (Fig. 8).
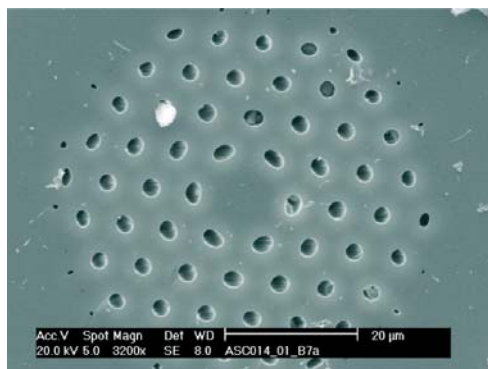

Fig. 9 SEM image of triangular structure of the photonic crystal fiber

Due to a complicated cross section area and some difficulties with shaping their faces needed at preparing mirrors we suggested to investigate the chromatic dispersion of PCF based on modified
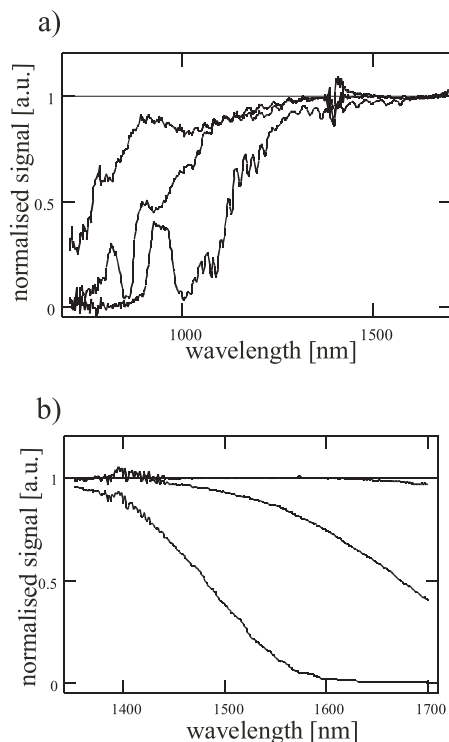

Fig. 10 Transmission functions of the PCF with different curvatures in the range (700-1700)nm and transmission functions of step-index fiber with different curvatures in the range (1350-1700)nm

interferometric method which allows to investigate the short as well as long sample of PCFs. This method is also appropriate for determination of bend (or temperature and so on) influence on the transfer function of the fibers [13].

In our experiments we investigated photonic crystal fiber made by Centaurus Technologies, Australia (Fig. 9). The core diameter was 13 μm, average hole diameter 2.6 μm and pitch 7.1 μm.

The spectral dependence of transfer function of the fiber depends on the radius of the fiber bending. The character of this dependence is quite different from that of conventional fibers - an increasing of bending radius of conventional fibers effects the decrease of the fiber transmittance mainly in the region of longer wavelengths, while the increase of bending radius decreases transmittance of PCF in short-wave region (Fig. 10). This fact clearly shows that the mechanism of guiding the light in conventional fibers and PCFs is not the same [14].

The study of the intermodal interference (Fig. 11) in the fiber can also be useful. For example it allows to determine the spectral dependence of the loss coefficient for the first higher order mode as well as to determine the difference of the phase constants of interfering modes [15 17].
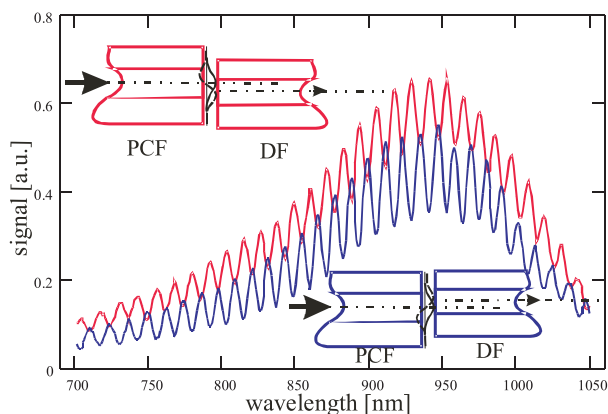
Fig. 11 Spectral dependencies of the signal interference measured in the PCF for two positions of the detecting fiber. The curves are not rectified for sensitivity of the detector and emissivity of the source

Knowledge of these parameters and their spectral dependence can provide useful information permitting a comparison between the fabricated optical fiber and the initial design upon which it was based. This includes potential information on the inhomogeneity of a fabricated fiber, which can be useful at their fabrication.

4. Conclusion

As it follows from above, the performed investigation of optical structures possessing the character of photonic crystals or photonic crystal fibers gave new results in the field of searching for

the mechanisms responsible for origin of such structures in the photorefractive materials, volume and planar photonic crystals occurring in nature, or preparing structures in laboratory in materials like, for example LiNbO$_3$ crystals and photoresistive materials. Also investigation of photonic crystal fibers brought new results showing differences between guiding mechanism in photonic crystal fibers and conventional optical fibers.

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COMMUNICATIONS

Josef Vican – Jozef Gocal – Branislav Melis – Peter Kotes – Patrik Kotula *

REAL BEHAVIOUR AND REMAINING LIFETIME OF BRIDGE STRUCTURES

The paper presents results of research activities of the Department of Structures and Bridges in the field of investigation of the real behaviour, reliability and durability of existing bridge structures. There are introduced two cases of these research activities in the paper having significant effects on the determining bridge remaining lifetime, which is the basic information from the viewpoint of bridge rehabilitation. The third presented research activity is devoted to the possibility of bridge RC beams strengthening using the FRP lamellas and strips from the viewpoint of determining the resistance of the strengthened RC beams.

1. Introduction

At present, the research activities of the Department of Structures and Bridges are oriented on the existing bridge reliability evaluation and investigation of their real behaviour respecting relevant damages of existing bridges and their rehabilitations in form of reconstructions. The main aim of the research is to propose the methodology of determining the bridge remaining lifetime from the viewpoint of ultimate limit states associated with strength and fatigue and to prepare reliability verification concept from the viewpoint of structural durability. To solve the problems, the experimental data related to the analysis of traffic load effects are collected by means of in-situ measurements and numerical simulations of the selected real bridge structures because the correct consideration of the service load effects plays a very important role in the reliability assessment of existing bridges.

Therefore, the fatigue remaining lifetime of the railway bridge stringers was analysed in order to observe and explain the occurrence and development of fatigue cracks of those bridge members caused by significant service load effects. In the case of fatigue life assessment, the service load effects are characterised in the form of stress range spectrum that is necessary to obtain by analysing service load effects.

The corrosion of structural steel is another relevant damage factor significantly influencing the bridge remaining lifetime from the viewpoint of ultimate limit states. In frame of the research activities, the corrosion effect on the buckling resistance and reliability of truss bridge compression chords was observed to determine their remaining lifetime. Because of stochastic character of the corrosion process and its influence on the compression chord reliability, the probabilistic approach was necessary to use respecting random variable input data.

In frame of research activities, the behaviour of the strengthened concrete beams with FRP lamellas was observed to determine the effect of strengthening on the beam shear and bending resistance.

2. Analysis of railway bridge stringer response to traffic load

The riveted connections of stringer and cross-beams in the case of railway bridges with open bridge decks are typical details prone to fatigue cracks. Webs of stringer and cross-beam are only connected without flange mutual connecting, so that the joint is verified on the shear and normal forces only. Due to connection arrangement, the bending moment is arising and producing normal stresses causing the crack in the form given in Fig. 1.

To determine the detail fatigue lifetime, the stringer response to service load was observed. Actual railway service load effects may be obtained either by experimental measurement or by numerical simulations. Usually, application of the first approach is financially and time demanding, and therefore it is mostly used for verification of accuracy and calibration of computational model for the second approach. Such a combined approach was applied to determine actual service load effects on the steel riveted stringers of the truss railway bridge, which is situated in km 309.309 of the railway track Kosice – Zilina.

The in-situ measurements oriented on collecting the time dependent normal stress response due to train passages were performed. The stresses were obtained by means of strain gauges whose arrangement can be seen in Fig. 2. The strains were measured and converted to stress records depending on time.

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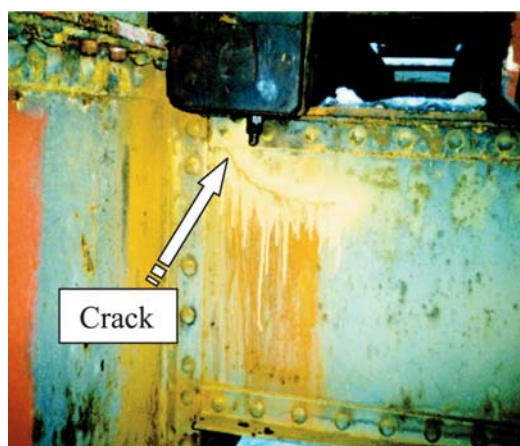
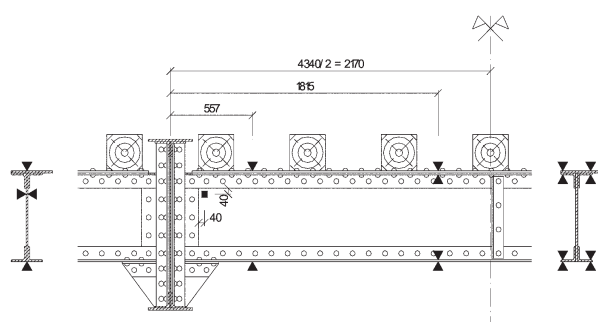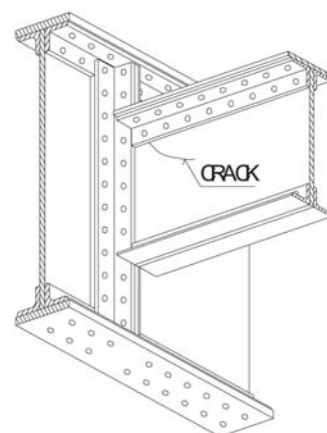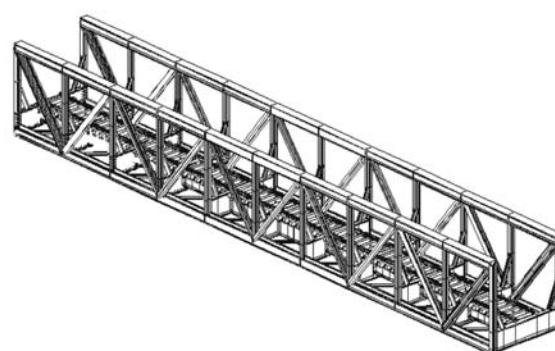*Fig. 1 The example of the observed crack in the stringer web*

*Fig. 2 Strain gauges arrangement of the observed bridge stringer and computational model
of the railway bridge used in numerical simulations*

Experimental observation of the stress response was used for verification of a numerical model. Passages of real trains were simulated on the numerical model developed using CAD software NEXIS 3.60 (Fig. 2). Due to time consuming computations, the combined spatial framed – solid numerical model was used.

Verification of the numerical model was realised by comparing normal and shear stresses calculated by means of numerical model and stresses obtained from in-situ measurements on real bridge stringer in monitored places. Regarding to time-consuming computations using spatial numerical model, influence line of stress response in the observed location – the second stringer flange rivet, was developed. It is a very typical location, from where the fatigue crack is starting in case of riveted bridge stringers. To create stress records, the computational algorithm in PASCAL language was developed. The algorithm allows obtaining stress records due to train passages using input influence line and trains data. Complete heavy trains data – their composition, frequency, weight and geometry of wagons were obtained from the ZSR IRIS-N information system. The passenger train passages data were considered according to the actual train schedule.

In accordance with the IRIS-N information system, passages of 163 freight trains and 174 passenger trains were simulated within

Statistical parameters of one-week real traffic and normal traffic according to EC1

Table 1

Statistical parameters	Real traffic	Normal traffic according to EC1
Average value	17.5648	20.3422
Average value fault	0.0740	0.0895
Median	17.0985	21.6000
Modus	1.2000	0.2250
Standard deviation	14.0017	16.6327
Data dispersion	196.0488	276.6454
Kurtosis	-1.1190	-1.1678
Skewness	0.2274	0.2429
Difference max-min	68.8439	51.6220
Minimum	0.0001	0.0150
Maximum	68.8440	51.6370
Number of values	35766	34552
Reliability level (95.0 %)	0.1451	0.1754

one-week sample of traffic. The stochastic stress records of simulation of train passages were obtained. From the viewpoint of fatigue assessment, it was necessary to transform stochastic stress record to the stress range spectrum. The classification was realized using the rain-flow method.
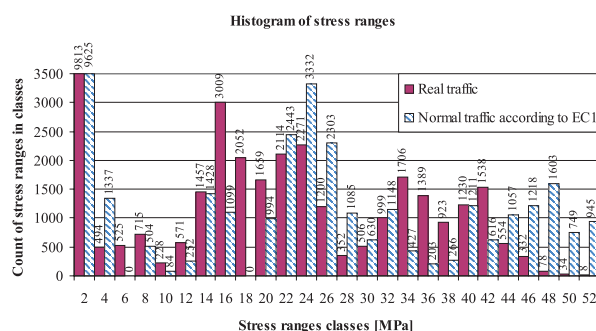
Histogram of stress ranges

Fig. 3 Histogram of stress ranges of one-week real traffic and normal traffic by EC1

Results obtained from the numerical simulation using IRIS-N were compared with the data calculated for one-week sample of normal traffic according to standard [1] using the same numerical model and calculation approach. Standard [1] introduces twelve types of service trains in all and it also prescribes their sequence in normal traffic (consisting of four types of passenger trains and four types of heavy trains) and heavy traffic (four types of heavy trains).

The comparison of one-week sample of real traffic and normal traffic according to standard [1] is presented in Tab.1 and Fig. 3. It shows that the average value of stress range due to standard normal traffic is about 15.8 % higher than the average value of stress range representing the real traffic. On the other hand, the total number of cycles due to standard normal traffic is about 3.5 % lower than the total number of cycles caused by the passes of the real trains within one week. Results obtained from the numerical

simulation will be further applied to determine the actual fatigue lifetime of the observed detail.

3. Resistance of compression chord under corrosion attack

The lateral stability of upper chords of truss bridge structures of small spans with lower bridge deck is only ensured by the transversal frames consisting of cross beams and stems (Fig. 4). Therefore, investigation of the compression chord buckling resistance is very important because of unstable mode of their failure having harmful consequences from the viewpoint of reliability. To assess their buckling resistance and lifetime a wide variety of degradation phenomena such as corrosion are to be taken into account regarding their ability to decrease substantially the overall reliability and durability of these elements or to mobilize consequently the instability phenomenon. Another aspect is to consider the possible influence of uncertainties in the material properties as well as differences between the theoretical and the actual geometry. All of the above-mentioned factors are stochastic in their nature, so the probabilistic approach to the structural reliability assessment should be required.

In order to solve the proposed task a non-linear finite element prediction of ultimate resistance of the compression chord of railway bridge subjected to corrosion, influenced by a random sample of initial imperfections and material properties were carried out. For the FEM modelling purposes, the existing railway bridge mentioned in chapter 2 having the cross-section according to Fig. 4 was chosen. The stochastic structural model required for probability analysis was created using multipurpose FEM software ANSYS [3] employing combination of BEAM4 for modelling the upper compression chord and BEAM188 by means of them other bridge components were modelled. Three types of the chord shapes presented in Fig. 5 were analysed in the parametric study. Probabilistic models of the considered random input variables are represented in form of the structural plate thickness having for particular cross-section types of the compression chord Gaussian distribution
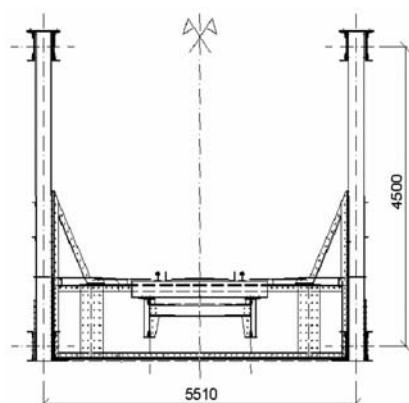

Fig. 4 Cross-section of the observed railway truss bridge and its computational model

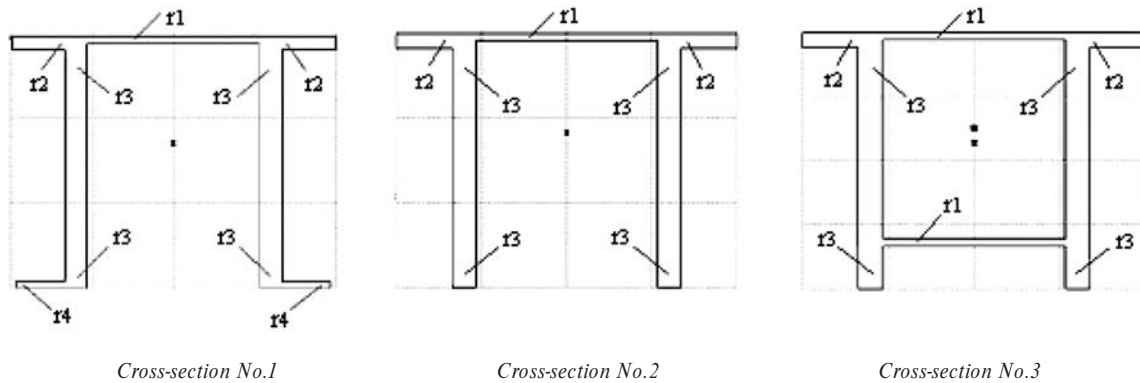Cross-section No.1   Cross-section No.2   Cross-section No.3

Fig. 5 Various shapes of upper chord cross-section and input parameters of geometry

and randomly varying normally distributed material properties (yield strength and ultimate strength).

The material model involved in the study is a non-linear rate independent Mises plasticity model with bilinear hardening. As a complement to this set of variables, the determination of the initial imperfections shapes was performed in two steps. Firstly, linear buckling analysis was carried out using the Block-Lanczos method for the predefined number of eigenfrequencies. Then, the amplitude of imperfection was calculated according to the standard [2]. Lastly, the shape of the elastic critical buckling mode was used to update the input geometry with an appropriate scaling factor [2], [9]. Then the non-linear analysis using Arc-Length method was applied for simulations of the ultimate strength of compression chord. Simulations of the ultimate resistance were carried out while searching for the limit point using explicit spherical iterations in the probabilistic framework. It involves consecutive repeating of calculations with randomly generated subsets of the input variables and evaluation of response in the form of so-called limit surface. The basic idea of the response-surface method is to approximate the exact ultimate resistance $R(X)$, where $X = [x_1, ..., x_n]$ is the vector of random input variables, by a polynomial function $\hat{R}(x)$ having the following general form

$$R(x) \cong \hat{R}(x) = a_0 + \sum_{i=1}^{N} a_i x_i + \sum_{i=1}^{N} a_{ii} x_i^2 +$$
$$+ \sum_{i=1}^{N} \sum_{j=1, \, j \neq 1}^{N} a_{ij} x_i x_j \qquad (1)$$

where the set of coefficients $a = [a_0, a_i, a_{ii}, a_{ij}]^T$, which correspond to the constant, linear, square and cross terms respectively, are to be determined. A limited number of evaluations of the resistance function are required to build the surface.

The investigation of the corrosion effect on the compression chord buckling resistance required to know appropriate corrosion model of structural steel. A lot of effort was exerted to evolve appropriate analytical models of corrosion mainly through extrapolation procedures processing the results of in-situ measurements. This has limited flexibility considering different environmental conditions what is their principal drawback. Current state-of-the-art models apply mechanistic or phenomenological approach to modelling corrosive lost of member thickness. However, these models are still of limited applicability. In our case, models presented in Tab. 2 were used. More detailed description of those models is in [4].

A parametric study considering effects of various corrosion models on the time dependent buckling resistance of the compression chord, analysed in form of an approximate function defining the stochastic response on random input variables was carried out. Investigated variants of corrosion influence on 3 types of cross-sections (Fig. 5) are shown in Fig. 6 for cross-section No 1. Variants A-D employ all corrosion models acting in different positions on the particular cross-sections, while variants E1–E2 and F1–F2 represent combination of max./min. corrosive effects onto flanges/webs and interior/exterior parts of the cross-sections mutually. Max/min effects are due to models by Melchers (red colour)/ Frangopol (blue colour).

Probabilistic models of corrosion damage   Table 2

Author	Mean value	Standard deviation	Distribution
Melchers [5]	$0.084 t^{0.826}$ [mm]	$0.056 t^{0.826}$ [mm]	Normal
Frangopol [6]	$32.07 t^{0.5}$ [µm]	$2.89 t^{0.045}$ [µm]	Normal
Qin-Cui [7]	$1.67[1 - \exp(-t/9.15)^{1.97}]$ [mm]	$0.00674[1 - \exp(-t/0.0181)^{0.0294}]$ [mm]	Normal
Guedes Soares [8]	$1.5(1 - \exp^{(-t/10)})$ [mm] where t is time in years		–

Results of the time dependent chord buckling resistance for cross-section No. 1 are presented in Fig. 7 and Fig. 8 for corrosion models A – D. The results show the time dependent change of mean value of the chord buckling resistance. However, in other cases of cross-sectional shapes the obtained results showed qualitatively similar tendencies regarding effects of the applied models
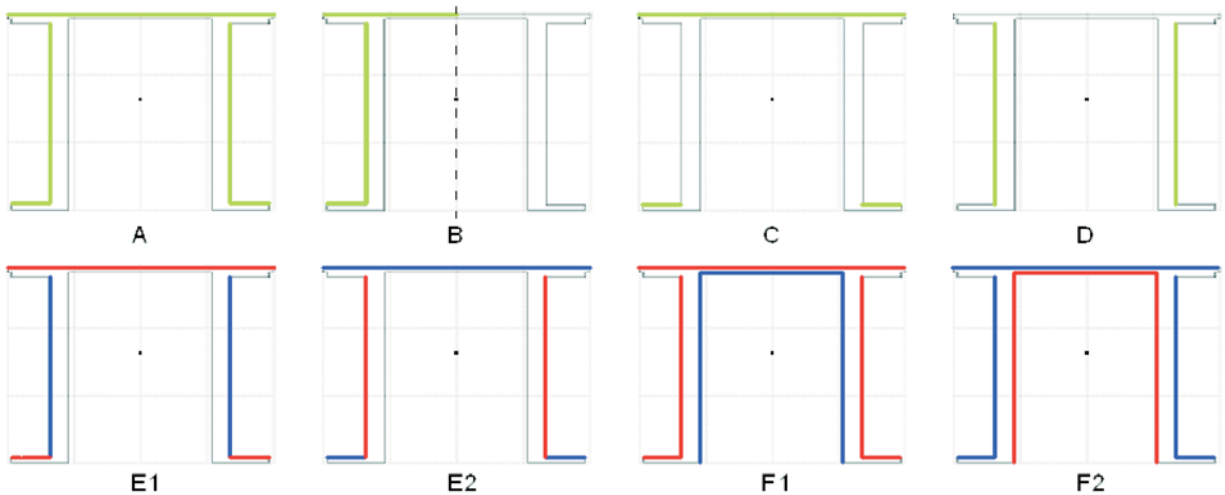
Fig. 6 Various types of corrosion attack of observed chord cross-section No. 1
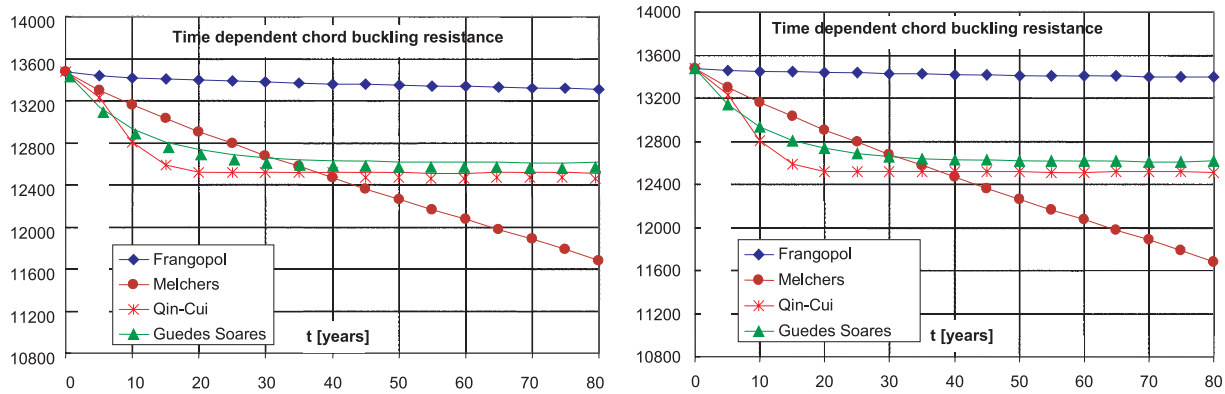
Fig. 7 Time dependent chord buckling resistance for the corrosion model A and B [kN]
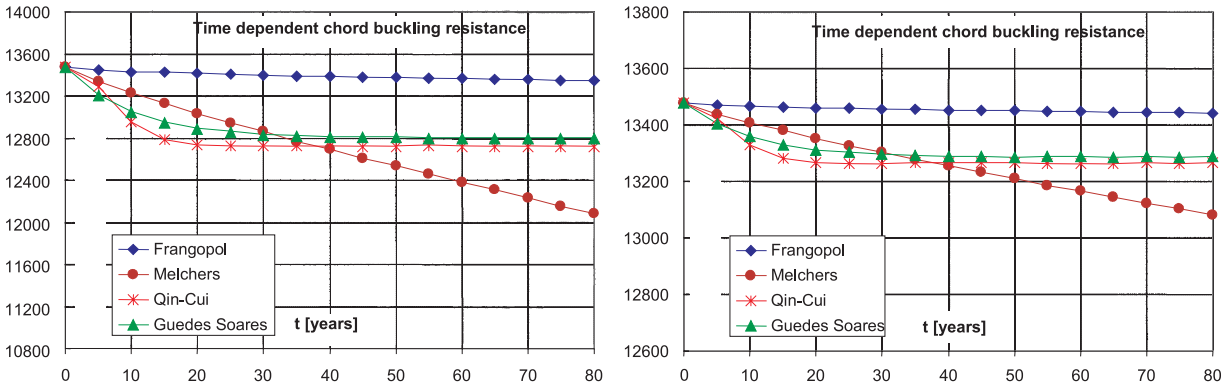
Fig. 8 Time dependent chord buckling resistance for the corrosion model C and D [kN]
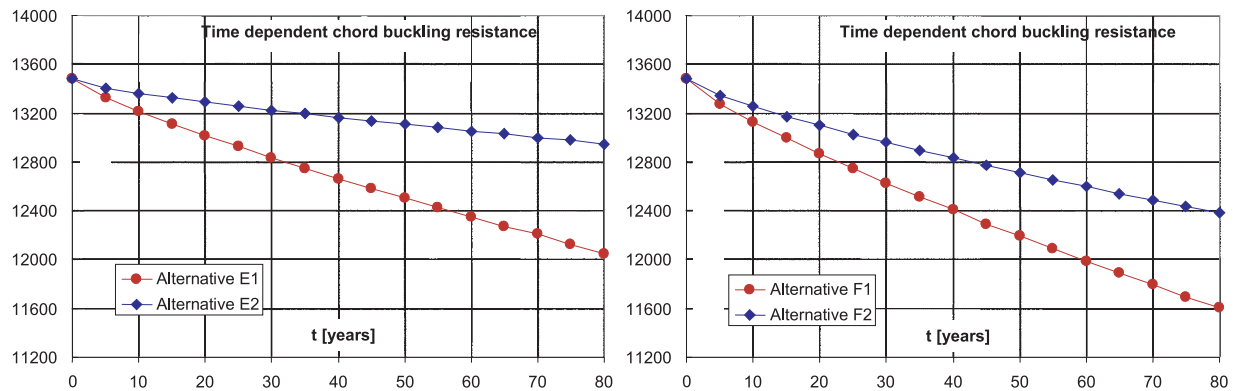
Fig. 9 Time dependent chord buckling resistance for the corrosion model E and F [kN]

of corrosion. Fig. 9 shows the results of the compression chord buckling resistance when combined corrosion models E and F according to Fig. 6 are applied. In the case of other investigated cross-sectional shapes presented in Fig. 5, the results were very similar. The results of the parametric study show very similar tendencies of the chord buckling resistance decrease due to various types of corrosion models. The Melchers' model seems to be the most aggressive. On the other hand, the Frangopols' model shows the most favourable results from the viewpoint of chord buckling resistance. Melchers' model also represents the biggest variability of the chord buckling resistance and from the viewpoint aforesaid it seems to be a very conservative and pessimistic model. However,

the maximum decrease of mean value of the compression chord buckling resistance due to corrosion attack does not exceed 14 % in any cases. Corrosion models according to Qin-Cui or Guedes Soares could be found as the optimal ones for practical use in our condition.

4. Experimental research of strenghtened RC beam resistance

The Department of Structures and Bridges is also orienting its activities in the field of the experimental and theoretical research
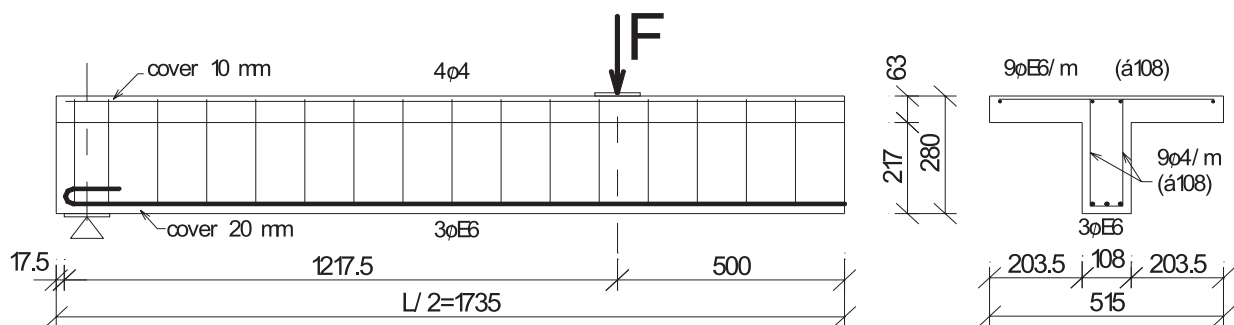
Fig. 10 Characteristics of the tested RC beam TO_0
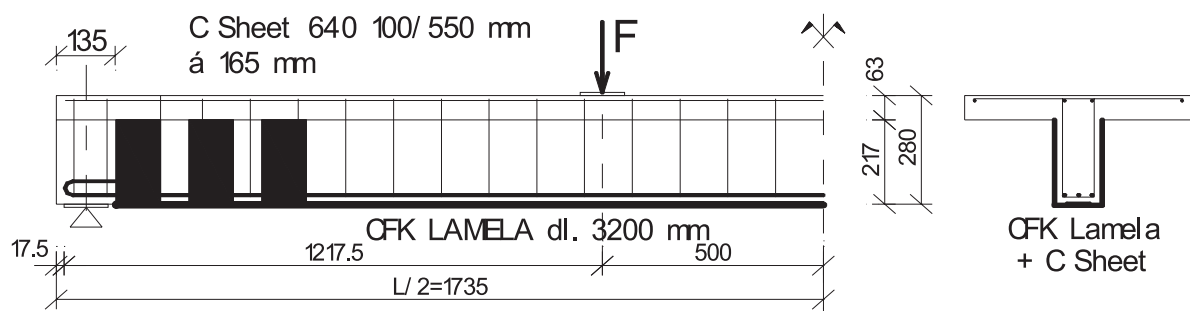
Fig. 11 Characteristics of the strengthened RC beams

of the bending and shear resistance of reinforced concrete (RC) beams strengthened by FRP. In the experimental analysis, two types of beam specimens were considered. Firstly, the non-strengthened RC girder TO_0 was made having comparison function. The beam TO_0 shown in Fig. 10 is three times reduced compared to the RC girder of the actually existing bridge located near the village Koláro-vice on the road I / 18. The 1/3 scale beam was considered because of the laboratory possibilities and the real beam geometry. Next, the five strengthened RC beams TO_1, TO_2, TO_3, TO_4 and TO_5 were fabricated (Fig. 11). The beams were strengthened by one MBrace® S&P CFK lamella 150/2000 for bending and by three S&P C-Sheet 640 of width 100 mm for shear. Three of these girders were statically loaded and two of these girders were dynamically loaded.

Firstly, the non-strengthened beam TO_0 was statically loaded till the failure in order to obtain the maximum force at the failure and to compare it with the maximum one at the failure of the strengthened girders. The dependence of the deflection on the loading force in the middle span is shown in Fig. 12. The maximum failure force was equal to $F_{max,TO\_0} = 30.0$ kN.
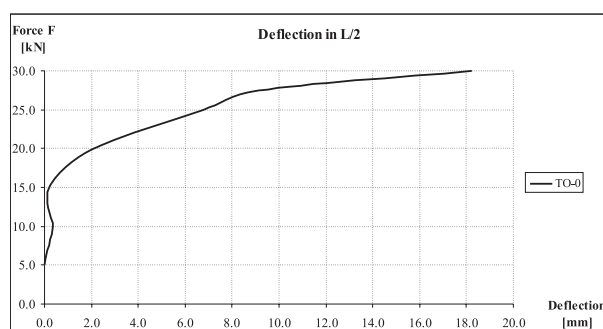

Fig. 12 The beam deflection in dependence on the load force – T-beam TO_0.

Next, three T-beams TO_1, TO_2 and TO_3 were statically tested. The resistance sensors and strain gauges were used to measure deflections and the strains.

The program of the static proof of the RC T-beams series TO consisted of:
1) Prior to testing, the non-strengthened girders TO_1, TO_2, TO_3 were cracked (the partial failure) in the bending area to simulate an overloading conditions which were corresponding to the limited bending crack width $w_{lim} = 0.02$ mm.
2) Strengthening the partially cracked RC T-beams TO_1, TO_2 and TO_3 using 1 lamella S&P CFK 150/2000 50/1.2 mm in the bending area, which was completed in the shear area by C-Sheet 640 strips.
3) Loading tests of the strengthened RC T-beams as far as bending failure.

The value of the maximum force at the T-beams failure after strengthening (step 3) was practically the same for all statically loaded beams, $F_{max,TO\_1} = 63.0$ kN. It means the load-carrying capacity increasing about 110%. The degree of the strengthening η, which is equal to the ratio between the resistance bending moment $M_{Rd,f}$ of strengthened cross-section and the resistance bending moment $M_{Rd,0}$ of non-strengthened cross-section, is higher than 2.0. Considering linear dependence between bending moment and load forces, the degree of strengthening may be defined by:

$$\eta = M_{Rd,f}/M_{Rd,0} = F_{max,TO\_0}/F_{max,TO\_1} = 2.10 \geq 2.0 \ (2)$$

The degree of strengthening should be less than 2.0. Therefore, the maximum load force, that should be practically used, is equal to $F_{max,TO\_1} = 60.0$ kN.

Two T-beams TO_4 and TO_5 were dynamically loaded firstly. The deflections and the strains were again measured by means of the resistance sensors and strain gauges. The harmonic loading having frequency 3Hz represented the dynamic load with 2 million cycles. In the case of the T-beam TO_4, the minimum force was 2.0 kN and the maximum one was 10.0 kN. In the case of the T-beam TO_5, the minimum force was also 2.0 kN, but the maximum one was 14.0 kN. This difference between maximum forces was used in order to observe the influence of the various loading on the cracks formation and the cracks development and on the load-carrying capacity of the T-beams. The maximum and the minimum deformations of the lifting spindle (TO_4) are shown in Fig. 13.
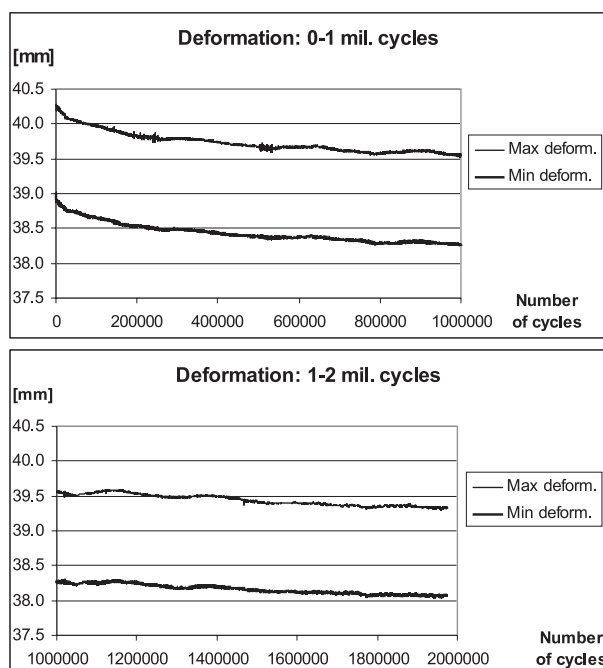
Fig. 13 The maximum and the minimum deformations of the lifting spindle – T-beam TO_4

After dynamic loading, T-beams TO_4 and TO_5 were statically loaded as far as their failure. The new cracks formation and

the cracks development were again observed. The values of deflection in the middle of the girder (L/2) span versus to loading force are shown in Fig. 14. The maximum force at the beam failure is equal to $F_{max,TO\_4} = 53.0$ kN in case of girder TO_4 and $F_{max,TO\_5} = 55.0$ kN in case of the girder TO_5.
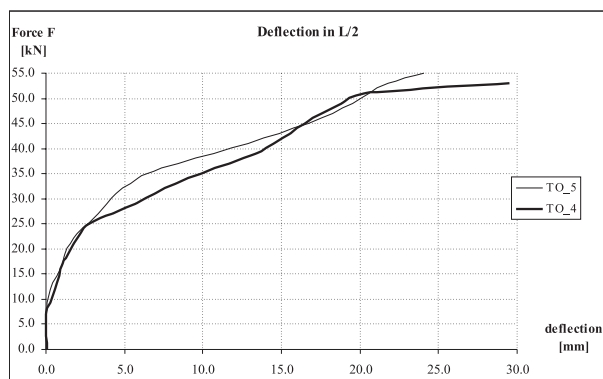
Fig. 14 Deflection of the beam middle spans depending
on the load force – T-beam TO_4, TO_5

5. Conclusion

In the paper, two cases of the research activities of the Department of Structures and Bridges were presented referring to the investigation of the bridge remaining lifetime determination from the viewpoint of fatigue and corrosion damage. Moreover, the experimental research results are presented related to the behaviour investigation of the bridge RC beams strengthened by means of FRP lamellas and strips from the viewpoint of determining their bending and shear resistance.

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COMMUNICATIONS

Jan Bencat – Milan Cibulka – Milan Hrvol *

EVALUATION OF THE SOIL ELASTIC MODULES BY MEANS OF BOX TESTS

The experimental measurements results of static and dynamic elastic modules (E_s, E_D) and initial tangent modules (G_o, E_o) of soils evaluated by means of box tests application techniques for calculation of these modules via static and dynamic loading half - space test means (SLT, DLT) and via shear wave velocity measurements tests are given. In the cross - hole method, body wave velocities are commonly evaluated from visual determinations of arriva timel of the waves at receivers. Additional time - determination techniques based on correlation and spectral analysis are discussed. The correlation technique is based on the cross - correlation function of waveforms recorded by two receivers. The spectral analysis technique is based on the phase of cross - energy spectral density function. The linear elastic and linear viscous - elastic behaviour of soils approach enables to measure soils damping parameters by mentioned box test means, too.

1. Initial Tangent Modules – E_o , G_o

Initial tangent modules (Richart, Hall, Woods [7]) of soils can be measured in the field or in the laboratory. If only low – amplitude strains occur (strain less than 0.001 percent), the key soil parameters are the initial tangent modules (E_o, G_o). The most direct field method and box test method for determining initial tangent modules is seismic testing. Shear and constrained modules are evaluated directly from shear (v_s) and compression (v_p) velocities, respectively, using the theory of elasticity.

In the *Impact - seismic method (ISM)*, body waves velocities are commonly evaluated from visual determinations of the wave arrival time at receivers or by time – determination techniques based on correlation and spectral analysis (Bendat and Piersol [4]).

1.1 Theoretical Studies

The linear elastic and linear visco - elastic behaviour of soils, respectively, generally in the case of small vibration amplitudes transmitted by ground are determined in theoretical works (Benčat [3], Martinček [5]). The visco - elastic model of simulation using complex modules E_o, G_o, K_o offers a very good approach to the actual soil behaviour. The complex modules are given by the formulae

$$E_o^* = E_o(1 + i\delta_E),\qquad(1.1)$$

$$G_o^* = G_o(1 + i\delta_G),\qquad(1.2)$$

where E_o, G_o are real components of initial (seismic) complex modules, (MPa); δ_E, δ_G are imaginary components of initial complex modules and represent the viscous properties of soil media. The relationship between a logarithmic decrement (ϑ), damping parameters ($\delta_E \approx \delta_G \approx \delta$) and a hysteretic damping ratio (D) is approximately given by formula

$$\vartheta \approx \pi\delta \approx 2\pi D.\qquad(1.3)$$

The determination of shear modulus G_o given by Rayleigh's (v_R) and shear (v_S) wave velocities, respectively, has been the object of experimental box tests. For linearly elastic medium (with low damping, like most soils at small strains) the shear modulus is derived as follows

$$G_o = v_S^2\rho,\qquad(1.4)$$

where ρ is unit mass (kgm^{-3}), v_S is shear wave velocity (ms^{-1}). The resilient modulus E_o is given by formula

$$E_o = 2(1 + v)v_S^2\rho,\qquad(1.5)$$

where v is Poisson's ratio.

1.2 Measuring Procedure

A special test facility was constructed for *ISM* tests in *Laboratory Dept. of Structural Mechanics, University of Žilina.* It incorporated a wooden box (Fig.1) with dimensions of 316 × 290 × 145 cm to examine the correlation between static, dynamic and

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seismic modules of the subballast and subgrade under simulated field conditions. The construction of the box, static loading test (*SLT*) and dynamic loading (impact) test (*DLT*) are fully described in Sec. 2.1.

It is common practice in the seismic methods (*ISM*) to use a line source – receiver array with one or two receivers located at a distance 1(*m*) from the source (Fig. 1).
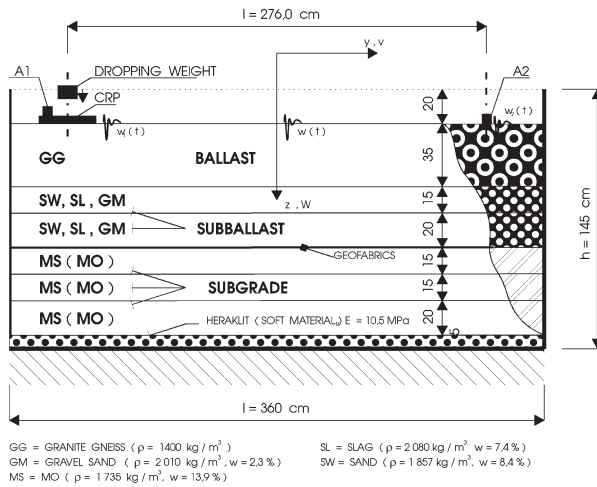

Fig. 1 Box Test Layout – Site View with Cutway Section

Propagation of body waves generated by the source is monitored with receivers (*A*1, *A*2) at the same depth as the source. The traditional approach used in the box tests to determine shear wave velocity (v_s) is based on identifying the time interval of the wave travelling between the first (*A*1) and second (*A*2) receivers. Once these times are determined, velocities are calculated via dividing distance (*l*) by appropriate times. Wave velocities determined by source to receiver measurements are termed the direct velocities method. Other techniques based on correlation and spectral analysis theories can be employed to determine body wave velocities in the *ISM* test (as well as other seismic tests like crosshole and downhole tests). This approach offers benefits in two areas. First, interval velocities have fewer potential errors than direct velocities (Stoke [9]). Second, the techniques can be fully automated. Furthermore, the retrieval of additional information such as strain rate effects and material damping is possible with spectral analysis. The first of this time – determination techniques is based on the cross – correlation function of waves travelling by two receivers. The cross – correlation $R_{xy}(\tau)$ of two functions $w_1(t)$ and $w_2(t)$ is given by the integral

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} w_1(t) w_2(t + \tau) dt, \qquad (1.6)$$

where $w_1(t)$ and $w_2(t)$ are time histories of the motion of the wave passing by the first and second receivers (waveforms); τ is the time delay and *t* is the variable of integration.

The two waveforms to be correlated are the same but one lags the other a time *t**, then the cross – correlation function will exhibit a maximum at τ equal *t**. If this two shifted but otherwise identical waveforms represent the waves recorded at two receivers, the time *t** will correspond to the time at which the peak of the cross – correlation function occurs and will represent the travel time of the wave between the receivers. If the distance between receivers is divided by time, the v_s velocity will be obtained. Another way to calculate wave velocities is based on the cross – energy spectral density function of the waveforms obtained at two different receivers (*A*1, *A*2). The cross – spectrum $G_{xy}(f)$, of two functions, $w_1(t)$ and $w_2(t)$, is

$$G_{xy}(f) = G(f) \overline{H(f)}, \qquad (1.7)$$

where $G(f)$ and $H(f)$ are the Fourier transforms of $w_1(t)$ and $w_2(t)$, respectively, *f* is frequency in Hz and " " indicates the complex conjugate.

Assume that the functions $w_1(t)$ and $w_2(t)$ are time records at the two receivers, for each frequency, the phase of the cross – energy spectrum of $w_1(t)$ and $w_2(t)$ will give the phase difference of the corresponding harmonic. Since the time period of that harmonic is known ($T = 1/f$), a travel time between receivers can be obtained for each frequency by

$$t(f) = \Theta_{xy}(f)/2\pi f, \qquad (1.8)$$

where $\Theta_{xy}(f)$ is the phase (in radians) of the cross – energy spectrum. The distance between receivers (*l*) is the known parameter. Therefore, the apparent velocity is

$$v(t) = 1/t(f). \qquad (1.9)$$

1.3 Case Studies

The records obtained from shear wave velocity measurements by *ISM* test at a box soil media are analyzed as an application of the travel time determinations explained above.

Details of the box set – up and equipment used can be found in reference Benčat [1]. The records obtained from shear wave velocity measurements by *ISM* test of the box soil media are analyzed as an application of the travel time determinations explained above. Details of the box set – up and equipment used can be found in reference Benčat [1]. The time records of shear wave motion obtained with two vertical acceleration transducers located at points *A*1 and *A*2 are presented in Figs. 2a and 2b. The time interval obtained from the first arrivals of the shear wave is *t* = 29.29 msec. This corresponds to a shear wave velocity v_s = 94.23 m/s and modulus for subgrade material (*MS*, ρ = 1735 kg/m³, *w* = 13.9 % *v* = 0.3) E_o = 40.05 MPa.

The cross – correlation function of the two time records is presented in Fig. 2c. The maximum value of this function occurs at time *t** = 29.40 m/s. The shear wave velocity calculated with
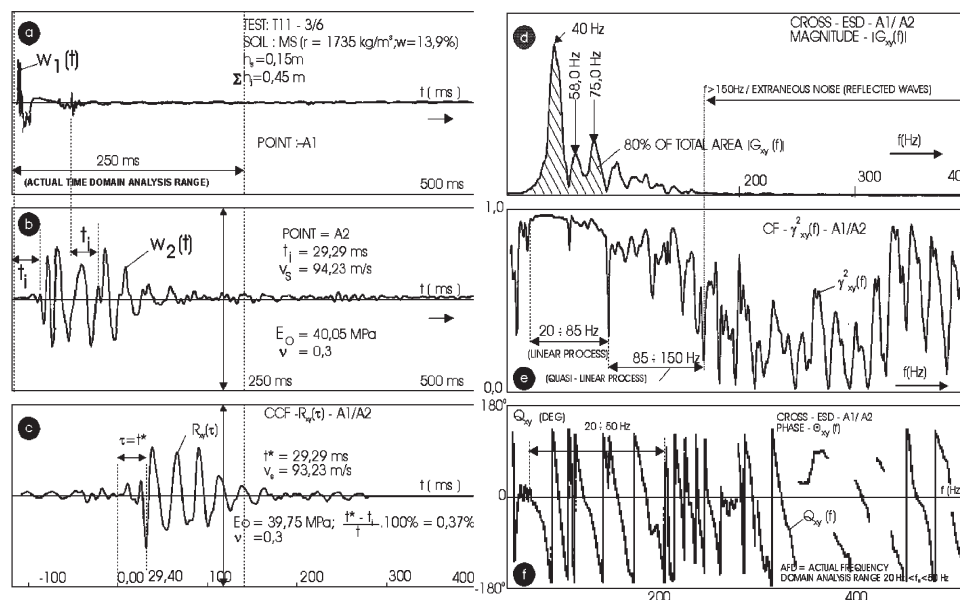
Fig. 2 Determination of shear wave velocity of Mo at level h3 = 45 cm
a) Time history at the first accelerometer; b) Time history at the second accelerometer; c) Cross correlation function; d) Magnitude of cross energy spectral density; e) Coherence function; f) Phase of cross energy spectral density

this time is $v_s = 93.87$ m/s and modulus $E_o = 39.75$ MPa. The phase of the cross – spectrum is shown in Fig. 2f, along with the coherence function in Fig. 2e and energy cross – spectrum in Fig. 2d. For a signal with no background noise, the coherence function $\gamma_{xy}^2(f)$ should have a value of one at any frequency if the system is considered linear. Comparison between mean value of the statistical value collection of all corresponding static and seismic resilient modules E_o/E_s at layer of soil under investigation (*MS, test T 11-3/6*) gives ratio: $E_o/E_s = 2.38$. There were also evaluated ratios E_o/E_s and E_D/E_s for each layer of the soils creating a corresponding combination of the subballast and subgrade in the box using the procedure described above.and in the following sections.

2. Static and Dynamic Modules - ES ,ED

Theory of an elastic half – space (Timoshenko and Goodier [10], Johnson [6]) provides formulation for the evaluation of elastic modules E_s, E_D. Pushing the rigid circular plate into the elastic half – space with uniform pressure (Hertz pressure) enables to develop the relationship for the evaluation of the resilient modulus $E_s = E_s (r, w, v)$ as follows:

$$E_s = \pi/2(1 - v^2)(r \cdot p, w) \qquad (2.1)$$

where: r is the radius of a rigid circular plate (mm), p (MPa) is static or dynamic pressure at the contact area of the rigid circular plate and the half – space, w is uniform normal displacement of the contact area (mm), E_s (MPa) is resilient modulus and is Poisson's ratio.

Eq.(2.1) is utilized in both the static and the dynamic experimental tests for the evaluation of static and dynamic resilient modules of soils in situ or by means of the box test. Based on the examination of the available models of the track substructure and considering of its desired features, the theoretical model based on an elastic approach provides satisfactory results for practical aims. To introduce a dynamic loading test (*DLT*) for in situ measurement of the resilient modules it was necessary to find the relationship between static (E_s) and dynamic (E_D) modules of the soils and substitute materials (e.g. granulated slag) which are used as in track subballast. Up to now, principles of the method of sleeper subgrade construction arrangement has been used at *ŽSR (Slovak Railways)* using modulus Es for each layer. To avoid the expensive experiments in situ both the static and the dynamic tests of soil parameters, the box tests were carried out in the laboratory. Several tests have been validated by comparison with field measurements from a test track, too. A special test facility was constructed for this purpose. It incorporated a wooden box (Fig. 3) with dimensions of $316 \times 290 \times 145$ (cm), to examine the correlation between static and dynamic resilient modules of the subballast and subgrade under simulated field conditions.

Fig. 3 also shows numbered layers of the subgrade and subballast and provides the description of the construction of the box and steel frame of the hydro mechanical equipment for the static loading test. At the contact areas both the bottom and side walls of the box were covered with soft composite material of 5.0 cm thickness and with modulus $E_s = 10.5$ MPa. Before starting the static test, the optimal regime of the electrodynamics plate vibrator acting to achieve an optimal degree of the soil compacting according to *Proctor standard test (P.S.T.)*. For each layer after
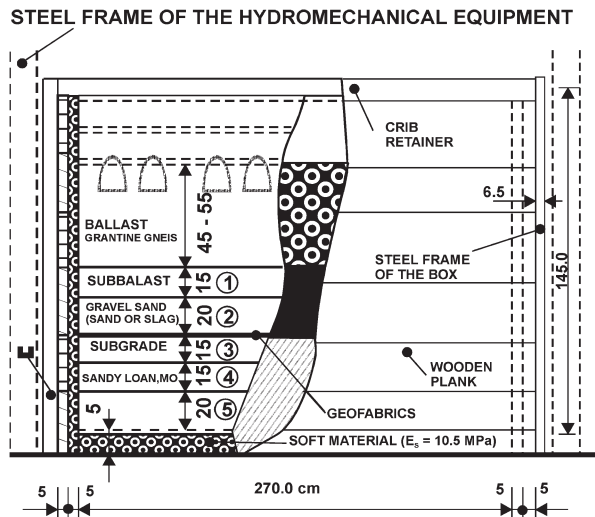
Fig. 3 Box test layout – Side view with cutway section

compacting specimens for standard laboratory test of the soil geotechnical properties (e. g. grain characteristics, specific gravity G_s, mass density (ρ), moisture (w) void ratio (e), compaction level, degree of saturation (S_R), Poisson's ratio (ν), etc.) were taken.

For cohesionless soils the optimal degree of compacting was expressed by volume mass with the application of TROXLER device (*radiometric probe half - space JGP 104*). Fig. 5 shows the location of the points (G_1, G_3) for taking specimens of soil at each layer of the substructure. The subgrade was successively created by three layers of $20.0 + 15.0 + 15.0 = 50.0$ cm of the cohesive soils and the subballast was created by the combination of the cohesionless soils imposed on the subgrade either with geofabrics or without the geofabrics. During the test the properties of the tested soils were approximately constant. Six combinations of soils and substitute materials which created the layered substructure were carried out. For each combination of soils the subgrade was made as a sandy loam (*MS*) with parameters $\rho = 1735.0$ kg/m³, $w_{OPT} = 13.9$ %. The layers of the subballast were successively created by gravel sand, sand and slag. The layers of the subballast in each test have thickness of $20.0 + 15.0 = 35.0$ cm.

2.2 Static Loading Test (SLT)

The static resilient modulus E_s (MPa) of the tested soils was evaluated by measuring the rigid circular plate vertical displacement w (mm) due to hydro mechanical equipment. The contact area of the plate for each test was $A = 1000.0$ cm². Fig. 4 shows the location of points $B1 ... B15$ for the *SLT*. The resulting vertical displacement w of the circular plate for each test was obtained as an average value of the displacement values measured in the points $S1$, $S2$ and $S3$ which were situated on the top of the plate. The vertical displacements were measured by inductive displacement transducers which were connected to the signal amplifier and via computer recorded and printed. Fig. 5 shows the interpretation of
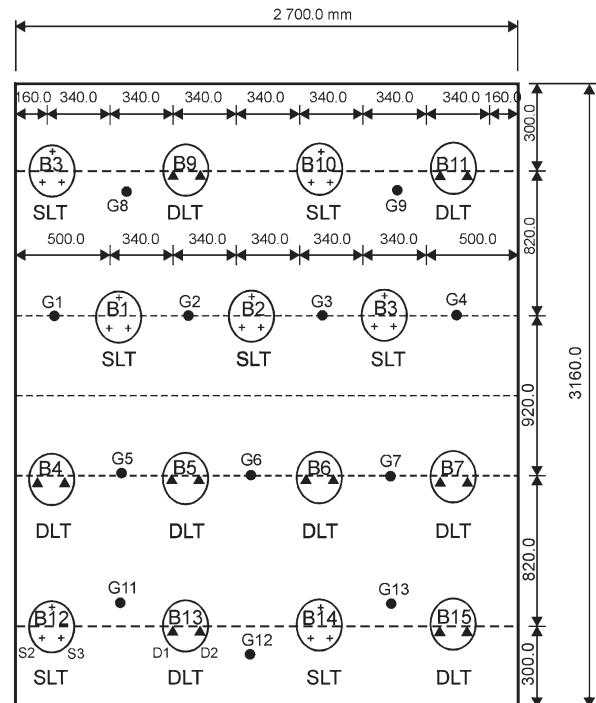
B1 ...B15 POINTS FOR LOADING TEST
G1 ...G3 POINTS FOR TAKING SPECIMENS (●)
SLT = STATIC LOADING TEST, S1,S2,S3 . . . MEASURED POINTS (+)
DLT = DYNAMIC LOADING TEST, D1,D2 . . . MEASURED POINTS (▲)

Fig. 4 Box test layout – Plan view with position of measured points

Fig. 5 Displacement time history of the static loading test

the displacement time history of the typical static loading test (*T6-4/4*).

The resilient modules E_s were calculated according to Eq. (2.1) where $w = \Delta w$ is the measured value of the vertical displace-

ment for the second cycle of the SLT due to pressure $p = 0.22$ MPa. This approach is sufficient because after a number of load repetitions,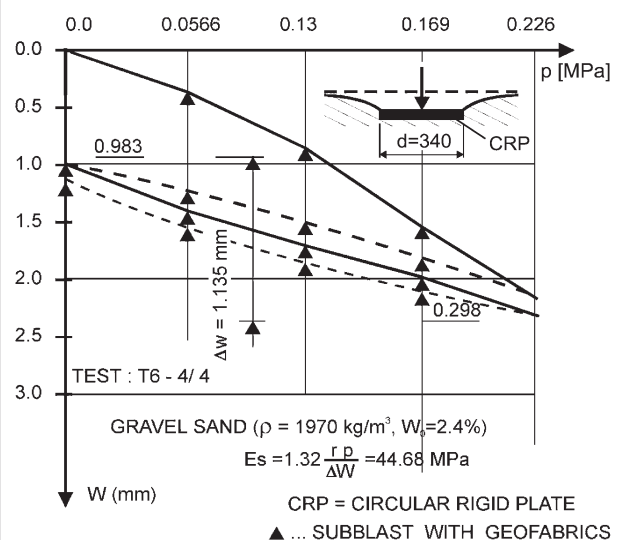 the soils behave nearly elastically as confirmed by the preceding tests. Then the resilient modulus E_s can be defined as a repeated stress deviator (Seed [8]) divided by the recoverable strain and it does not usually change significantly after a large number of cycles.

2.3 Dynamic Loading Test (DLT)

Dynamic resilient modulus E_D (MPa) was evaluated in the same way as a static modulus E_s, but the dynamic load was performed by impact loading test device, Fig. 6. This device consists of the circular rigid plate (1) with the contact area $A = 1000.0$ cm^2, dropping weight (2) with the mass $Q = 12.5$ kg, indention for setting the height of the weight (3), springs (4), plunger (5) guide rod (6) casing (7) and safety pin (8). Dl and $D2$ are points where the dynamic vertical displacements were measured by inductive displacement transducers. The measuring of dynamic deflections was performed by the same set of the apparatus as in the case of static tests. Fig. 4 shows location of the points where $DLTs$ were performed. In each dynamic test 6 impacts in the measured spot were introduced by dropping weight from the constant height h. The height h was set experimentally to achieve the constant area impact stress $p = 0.22$ MPa. In this case the dynamic modules were calculated according to Eq.(2.1), where w is the mean displacement value, obtained from the measurements at points Dl and $D2$ situated on the top of the circular plate (from the 5 last values of the 6 performed impacts). Fig. 7 shows typical displacement time history of the dynamic deflection during the impact caused by dropping weight Q.

Fig. 7 Displacement time history of the dynamic loading test

2.4 Experimental Results

Seven static and eight dynamic loading tests were performed in each layer of the soils creating a corresponding combination of the subballast and subgrade in the box. Finally, we obtained 35 values of the E_s and 40 values of the E_D, for the soil combinations in each box test. There were carried out 6 combinations of soils with and without geofabrics at the top of the subgrade. Fig. 8 shows the ratio between dynamic and static (E_D/E_s) and initial tangent and static resilient modules (E_o/E_s) for individual soils in the box test combination, (Benčat et.al.[2], Benčat[11]). Fig. 8 also

Fig. 6 Dynamic loading test device
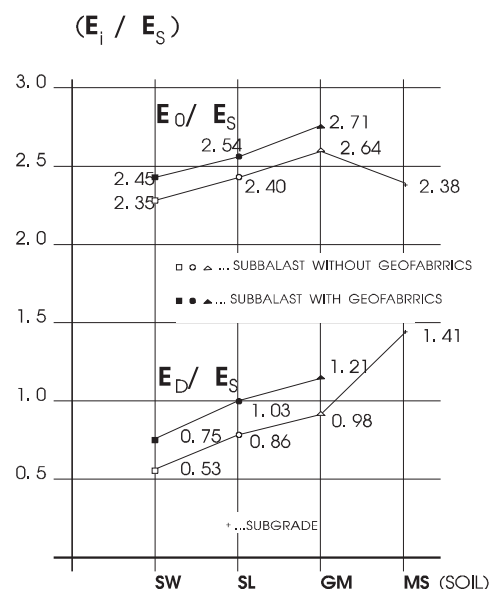
Fig. 8 Ratio E_o /E_s and E_D /E_s for individual soils in the box test combination

shows the interpretation of the ratio E_D/E_s, which was calculated as a mean value of the statistical value collection of all corresponding tests. The correlation coefficient k varied from 0.85 to 0.92.

3. Conclusions

The box test results confirmed the results obtained from in situ tests with the same experimental set – up apparatus. The advantages of the box tests are in the constant conditions during the test performance, possibility to change combinations of soils creating a railway substructure and in lower expenses in performing the study of the track structure material properties. The results will be utilized in performing *DLT* in the evaluation of bearing capacity of substructure layers in present and newly built *Slovak Railways* networks. The technique to determine travel time of body waves in geotechnical seismic testing has received little attention. Travel time is required to calculate shear wave velocities from which initial tangent modules (G_o, E_o) are determined. With the use of portable signal processing equipment and other techniques for travel time determination, such as correlation and spectral analysis, can be effectively implemented in the field tests, too.

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COMMUNICATIONS

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PAVEMENT DIAGNOSIS AS INTEGRANT OF THE PAVEMENT MANAGEMENT SYSTEM

The paper integrates the most important scientific information of the authors in the field of diagnostics of road pavements. Basic attributes of pavement serviceability (evenness, skid resistance) and pavement bearing capacity are the core of the paper. The results of simulations of dynamic systems, outputs of the models developed at the authors place, the correlations to the world established parameters of evenness and skid resistance are presented in the paper. Objective scientific outputs have formed an effectual base for development the Slovak Pavement Management System (PMS). The principles of the Slovak PMS are described.

1. Introduction

The Slovak Pavement Management System (PMS) [10] is a tool for effective dividing of budget for the management of road rehabilitation. The system includes processes for effective maintenance, repairs and renewal of road surfaces and structures. The processes are based on diagnostics of the pavement surface parameters (serviceability level of pavement) and bearing capacity. These parameters input into PMS as follows:

- Surface failures [2] - input by *Index of surface deterioration (ISD)* describing ratio of failures area to surface area. Evaluation criteria include five levels of quality (from excellent to emergency).
- Longitudinal unevenness [2] - input by *International Roughness Index (IRI)* describing longitudinal unevenness quality in five levels.
- Transversal unevenness [2] - input by *Ruts depth* represents the transversal unevenness. The quality is evaluated in five levels scale as for longitudinal unevenness.
- Skid resistance [2] - input by *Skid Resistance Index (SRI)* describing skid resistance in relation to microtexture and macrotexture quality. The methodology of quality evaluation uses three levels of assessment.
- Bearing capacity uses deflection bowls measured by FWD Kuab as an input for the program CANUV [1] that gives the possibility to determine modulus of pavement layers to calculate residual pavement life and overlay.

The technology of pavement rehabilitation is chosen according to all the parameters. Evaluation of these variable parameters (except surface failures) in scope of the Slovak PMS and basic principles of PMS are presented in the next part of the paper.

2. Longitudinal unevenness

The discretionary evaluated road sections, which are homogenous from the point of view of construction and degradation conditions, can be evaluated through the medium theory of stationary random process. This type of random process can be best characterized by a correlation function or power spectral density (PSD). The correlation function $K_h(\lambda)$ for this type of process is expressed in linear domain by equation

$$K_h(\lambda) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} [h_1(l) - E_h] \cdot [h_2(l - \lambda) - E_h] \cdot$$
$$\cdot f_2(h_1, h_2) dh_1 \cdot dh_2 \qquad (1)$$

where: λ - linear lag [m],
     E_h - expected value of stochastic unevenness; $E_h = 0$,
     $h(l)$ - stochastic unevenness,
     $f_2(h_1, h_2)$ - combination density of expectation.

Stochastic unevenness is computed as a difference between a real and theoretical profile. In our case we must identify elevations of longitudinal profile per 0.25 m and longitudinal unevenness are evaluated through the standardized correlation function $\rho_h(\lambda)$ (left hand side of Fig. 1).

For the purpose of unevenness assessment it is more appropriate to use power spectral density (PSD) $S_h(\Omega)$ (right hand side of Fig. 1), which can be expressed from the correlation function by means of Wiener Chinchine equation:

$$S_h(\Omega) = 2/\pi \cdot \int_0^\infty K_h(\lambda) \cdot cos(\Omega\lambda) \cdot d\lambda \qquad (3)$$

where: D_h - dispersion of an stochastic unevenness [m²],
     Ω - angular spatial frequency [rad.m^{-1}],
        $\Omega = 2 \cdot \pi/L$ (4)
     L - unevenness wavelength [m].

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$$\rho_h(\lambda) = K_h(\lambda)/D_h \qquad (2)$$
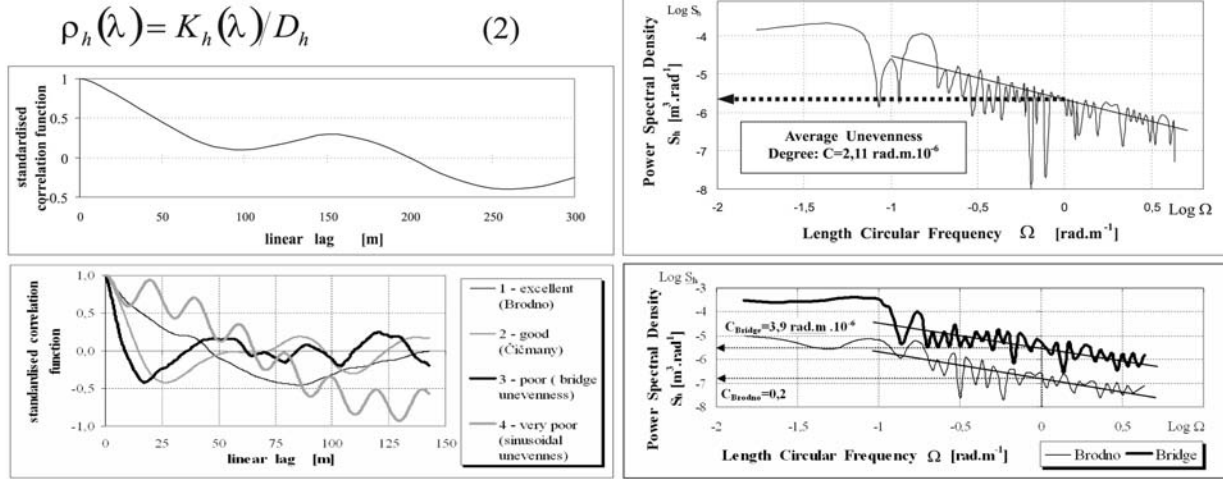
Fig. 1 Standardized correlation function and PSD of stochastic unevenness
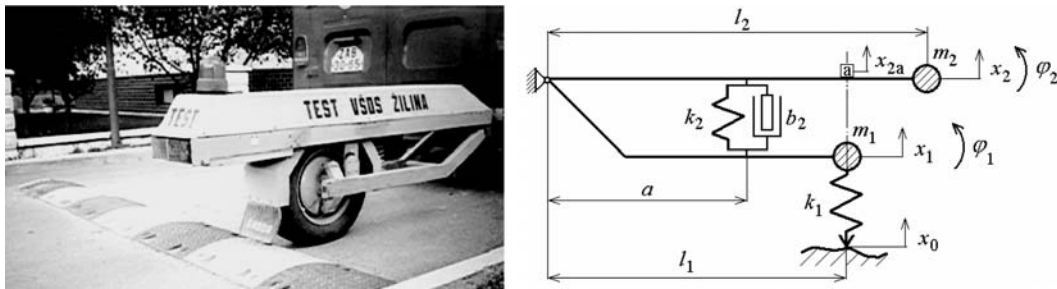
Fig. 2 Single-Wheel Vehicle of the University of Žilina

The measuring set called the Single-Wheel Vehicle of the University of Žilina (Slovak abbr. JP VŠDS) was designed on the DMS (double-mass measuring set) principle. This equipment represents a model of a quarter of the passenger vehicle and its basic parts are presented in Fig. 2.

For vertical vibrations of a quarter car model around the static equilibrium positions we obtain the following equations of motion. The equations of motion for small vibrations are given by [3].

The vertical displacement (velocity, acceleration) of mass m^2 can be expressed as a function of geometry and displacement (velocity, acceleration) of accelerometer as follows:

$$x_2 = (l_2/l_1) \cdot x_{2a}, \ \dot{x}_2 = (l_2/l_1) \cdot \dot{x}_{2a}, \ \ddot{x}_2 = (l_2/l_1) \cdot \ddot{x}_{2a} \qquad (5)$$

After substituting (5) to equations in [3] we can rewrite these relations as:

$$m_1 \ddot{x}_1 + b_2 \frac{a^2}{l_1^2}(\dot{x}_1 - \dot{x}_{2a}) + k_2 \frac{a^2}{l_1^2}(x_1 - x_{2a}) + \\ + k_1 x_1 = k_1 x_0 \qquad (6)$$

$$m_2 \ddot{x}_{2a} + b_2 \frac{a^2}{l_2^2}(\dot{x}_{2a} - \dot{x}_1) + k_2 \frac{a^2}{l_2^2}(x_{2a} - x_1) = 0. \qquad (7)$$

The unevenness degree C [rad.m.10^{-6}] of an evaluated road section is expressed from the basic relation [9] that was modified for our mode of unevenness identification by JP VŠDS.

$$C = \frac{D_y}{I \cdot \frac{1}{N} \sum_{i=1}^{n} v_i} \qquad (8)$$

where: D_y – dispersion of sprung mass acceleration – left hand side of Fig. 3 [m^2.s^{-4}],

I – parameter of dynamic transfer [rad^{-1}.s^{-3}],

C – unevenness degree – right hand side of Fig. 3 [rad.m],

v_i – digital values of a measured velocity – right hand side of Fig. 3 [m.s^{-1}].

The correlations between unevenness degree C and IRI (International Roughness Index) have been determined for a consideration of theoretical dynamical response of RQCS (Reference Quarter Car Simulation) [8] and experimental measurements (Fig. 4).
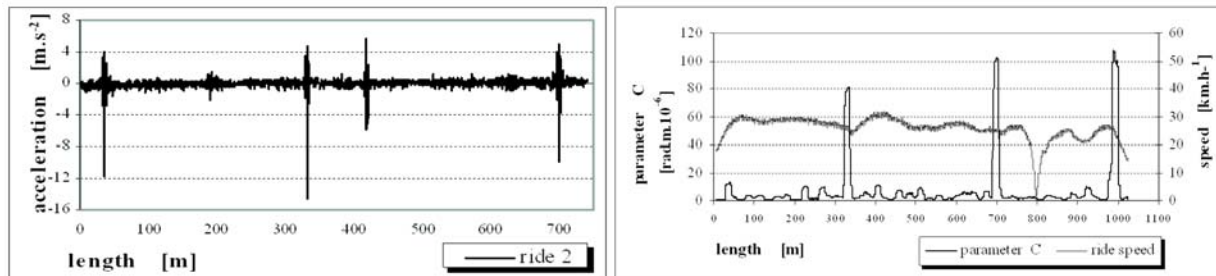
Fig. 3 Vertical acceleration, ride speed and parameter C evaluated by JP VSDS
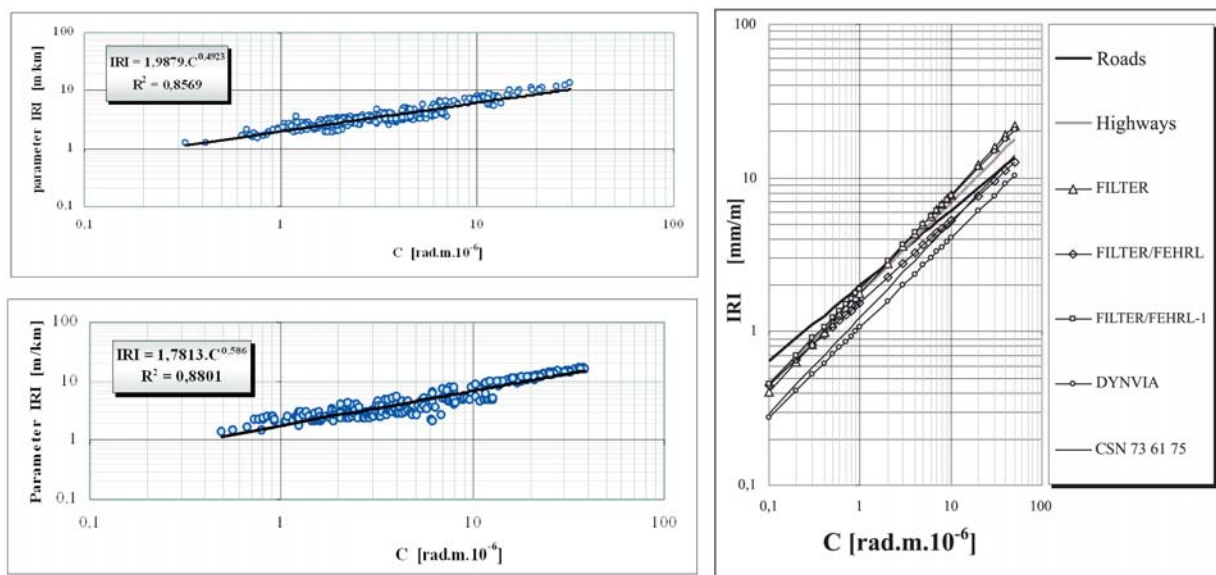
Fig. 4 Comparison of Slovak and World-wide correlation dependences between unevenness degree C and IRI

The comparison of our correlation dependences (roads – ride speed of 90 kph and highways of 130 kph) with results of the Second International PIARC-WRA experiment (including FILTER programme) [3], [4], [11] can be seen in Fig. 4.

3. Skid resistance

Pavement skid resistance was in the past evaluated by different parameters. The parameters were oriented to macrotexture qualify or to detection of skid resistance of pavement surface. At present there are a lot of efforts to unify the measurement evaluation to acquire a common index which is necessary for objective characterization of pavement quality in dependence on its skid resistance properties.

SRI (Skid Resistance Index) is a new methodology mostly used for measurement and evaluation of pavement skid resistance. The Slovak Road Administration devices (SKIDDOMETER BV 11 and PROFILOGRPH GE) are used in Slovakia. Following inputs for estimation of SRI are needed:

- Friction evaluated by Mu from device SKIDDOMETER BV11,
- Operating speed (v) by SKIDDOMETER BV 11,
- Macrotexture evaluated by MPD from device PROFILOGRAPH GE, or by MTD from the volumetric sand patch method.

Taking into account the results of the international harmonization experiment which was focused on measurement and evaluation of pavement skid resistance the estimation of the Skid resistance index (SRI) should be computed by the following equation

$$SRI = A + B \cdot FRS \cdot e^{(S - S_R)/S_0} \qquad (9)$$

where: A, B – particular parameters for each device,
  FRS – reported friction value from the test device (an average value for a chosen interval),
  S_r – reference slip speed determined to 30 kph,
  S_0 – speed parameter linearly related to the result of a macrotexture measurement,
  S – slip speed (relative speed between the tire and the travelled surface at the contact area).

Slip speed is calculated as

$$S = v * \text{slip ratio} \qquad (10)$$

where: v – operating speed of the device [kph],
slip ratio – quotient of the slip speed divided by operating speed (for SKIDDOMETER BV11 is $S = 0.17*v$).

The speed parameter is calculated from the equation

$$S_0 = a + b \cdot MPD \qquad (11)$$

where: MPD – Mean Texture Depth from the device PROFILO-GRAPH GE [mm],
a, b – particular parameters for each device.

The parameters a, b were determined from comparative measurements performed in Slovakia with a particular device to achieve the best correlation with the sand patch test method (MTD). Figure 5 shows the correlation between MTD and MPD measured by PROFILOGRAPH GE. On the base of the determined relation the value S_0 (12) is calculated. The value is necessary for estimation of SRI (13).
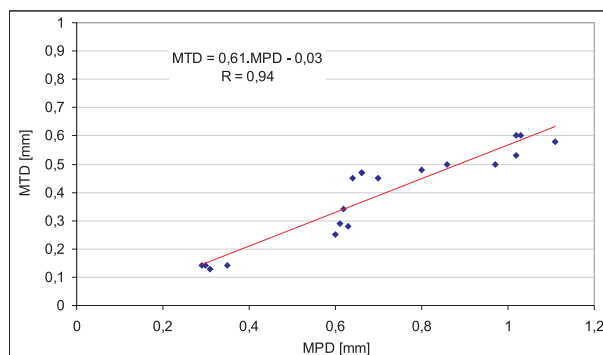
Fig. 5 MPD by PROFILOGRAPH GE versus MTD by Sand Patch Method

Final relations for estimation of SRI for devices which have been used in Slovakia could be defined as follows:

$$S_0 = 68.16 \cdot MPD - 15.0 \qquad (12)$$

$$SRI = 0.101 + 0.78 * Mu * e^{-(S-30)/S_0} \qquad (13)$$

These equations are defined for the estimation of the Skid Resistance Index for the SKIDDOMETER BV11. The SRI shows a good correlation with other pavement skid resistance measurement devices with an operating speed allowance. Figure 6 shows the correlation between the British Pendulum Tester which is one of the mostly used devices in the world.
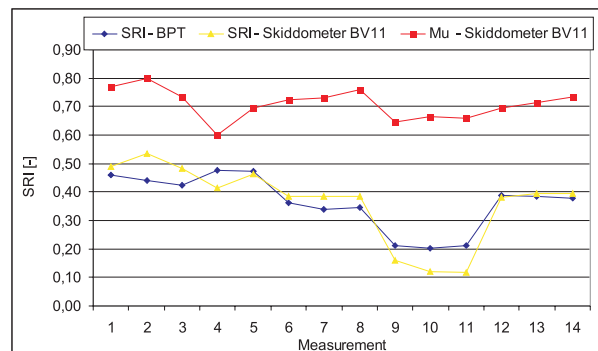
Fig. 6 Relation between SRI and Mu by SKIDDOMETER BV11 and SRI by British Pendulum Tester

3. Pavement bearing capacity

Diagnostics and evaluation of pavement bearing capacity is the important element in the majority of the Pavement Management Systems. Non-destructive testing methods based on the measuring of pavement surface deflection are the most extended in this field. Measured values of deflections are used for an analysis of pavement structure response to an applied load. Characteristics necessary for the pavement evaluation are derived from deflections using some theoretical assumptions. Majority of the used methods replace the real pavement structure by the physical model of linear elastic half-space or linear elastic multi layered half-space. Measured deflections are then considered as vertical deformations of the physical pavement model. These assumptions give the possibility to determine the surface modulus of pavement structure or the modulus of individual pavement layers from the back calculation.

3.1 Evaluation according to surface modulus

Basic theory of linear elastic half space was used for calculation of the pavement surface modulus. The model of static loading of linear elastic half-space by a vertical uniform load was used and dynamic effect of the applied load was neglected. It was assumed that vertical deformations (deflections) of the pavement surface correspond with vertical deformations caused by a static load. Then, within the ambit of a linear elastic half-space theory, when the load is distributed through the elastic load plate of diagnostics equipment the surface modulus of pavement in the centre of load can be calculated from the equation

$$E = \frac{2 \cdot (1 - \mu^2) \cdot p_0 \cdot a^2}{w(0)_{z=0}} \qquad (14)$$

and the pavement surface modulus at the distance "r" from the centre of load (but $r > a$) as follows

$$E_{(r)} = \frac{(1 - \mu^2) \cdot p_0 \cdot a^2}{w(r)_{z=0} \cdot r} \qquad (15)$$

where: μ – Poisson's ratio,
     p_o – contact pressure [MPa],
     a – radius of loading plate [m],
     r – distance from the centre of load [m],
     $w(0)z=0$ – deflection of pavement surface at the axis of load [m].

The surface modulus of pavement calculated according to (14) and (15) is the simplest characteristic of the pavement bearing capacity and it is very often used for evaluation of the pavement structure in terms of its mechanical efficiency and homogeneity. The same principle was used for determination of a criterion for the pavement bearing capacity evaluation in Slovakia at the network level [9].

3.2 Computer program for evaluation

The surface modulus is not sufficient for evaluation at the project level. In this case it is necessary to determine the modulus of individual pavement layers. For this purpose the program CANUV [9] was developed at the Highway Engineering Department of Civil Engineering Faculty. Basic input data are deflection bowls measured by the Falling Weight Deflectometer KUAB. The program uses a back-calculation procedure to determine the modulus of tree main pavement layers (asphalt layers, subbase, subgrade). The principle of back calculation of the modulus is to fit a measured deflection bowl to a theoretical bowl calculated for a set of layer modulus. The theory of an elastic multi-layered system is used for the calculation of the theoretical bowls (deflections). The back-calculation procedure finishes when the following criteria are met:

$$\frac{w_{m,1500} - w_{c,1500}}{w_{m,1500}} \times 100 < 2 \tag{16}$$

$$\frac{(w_{m,0} - w_{m,300}) - (w_{c,0} - w_{c,300})}{(w_{m,0} - w_{m,300})} \times 100 < 1 \tag{17}$$

$$\frac{w_{m,900} - w_{c,900}}{w_{m,900}} \times 100 < 4 \tag{18}$$

where: $w_{m,i}$ – measured deflection in distance "i" from the centre of load [μm],
     $w_{c,i}$ – calculated deflection in distance "i" from the centre of load [μm].

After the modulus back-calculation the reliability of the pavement structure is evaluated. The relation between the bending tensile stress and the material strength with regard to the material fatigue is applied in this process for bound pavement layers

$$S = \sigma / (S_n \cdot R) \tag{19}$$

and the pavement subgrade reliability evaluation is calculated from the formula

$$S = \sigma_z / \sigma_{z,m} \tag{20}$$

where: S_n – material fatigue coefficient ($S_n = a - b \cdot \log N$),
     R – tensile-bending strength of the material in the critical layer [MPa],
     σ – bending tensile stress at the bottom of the layer [MPa].

Provided that $S = 1$ the residual performance of pavement bound layers is calculated as

$$\log N_b = (a \cdot R - \sigma) / b \cdot R \tag{21}$$

where: N_b – residual number of standard axles for bound layer,
     a, b – fatigue coefficients of bound layer

and the residual performance of subgrade is calculated as

$$\log N_S = \frac{0.00346 \times E_s - \sigma_z}{0.7 \cdot \sigma_z} \tag{22}$$

where: N_s – residual number of standard axles for subgrade,
     z – computed vertical stress on the top of subgrade [MPa],
     E_s – modulus of subgrade determined by back calculation [MPa].

The lowest computed value from N_b and N_s determines the "critical layer" of the pavement. The residual performance in standard axles in future years is calculated for this layer from

$$N_R = N_i - \sum_{r=1}^{n} N_n \times \delta_i \tag{23}$$

where: N_R – residual performance of critical layer in the year "r",
     N_i – total residual performance of the critical pavement layer,
     N_n – annual number of standard axles in a design section,
     δ_i – annual increase of the traffic in a design section [%].

For the first $N_R < 0$ residual life of pavement (RL) in the years is calculated from the value of index "r" as $RL = r - 1$. The pavement reliability in term of bearing capacity is classified according to the value of residual life.

4. Slovak Pavement Management System

The most important benefit of PMS is a more objective distribution of financial resources based on an economic analysis, the construction and user costs included. The Slovak PMS works at two levels:
- the road network level solving the selection of sections for rehabilitation, budget requirements and distribution of financial resources;
- project level solving a proposal for maintenance and repairs; it defines requirements for maintenance and repair costs of selected sections.

Decision-making processes of both levels of the PMS use the surface properties and bearing capacity of the pavement as neces-

sary input parameters (most of them are described in the previous part of paper).

Two decision-making procedures could be used generally, priority and optimization, respectively. The former determines technology for pavement rehabilitation according to the current state of pavement serviceability and bearing capacity characteristics.

4.1 Priority procedure

The procedure uses results of evaluation for the pavement serviceability characteristics (evenness, skid resistance, surface failures) but the main criterion is the strengthening layer thickness (overlay) calculated for required residual life of the pavement.

If the strengthening thickness (h_z) is not more than 30 mm a proposal of rehabilitation technology takes into account evaluation of the pavement surface state and pavement unevenness and one of pavement maintenance technologies is proposed (surface dressing, emulsion micro carpet, thin asphalt layer etc.).

If the bearing capacity of pavement is not sufficient for required residual life of the pavement ($h_z > 30$ mm) two cases are possible. A total pavement reconstruction is proposed ($h_z > 100$ mm) or if the calculated hz varies between 40 mm and 100 mm then the final value of hz depends on the pavement surface state classification according to failures and transversal unevenness.

When possible rehabilitation technologies are known an economic analysis has to be carried out. The special software SEH PS [5] analyses an economic effectiveness of the proposed rehabili-

tation technologies. The SEH PS calculates the Internal Rate of Return (IRR) on the base of construction costs of rehabilitation technology and user costs on the base of the pavement surface properties (traffic loading, structural parameters of the road, cost for repairs and maintenance of vehicles, tire wearing, fuel and lubricants costs, lifetime of technology, and other transport costs).

4.2.2 Optimizing procedure

The principal point of optimizing procedure is a determination of optimal time for pavement strengthening. Technology of rehabilitation is proposed in relation to time interval between pavement diagnostic and optimal time for pavement strengthening and current state of pavement serviceability.

An optimizing process of rehabilitation time needs, apart from the above described parameters the degradation models of all the parameters. The degradation models are derived from long time measurements on experimental sections and in laboratories. The quality of the model is a basic assumption of optimizing process precision.

5. Conclusions

The article describes the principles of the decision making processes in frame of the Slovak PMS. The single parameters of the pavement serviceability and the pavement efficiency are permanently observed and evaluated. The results will use for next specification of the decision criteria that are used for determination of the maintenance and rehabilitation methodology.

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COMMUNICATIONS

Martin Matas – Andrej Novak *

MODELS OF PROCESSES AS COMPONENTS OF AIR PASSENGER FLOW MODEL

The paper presents the model of air passenger processes at airports for single source and two source processes. It shall be used as a component for the development of passenger flow model. In the paper the single source model is applied on passenger check-in process. It is based on deterministic queuing model where cumulative diagrams of the flow are used for calculations. As input data the model requires the arrival and service profile of passengers at the check-in. The arrival profile is a cumulative number of passengers arriving to the server during the time. The service profile is a cumulative number of passengers which can be served by the check-in process. The calculation of average waiting time and average length of the waiting queue is presented in the paper. In latter part of the paper the case of two source process is discussed.

Key words: Air passenger flow model, airport passenger processes, deterministic queuing, cumulative diagrams

1. Introduction

The context of our research is the evaluation of future airport concepts in which the airport airsides and the landsides are separated. This idea was presented in [1] and further elaborated in [2]. For the evaluation of the concepts we decided to develop a model of Airside-Landside Separated airports (ASLS). The model should simulate air passenger movements from the city air terminal (landside terminal) to the airside gates while passing through standard passenger-related airport processes such as the check-in, the security, etc. In the literature models do exist for passenger flows at airports. The queuing theory based model presented in [3] is oriented on passenger waiting times in an air terminal based on passenger arrivals in relation to the capacity of passenger processors (i.e. check-in, passport control, departure lounge and embarkation). This queuing theory based approach is too constraining for the development of our model since it requires strong assumptions such as the steady arrival rate. Thus it does not capture well the dynamics of passenger arrivals. On a microscopic level [4], [5], [6] developed the simulation model in ARENA simulation software. An analytical aggregate model for estimating capacity and delays at airport terminals has been presented in [7]. Neither of above mentioned models does capture passenger movement or passenger processes prior to their arrival to the terminal. The models in the articles have not been described in the detail, and often processes have been pre-programmed in the simulation software. Thus it is not transparent for us how the passenger process has been modelled in the detail. For these reasons we decided to develop our own model based on the knowledge about the passenger behaviour. We classified the air passenger activities into passenger processes and passenger movements among the processes.

To get an overview of which models exist that are able to represent passenger processes and movements we conducted a literature review. For the passenger movements the identified models where classified based on the modelling approach to mircosimulation models, cellular automata models, queuing theory based models, gas-kinetics based models and continuum based models, whereas this classification was adopted from [8]. For passenger processes the modelling approaches identified where stochastic models based on queuing theory [3], deterministic models using cumulative diagrams [9], [10] and simulation models [11]. This paper focuses on the processes while the key question is what modelling approaches would best fit for the development of our passenger flow model. The discussion about existing models and the prototype of our model for processes based on the deterministic queuing model is presented.

2. Modelling of Passenger Processes at Airports

2.1 Single source processes

For the passenger processes we considered three options as it is mentioned in the introduction: queuing theory based stochastic models, deterministic queuing models and microsimulation models. In our case the queuing theory models require too unrealistic assumptions about passenger behaviour. For example it requires constant passenger arrival rate while at airports passengers often arrive at variable rate according to the so called arrival profile mentioned in [12]. On the other hand the microsimulation can capture individual characteristics of a single passenger. However, it often requires large amount of data as an input especially about

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the passengers and the airport layout. The model may become too large in terms of input data required and too complex in terms of an amount of objects in it. Therefore for the modelling of the processes we decided to use deterministic queuing model based on cumulative diagrams. This approach is at a mesoscopic level of detail, because it can not represent individual passenger behaviour as microscopic model could but, on the other hand, it still can capture the characteristics of a group of passengers in terms of their arrival behaviour as an advantage against too rigid stochastic queuing models. The model requires that the passenger arrival profile and the server service profile are known. It could be represented by $A(t)$ and $S(t)$ functions for arrival and service profile respectively. These functions are used to produce departure profile $D(t)$ representing passenger departures from the server. By using $A(t)$ and $D(t)$ functions other operational variables could be calculated. Every passenger waits in the line certain time ranging from zero to some value. Sum of all waiting times could be calculated as an area bounded between $A(t)$ and $D(t)$ functions:

$$T_{wait} = \int \big(A(t) - D(t)\big)dt \qquad (1)$$

Cumulative number of passengers at the time t is represented by $N(t)$. Then the average waiting time per passenger until the time t is:

$$t_{wait\_avg} = \frac{T_{wait}}{N(t)} = \frac{\int \big(A(t) - D(t)\big)dt}{N(t)} \qquad (2)$$

Analogically, the average length of the queue as an area bounded between $A(t)$ and $D(t)$ divided by elapsed time between the arrival of the first passenger (t_0) and the facility(e.g. check-in) closing time (t_{close}):

$$n_{wait\_avg} = \frac{T_{wait}}{(t_0 - t_{close})} = \frac{\int \big(A(t) - D(t)\big)dt}{(t_0 - t_{close})} \qquad (3)$$

Other queuing operational characteristics could be calculated but these two are of our particular interest since passenger waiting time could be used for calculating the total travel time and the length of the queue could be used for determining the space capacity of the facility area.

Based on the above mentioned modelling approach we developed a prototype of the passenger process model with a working name "PaxMod". At this stage of development the model requires the arrival and departure profiles to be stepwise linear. Therefore it takes as an input the set of integer data representing the cumulative number of passengers arriving and departing from the server during the time. An example of such data input is presented in Table 1 and Table 2. The data from the tables were collected at Brno-Turany international airport in the Czech Republic on Sunday 20th of May 2007 on the scheduled flight of Ryanair to London Stansted airport. The data inputs are then visualised in a discrete cumulative diagram as it is depicted on the upper part of Fig 1.

CUMULATIVE NUMBER OF PASSENGERS ARRIVING TO THE CHECK-IN AT A GIVEN TIME	Table 1
Time from STD (min)	Cumulative arrivals (pax)
−200	0
−190	4
−170	7
−160	8
−140	15
−130	18
−120	40
−100	65
−90	107
−70	124
−60	162
−40	179
−30	180
−10	184
0	184

CUMULATIVE NUMBER OF PASSENGERS DEPARTING FROM THE CHECK-IN AT A GIVEN TIME	Table 2
Time from STD (min)	Cumulative departures (pax)
−140	0
−120	12
−100	50
−90	84
−70	119
−60	156
−40	184
−30	184
−10	184
0	184

Fig 1 Evolution of passenger arrivals, departures and waiting times at the Check-in (screen shot from PaxMod).

2.2 Processes with two sources

The above mentioned model can represent the evolution of passenger arrivals and departures from the server however it can neither handle characteristics of individual passenger nor a group of passengers. As it has been mentioned earlier we are not interested in individual passengers, however, we are still interested in a group of passengers especially in terms of their waiting times. For instance, we need to distinguish between business and leisure passengers. Moreover the passengers may arrive from different sources. For example, there might be transit passengers arriving to the gate together with departing passengers thus creating one single arrival profile to the gate. To be able to distinguish between the two or more groups we chose an approach explained in the following paragraphs.

Let us suppose that there are two different groups of passengers arriving to the server. One group would be represented by the letter A and the other one by the letter B the frequency of arrivals of the passengers from the groups A and B is represented by the functions $a(t)$ and $b(t)$. Arrival profiles for both groups are integrals of the arrival-frequency functions:

$$A(t) = \int_{t_0}^{t} a(\tau) d\tau \qquad (4)$$

$$B(t) = \int_{t_0}^{t} b(\tau) d\tau \qquad (5)$$

These arrival profiles are combined together in one single arrival profile

$$C(t) = A(t) + B(t) \qquad (6)$$

The service profile of the server is represented by the function $S(t)$. From $C(t)$ and $S(t)$ it is possible to calculate performance variables for the mixed group consisting of the passengers A and B. However we need to calculate these variables for each group individually. Therefore, we need to know what part of the service profile is used by the group A and what part is used by the group B. Thus we divide service profile $S(t)$ on two profiles $SA(t)$ and $SB(t)$ so that $S(t) = SA(t) + SB(t)$. $SA(t)$ and $SB(t)$ could be calculated as follows.

During the short period of time Δt starting at t_i the server with the service profile $S(t)$ can process Δn passengers who stand behind the position n_i in the waiting line (Fig 2). The service rate at which the passengers are processed is denoted by $s(t)$ where $s(t) = dS(t)/dt$. This service rate is proportionally distributed between A and B passengers: $s(t) = s_A(t) + s_B(t)$. The proportion of the A or B passengers in the small group of Δn passengers is denoted by $\rho_{NA}(n_i) \cdot \Delta n$ and $\rho_{NB}(n_i) \cdot \Delta n$ whereas $\rho_{NA}(n_i)$ and $\rho_{NB}(n_i)$ represent the proportion of A or B passengers in the Δn. Thus

$$s_A(t_i) = \rho_{NA}(n_i) \cdot s(t_i) \qquad (7)$$

$$s_B(t_i) = \rho_{NB}(n_i) \cdot s(t_i) \qquad (8)$$
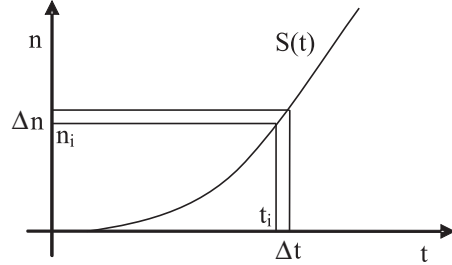
Fig 2 Example of passenger service profile as a function of time.

Now we need to calculate the $\rho_{NA}(n)$ and $\rho_{NB}(n)$ functions. They represent kind of distribution of the passengers A and B in the waiting line. This distribution depends on $a(t)$ and $b(t)$ arrival profiles.
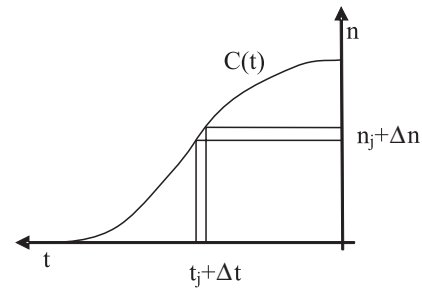
Fig 3 Example of passenger arrival profile as a function of time.

During the short period of time Δt starting at t_j there are Δn passenger arrivals to the waiting line (Fig 3). The proportion of A or B passengers is dependent on the ratio between their arrival rates in a way that

$$\rho_{TA}(t) = \frac{a(t)}{a(t) + b(t)} \qquad (9)$$

$$\rho_{TB}(t) = \frac{b(t)}{a(t) + b(t)} \qquad (10)$$

for every time moment t there exist a position n in the line according to the $C(t)$ where as

$$n = C(t) \qquad (11)$$

To obtain time moment t from (11) we need to use inverse funtion from $C(t)$ denoted by $C^{-1}(n)$ so that

$$t = C^{-1}(n) \qquad (12)$$

If we use (12) as an input to our $\rho_{TA}(t)$ and $\rho_{TB}(t)$ functions we obtain passenger distribution in the waiting line:

$$\rho_{NA}(n) = \rho_{TA}(t) = \rho_{TA}(C^{-1}(n)) \qquad (13)$$

$$\rho_{NB}(n) = \rho_{TB}(t) = \rho_{TB}(C^{-1}(n)) \qquad (14)$$

Later when the service starts $n = S(t)$ which can be put into (13) and (14) to obtain (15) and (16)

$$\rho_{NA}(n) = \rho_{TA}(C^{-1}(S(t))) \qquad (15)$$

$$\rho_{NB}(n) = \rho_{TB}(C^{-1}(S(t))) \qquad (16)$$

To obtain a service rate for the group A and B we combine (7) with (15) and (8) with (16)

$$s_A(t) = \rho_{TA}(C^{-1}(S(t))) \cdot s(t) \qquad (17)$$

$$s_B(t) = \rho_{TB}(C^{-1}(S(t))) \cdot s(t) \qquad (18)$$

Service profiles are parametrical integrals of service rates:

$$S_A(t) = \int_{t_0}^{t} s_A(\tau)d\tau \qquad (19)$$

$$S_B(t) = \int_{t_0}^{t} s_B(\tau)d\tau \qquad (20)$$

If the service time is relatively small in comparison to the waiting time we can use service profile as a departure profile. Then using $A(t)$, $B(t)$, $S_A(t)$ and $S_B(t)$ we can calculate performance variables for each group individually the same way as for service with only one arrival group (par. 2.1).

3. Conclusion

The general aim of our research is to develop a passenger flow model. This would be based on elementary models of passenger processes and passenger movements. So far we have developed a prototype of passenger process applied on the airport check-in. We chose the deterministic queuing approach based on cumulative diagrams. It does not track individual passengers, but it can capture the global performance variables such as an average waiting time, average queue length, average service rate etc. The paper presents how to divide the service profile in two service profiles if there are two sources of arrivals. It is useful when each group of passengers needs to be tracked even in case they mix with other groups in arrival queue.

4. Future Work

At this stage of our research we have been focusing on passenger processes for the development of the passenger flow model. As it is mentioned in the introduction we conducted literature review for existing models of passenger movements. We have not yet developed the model for movements although we have drawn following conclusions from the literature review. For the evaluation of future airport concepts we are interested in passenger flows rather than in individual behaviour of a passenger. Therefore, microscopic models such as microsimulation models or cellular automata models are too detailed and require too much data about the modelled object. On the other hand, macroscopic models may not be able to capture flow dynamics realistically since they are often based on strong assumptions which tend to be unrealistic. For these reasons we decided to make a compromise between the two extremes of having a too detailed model or too unrealistic assumptions. Passenger movements are to be modelled as a flow between the processes where the speed of the passenger flow would depend on the free flow speed and the density of passengers in the corridor. It could be represented using fundamental diagrams elaborated in [13]. The output from the processes would be the input for the models of movements and vice-versa. Therefore, the level of detail for the model of passenger movements would match the level of detail of the model used for passenger processes. Once we develop models of passenger movements at airports they, in turn, could be used as components to connect air passenger processes thus creating the basic structure of a passenger flow model.

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COMMUNICATIONS

Jozef Gnap – Vladimír Konecny *

THE IMPACT OF A DEMOGRAPHIC TREND ON THE DEMAND FOR SCHEDULED BUS TRANSPORT IN THE SLOVAK REPUBLIC

Scheduled bus transport is an important and irretrievable transport system from society point of view. All age groups of population use this transport system to satisfy their own transport requirements. The performance of scheduled bus transport has markedly decreased recently. The performance level is affected not only by fare prices and incomes of population but also by the change in population structure. Population represents potential demand for scheduled bus transport services.

Key words: scheduled bus transport, demographic trend, demand, passengers, inhabitants

1. Introduction

Concrete groups of passengers have different transportation requirements in relation to demand for scheduled bus transport services. Their demand for transportation services is influenced by various factors. These factors consequently affect the possibility of substitution of the bus transport services by individual motoring. It is therefore necessary to investigate not only the total demand for scheduled bus transport but also individual demands of concrete age groups.

The paper deals only with scheduled bus transport (besides urban mass transport), because only this kind of mass transport has provided performance and revenue data in relation to concrete passenger groups in the Slovak Republic.

2. An analysis of performance and revenues of scheduled bus transport in the SR

Performance and revenues of provided transport services are a result of passenger demands for concrete transport services. Fares and incomes of population are the most influential factors that affect demands for scheduled bus transport. The impact of these factors is elaborated in detail at [4], [5] and [6]. This paper is focused on demographic development in the SR as an important factor of the demand.

In the case of fare prices and incomes changes the passengers search other possibility for transportation. It means a change in the scheduled bus transport services and their substitution by other kind of transport, often by individual motoring. Calculation of growth rates for transported passengers was based on data[1] included in table 1. Figure 1 was elaborated on the basis of these data, too.

The number of passengers transported by school reduced fares decreased on the average by 8.5 % and by full fares decreased on the average by 6 % from 2000 to 2004. On the other hand the number of passengers transported by other reduced fares increased on the average by 16.1 % from 2000 to 2004. Pensioners were included into this group of passengers.

The structure of passengers transported by scheduled bus transport (thousands of passengers) according to the fares in the Slovak Republic from 2000 to 2004

Table 1

| Year | Number of passengers transported by | | |
	school reduced fares	full fares	other reduced fares
2000	107 100	309 345	9 181
2001	106 120	281 110	18 589
2002	109 244	269 600	11 829
2003	98 756	244 296	14 800
2004	75 201	240 963	16 662

Source: Elaborated on the basis of [9]

The shares of fares on revenues in scheduled bus transport in the SR in 2000 were:
– 92 % in case of full fares,
– 5 % in case of school reduced fares,
– 3 % in case of other reduced fares.

The situation changed considerably within 5 years, the shares in 2004 were:
– 84 % in case of full fares,

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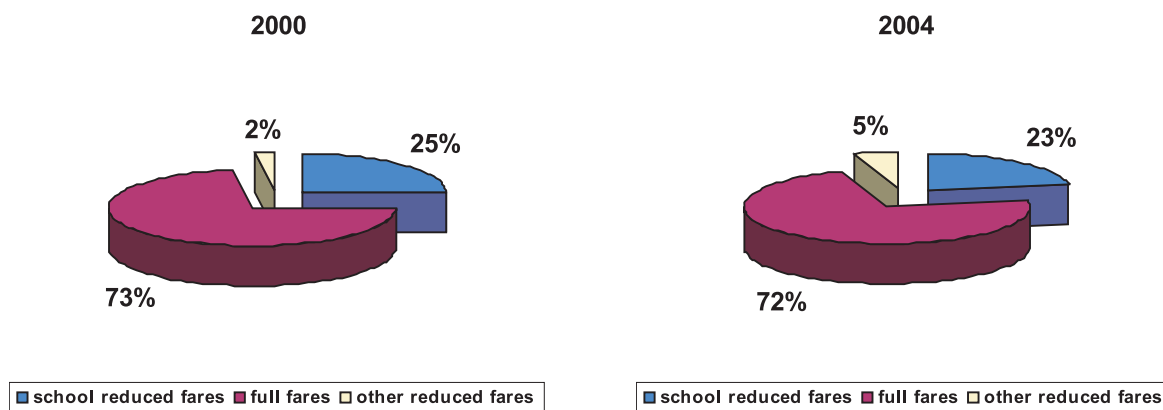
2000

2% 25%

73%

□ school reduced fares ■ full fares □ other reduced fares

2004

5% 23%

72%

□ school reduced fares ■ full fares □ other reduced fares

Fig. 1 Comparison of scheduled bus transport performance split by the fares in the SR in 2000 and 2004

– 10 % in case of school reduced fares,
– 6 % in case of other reduced fares.

For more details see table 2.

The structure of revenues in scheduled bus transport (thousands of SKK) according to the fares in the Slovak Republic from 2000 to 2004

Table 2

Year	School reduced fares	Full fares	Other reduced fares
2000	213 772	3 789 260	105 427
2001	305 335	4 162 912	164 937
2002	288 103	4 138 212	127 698
2003	476 295	4 312 938	139 410
2004	519 489	4 160 039	277 488

Source: Elaborated on the basis of [9]

3. A demographic trend in the Slovak Republic

Aging of population is an all-society problem not only in the Slovak Republic territory in recent years. It caused changes in potential demand of concrete population groups for scheduled bus transport. Fig. 2 depicts aging of population in the SR territory; Fig. 3 includes comparison of population split in the SR in 2000 and 2004.

The structure of transported passengers had to be synchronized with aging structure of population split realized by the Statistical Office of the SR in relation to the investigation of demand and performance of scheduled bus transport. For that purpose the following classification was realized:
– age group from 5 to19 years – school reduced fares,
– age group from 20 to 64 years – full fares,
– age group from 65 and above – other reduced fares.
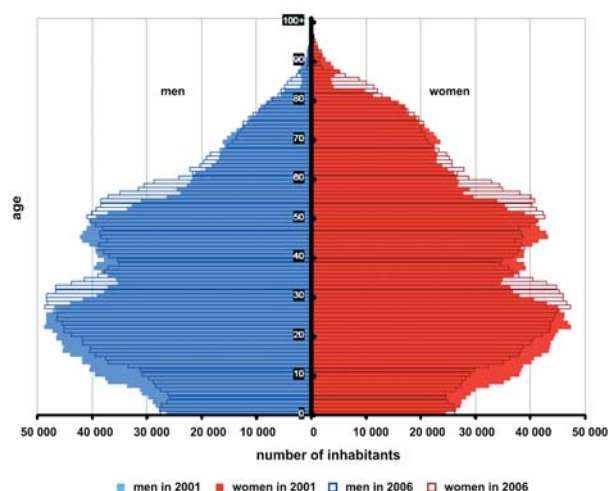
Source: The Statistical Office of the SR

Fig. 2 Population split by the age group in the SR, 2001 and 2006 (December 31)

The structure of population by the age in the SR from 2000 to 2004 (persons)

Table 3

Age group (years)	2000	2001	2002	2003	2004
0 – 4	285 562	274 648	267 273	262 203	259 974
5 – 19	1 194 678	1 175 603	1 142 661	1 109 856	1 076 827
20 – 64	3 302 723	3 316 015	3 353 002	3 387 683	3 422 079
65 and above	619 584	612 685	616 225	620 311	625 942

Source: Elaborated on the basis of [7]

In relation to the structure of age groups the percentage share of pupils and students (5–19 years) decreased by 2 % in a relatively

2000

2004

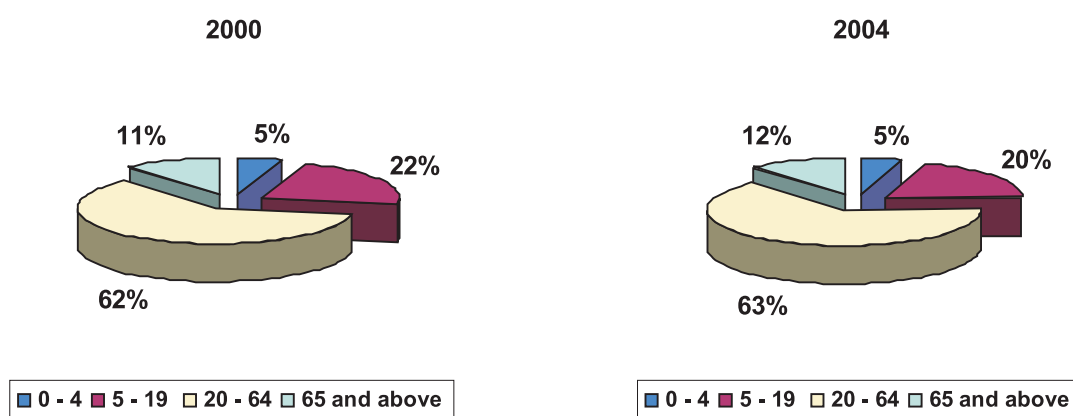*Fig. 3 Comparison of population split by the age groups in the SR in 2000 and 2004*

short time period. Within the mentioned population groups the following trend was observed:
– age group from 5 to19 years – average year to year decrease by 2.6 %,
– age group from 20 to 64 years – average year to year increase by 0.9 %,
– age group from 65 and above – average year to year increase by 0.3 %.

Prognosis of population by the age in the SR is included in table 4. A marked increase in people over 65 years old is expected in the future. The number of pupils and students will decrease to 2015. It will increase again from 2015. The age group from 5 to 19 years (pupils and students) will reach 16 % share of population, working age inhabitants (from 20 to 64 years) will reach 67 % share and pensioners will reach 12 % share of population in 2010.

The years 2020 and 2025 will be extreme and significant in relation to demography because the number of pensioners will be higher than the number of pupils and students. In 2025 the following percentage shares of population are expected:
– 16 % in case of pupils and students,
– 60 % in case of people in working age,
– 19 % in case of pensioners.

The prognosis of population by the age groups   Table 4
in the SR (persons)

Age group (years)	2010	2015	2020	2025
0 – 4	275 816	291 002	284 774	260 885
5 – 19	894 853	821 665	830 320	856 514
20 – 64	3 577 152	3 583 514	3 473 333	3 356 876
65 and above	675 883	775 472	921 798	1 047 470

Source: Elaborated on the basis of [1]

4. A relationship between the demographic trend and the performance of scheduled bus transport in the SR

The demographic process determines a demand for mass passenger transport not only in the SR. Pupils and students represent the greatest potential demand for transportation because they use this kind of transport for travelling to and from school. Their potential demand for transportation is influenced by decrease in their population.

This predication is confirmed by correlation between two variables – the number of people included in the age group from 5 to 19 years and the number of passengers transported by school reduced fares from 2000 to 2004. The correlation reaches the value of minus 0.83, it means a strong negative correlation (dependence) between the investigated variables.

A decrease in pupils and students transported by bus transport is expected in the future similarly as the population of pupils and students decreases. Their habits and activities did not change. This fact will eliminate the possibility of their movement rising (number of trips) in the future. Movement of pensioners achieved a lower level in comparison to other inhabitant groups.

The formula for calculation of future demand for schedule bus transport services in the SR based on the use of average movement of pupils and students within 5 years (from 2000 to 2004):

$$FD_i = NI_i \cdot AMP_{SRF} \qquad (1)$$

where: FD_i means a future demand for schedule bus transport services in the year i, (trips per year or passengers per year),

NI_i means a number of inhabitants included in the age group from 5 to 19 years for the year i,

AMP_{SRF} means an average movement of passengers transported by scheduled bus transport, the variable reaches the value of 86.87 trips per capita and year[2]

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[2] the value was calculated on the basis of [9] as an average of movements of pupils and students for every year during the period from 2000 to 2004

The future demand for scheduled bus transport is calculated on the basis of application of formula (1). The number of passengers transported by school reduced fares is expected to reach 77 736 thousands of passengers in 2010 and 74 405 thousands of passengers in 2025 (in case of changelessness of pupils' and students' demand for scheduled bus transport). This calculation approach respects only a demographic trend, not e. g. changes in fare prices.

The future pupils' and students' demand for scheduled bus transport services can be estimated without respecting the demographic process, too. It can be realized through the application of an average decline rate for passenger transported by school reduced fares. In 2010 44 131 thousands of passengers and in 2025 only 11 643 thousands of passengers will be transported using the above mentioned approach. Respecting the demographic trend in the SR the average movement of pupils and students will be 43.32 trips per year and capita in 2010, it will be only 13.59 trips per year and capita in 2025. These values of movement of pupils and students in 2010 and in 2025 are unrealistic. There is a wide difference between the future movement and average movement value within 5 years (from 2000 to 2004). The future movement in comparison to the current average movement could not be so low because pupils and students travel mainly for education. The education process will not markedly change in the future.

Other approach to calculation of future pupils' and students' demand for scheduled bus transport services is based on the application of regression function that expresses the relationship between the number of passengers transported by school reduced fares and the number of persons included in the age group from 5 to 19 years (see Fig. 4).

Working age inhabitants represent the largest population group. The increase in an average wage is a significant factor affecting demand of people in working age for scheduled bus transport in comparison to the demography as another important factor. The relationship between the average monthly wages and the number of passengers transported by full fares expressed as a correlation reaches the value of minus 0.95, it means a strong negative dependence. The correlation between the number of working age inhab-

itants and the number of passengers transported by full fares reaches the value of minus 0.94, it means a strong negative dependence, too. If the prognoses of demographic trend and average wage level are known, the future demand could be estimated using the regression functions (see Fig. 5). The increase in an average wages influences the decrease in a demand for scheduled bus transport and the increase in a demand for individual motoring.

Fig. 5 Regression functions of relationship between the number of passengers transported by full fares and average monthly nominal wage (the left side of Fig. 5), and the number of inhabitants included in the age group from 20 to 64 years (the right side of Fig. 5)

The possibilities to drive or use the passenger cars are different for concrete population groups. The working age inhabitants have the greatest potential for the substitution of scheduled bus transport services using the individual motoring, followed by the pensioners. The pupils and students till the age of 18 have almost no possibilities to drive own cars (only as fellow passengers).

The working age inhabitants can own driver's licences and can use their cars. This activity is supported rising the average monthly wage in the SR in recent years. Within 7 years (from 2000 to 2006) the average monthly nominal wage increased by 8.6 % per year and the average monthly real wage increased by 9.9 % per year. Even the growth of fuel prices did not reduce the use of passenger cars in the SR till 2006 [6]. An inhabitant of the SR could buy 331 litres of gasoline (price 34.50 SKK per litre) or 366 litres of diesel (price 31.20 SKK per litre) in case of spending the average monthly wage of 11 430 SKK in 2000. In 2004 447 litres of gasoline (price 35.40 SKK per litre) or 496 litres of diesel (price 31.90 SKK per litre) could be bought in case of spending the average monthly wage of 15 825 SKK.

A lot of pensioners (over 65 years) can use their cars, too but they are limited by the level of pensions. Many of them face the health troubles that make them incapable of driving passenger cars. Their incomes are markedly lower in comparison to the incomes of working age inhabitants. 157 litres of gasoline or 194 litres of diesel could be bought in case of spending the average monthly pension of 5 412 SKK in 2000 and 199 litres of gasoline or 221 litres of diesel in case of spending the average monthly pension of 7 046 SKK in 2004 (see Fig. 6).

There is a difference between the average income of working age person and the average income of pensioner in the SR. The
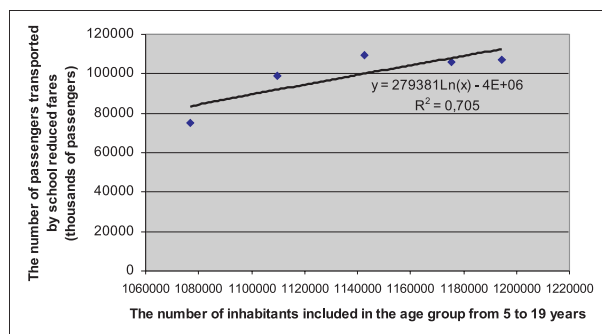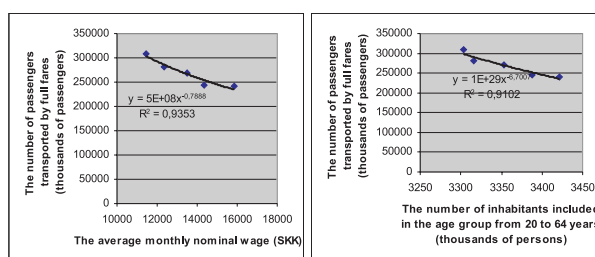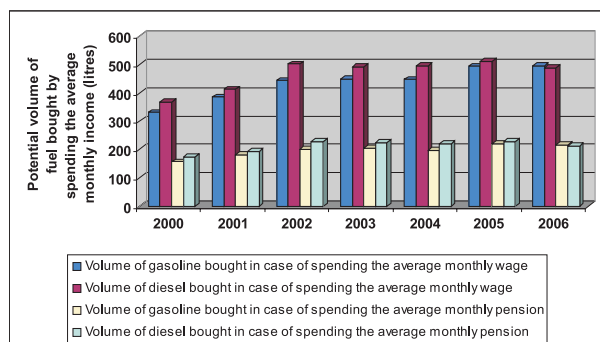

Fig. 4 Regression function of relationship between the number of passengers transported by school reduced fares and the number of inhabitants included in the age group from 5 to 19 years

Fig. 6 Calculation of bought fuels in case of spending the average monthly income

the Slovak Republic economy (in GDP) and by the growth of motorization[3] in the SR. An indicator of motorization does not reach even half of the EU 25 countries' value.

The proportionality of shares of passenger groups based on the full and reduced fares changed in last years. The number of passengers transported by full fares decreased faster in comparison to another groups of passengers. This trend influences and will influence operators` revenues and benefits. It will cause the increase in required sources for financing of public interest transportation performance in the future.

For the expectation of future development in the Slovak Republic, we can define the significant risks as insufficient capacity of road infrastructure, lack of parking places in town residential areas, increase volume of exhaust emissions, higher accident rate in the field of road transport etc. The slowdown of decrease in scheduled bus transport performance could be realized through providing transport services characterized by a higher quality level (higher quality often gives rise to higher operator`s costs and is followed by higher fares) or through the development of integrated transportation systems.

income gaps are deeper in case of comparison between the households consisting of working age inhabitants and pensioners.

5. Conclusion

A decrease of performance of schedule bus transport is expected as a result of the aging of inhabitants and changes in the population structure.

The economy growth and increase in incomes cause the movement of working age inhabitants from using the scheduled bus transport to the individual motoring. A decrease in transported passengers is expected. It is influenced mainly by the growth of

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[3] the number of passenger cars per 1000 inhabitants

COMMUNICATIONS

Milan Gregor – Stefan Medvecky *

ZILINA UNIVERSITY CONCEPT OF DIGITAL FACTORY

This paper focuses on the Digital Factory technologies, the areas of their application, as well as its benefits for industry. The paper presents the Digital Factory concept built in the framework of co-operation of the University of Zilina and the Central European Institute of Technology (CEIT).

Key Words: Digital Factory, Virtual reality, Simulation, Digital Mock Up (DMU)

1. Introduction

Digital manufacturing and product lifecycle management could be the edge that manufacturers in a global economy need to deliver high-quality products with the quickest time to market. Digital manufacturing enables to eliminate errors in the production line, human or mechanical. Product Lifecycle Management (PLM) represents above introduced vision.

Different types of software are linked in PLM solutions, which control different parts of the manufacturing cycle. Computer Aided Design (CAD) systems define what will be produced. Manufacturing Process Management (MPM) defines how it is to be built. Enterprise Resources Planning (ERP) answers when and where it is built. Manufacturing Execution System (MES) provides shop floor control and simultaneously manufacturing feedback. The storing of information digitally aids communication, but also removes human error from the design and manufacture process.

The products innovations are the topic of current discussions. There exists almost no discussion about innovations of production and assembly systems, in spite of the fact that the majority of foreign investment in Eastern Europe was focused in production and assembly.

The Virtual Reality can be used as by the product development as by the design of production processes, workplaces, production systems, etc. The utilization of Virtual Reality by the design and optimisation of production processes and systems is often entitled as Digital Factory [1].

2. Digital Factory

Digital Factory entitles virtual picture of a real production. It represents the environment integrated by computer and information technologies, in which the reality is replaced by virtual computer models. Such virtual solutions enable to verify all conflict situations before real implementation and to design optimised solutions.

Digital Factory supports planning, analysis, simulation and optimisation of complex products production and simultaneously creates conditions and requires team work. Such solution enables quick feedback among designers, technologists, production systems designers and planners.

Digital Factory represents integration chain between CAD systems and ERP solutions, as it is shown in the following Figure.

Fig. 1 Integration of Information Systems in Production [3]

One of very important properties of Digital Factory is the vision to realize process planning and product development with parallel utilisation of common data.
- Digital Factory principle is based on three parts:
- digital product, with its static and dynamic properties,
- digital production planning and,
- digital production, with the possibility of utilisation of planning data for ,
- enterprise processes effectiveness growth.

It is very important to gain all required data only one time and then to manage them with the uniform data control, so that all software systems will be able to utilize it. The integration is one of the main conditions for the implementation of Digital Factory.

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2.1 Digital Factory application area

Digital Factory is appropriate mainly as a support for the batch manufacturing of high sophisticated products, their planning, simulation and optimisation. Its main current application area is automotive industry, mechanical engineering industry, aerospace and ship building industry as well as electronics and consumer goods industries [2].

3D digital model of products (DMU – Digital Mock Up) creates currently basic object for the work in digital manufacturing environment [5]. There exists possibility to optimise products, processes and production systems even by the development phase with the utilisation of 3D visualisation and modelling techniques. This solution brings time to market reduction and significant cost reduction [4]. Such complex 3D models are currently known as so called FMU (Factory Mock Up).

The system for the design of shop floor 3D layouts and generation of 3D models of production halls is missing in current Digital Factory solutions [6]. It is possible to create the 3D model of production hall directly in CAD systems. Such solution is advantageous by new layouts or by new production systems designs.

But, production halls do exists, in majority of real cases. By such conditions, it is often more effective to create 3D model of production hall with the utilisation of Reverse Engineering technologies and 3D laser scanners [5].

The material flow simulation enables to optimise the movement of material, to reduce inventories and to support value added activities in internal logistics chain [7].

The subsystems for effective ergonomics analysis utilise international standards as The National Institute for Occupational Safety and Health (NIOSH), Rapid Upper Limb Assessment (RULA), etc., which enable right planning and verification of man-machine interactions on the single workplaces.

The highest level of analysis is represented by a computer simulation of production and robotics systems which enables optimisation of material, information, value and financial flows in the factory [5].

The Digital Factory solutions enable visualisation and 3D modelling in ergonomics analyses. They offer simultaneously all international standards for ergonomics analyses.

2.2 Digital Factory Advantages

Digital Factory implementation results directly in economic as well as production indicators improvement. Any slight saving realised in a design and planning phase can bring huge cost reduction in a production operation phase. Thanks to this is payback period by investment in Digital Factory very short.

Digital Factory advantages [5]:
- reduction of entrepreneurship risk by the introduction of a new production,
- processes verification before start of production,
- possibility of virtual "visit" of production halls,
- validation of designed production concept,
- optimisation of production equipment allocation,
- reduction in required area,
- bottlenecks and collisions analysis
- fast changes,
- better utilization of existing resources,
- machines and equipment off line programming saving time and resources,
- reduction or full elimination of prototypes,
- ergonomics analyses, etc.

Digital Factory enables to test and reveal all possible production problems and shortages before start of production.

The highest potentials for high quality and low costs of products are in product development and production planning phases. The statistics show that product design and production planning influence about 80 % of production costs [3].

Digital Factory enables product launching time reduction up to 25-50 %. Estimated cost savings are supposed from 15 to 25 %. According to some studies done in industry, using digital manufacturing techniques, twice the amount of design iterations can be processed in 25 percent of the time.

The current production equipment is often inflexible by quick changes. That is why the designers of such equipment are looking for new solutions (automatic reconfiguration of production machines) with fully automated control systems, which will be able to find optimized production process and parameters after production task definition.

According to CIMdata report (March 2003), Digital Factory enables to achieve following financial savings:
- Cost savings by assets reduction about 10 %,
- Area savings by layout optimisation about 25 %,
- Cost savings by better utilisation of resources about 30 %,
- Cost savings by material flows optimisation about 35 %,
- Reduction in number of machines, tools, workplaces about 40 %,
- Total cost reduction about 13 %,
- Production volumes growth about 15 %,
- Time to market reduction about 30 %.

3. Digital Factory implementation methodology

Rough procedure of Digital Factory implementation is as follows [3]:

Definition of total standards and production principles for entire planning operations, creation of primitives and customer databases,

First data collection and organisation with the utilisation of data management system. All responsible persons have direct access to the date, their addition, inspection and changes,

In this phase, Digital Factory system improves co-ordination and synchronisation of individual processes throughout their "networking" supported by workflow management system,

In the fourth phase, Digital Factory system takes automatically some routine and checking activities, which are very time consuming in common systems. Implemented system insures high quality of all outputs.

4. Digital Factory at the University of Zilina

The University of Zilina became the first university in Eastern Europe which uses software solutions for Digital Factory in education and research [5]. The University of Zilina in co-operation with the Central European Institute of Technology started to build their own Digital Factory concept whose structure is shown in Fig. 2.
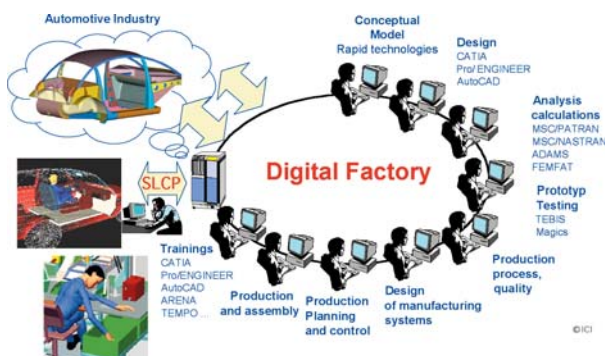

Fig. 2 Digital Factory Concept Built at the University of Zilina [5]

This concept increases the borders of current Digital Factory solutions. It endeavours to integrate activities conducted by designers, technologists, designers of manufacturing systems, planners. It simultaneously tries to increase the offer of individual existing modules. The concept design goes from theoretical studies as well as practical experience gained in industry in Slovakia (VW Slovakia, Whirlpool Slovakia, PSL, etc.).

5. The Digital Factory supports the choice of an appropriate shop floor control strategy

There exist a plenty of production control strategies. Among them, the most known are: Material Requirements Planning (MRP), Load Oriented Control (LOC), Drum Buffer Rope (DBR), Constant Work In Process (CONWIP), KANBAN and Input/Output Control. Fig. 3 describes the basic principles of CONWIP and DBR production control strategies.

The Digital Factory offers the simulation and virtual reality, in general, as the support tools for the analysis of complex systems. The authors developed and validated the procedure for the choice of an appropriate shop floor control strategy for a given production system configuration. This procedure was than applied in a decision process by the choice of an appropriate production control strategy in industry. Further on the authors used metamodelling to simplify the chosen control of the given production system.

The production managers can operate production system either by low throughput times and inventories or by high utilization of capacities. Those control strategies are mutual exclusive. Fig. 4 shows the so called "Production Control Dilemma".

Modelling of large systems such as hierarchical models of entire enterprises require high computing power which is multiplied by utilisation of 3D animation with virtual reality features. Simulation is a time consuming technique. There exists possibility
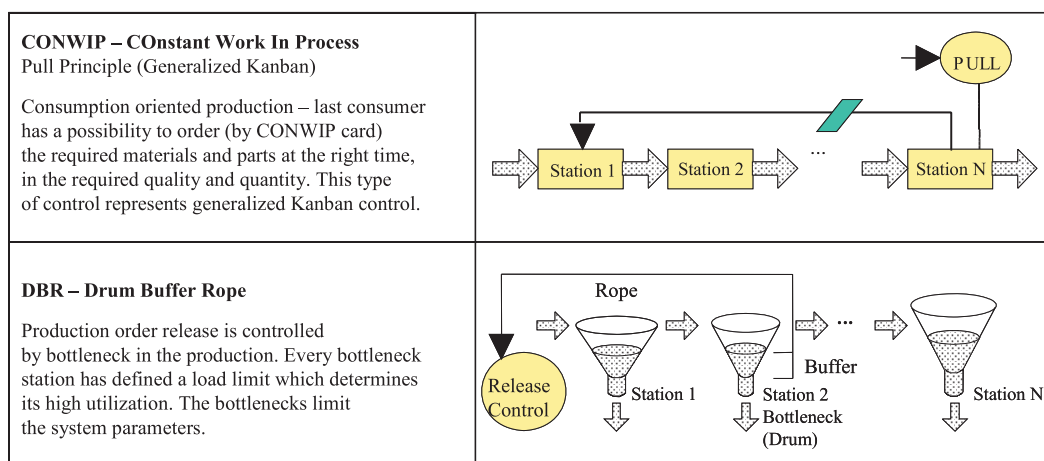

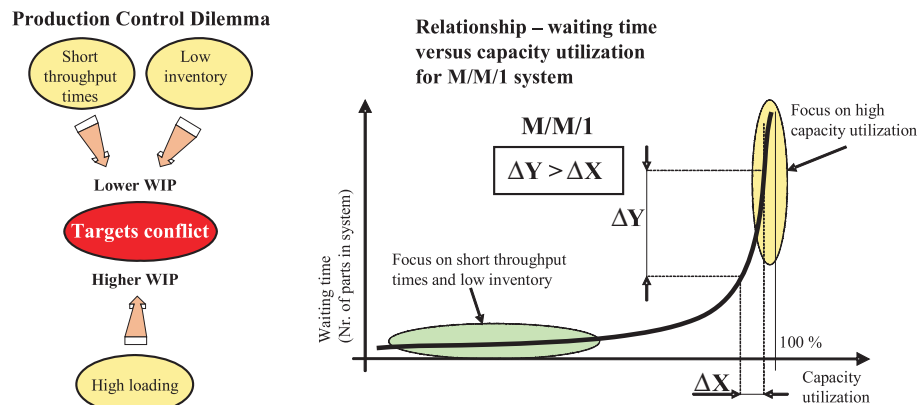Fig. 3 CONWIP and DBR Production Control Strategies [3]

Production Control Dilemma

Relationship – waiting time versus capacity utilization for M/M/1 system

Fig. 4 Production Control Dilemma

to replace very complicated and complex simulation models by validated metamodells and on this way to fasten the decision making process in industry. Metamodelling offers practical approach to the statistical summarisation of simulation results. It enables a given extrapolations in the framework of simulated conditions borders and the fast approximate manufacturing systems control.

The principle of metamodelling is similar to the hierarchical modelling (see following Fig. 5).
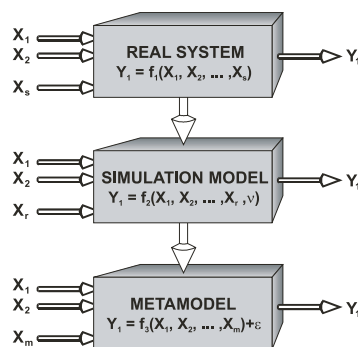

Fig. 5 Metamodellling Principles

Following Fig. 6 shows the proposed procedure for the design and validation of metamodells.

To be able to achieve a short response in forecasting of manufacturing system behaviour under a given control strategy, the below described manufacturing system metamodell was developed. Simulation was used for testing the responses of the production system to the proposed changes of chosen control factors. The set of possible control strategies (KANBAN, CONWIP, DBR, LOC and MRP) was tested with using of the proposed metamodell.

Following part, taken from a comprehensive theoretic and applied study conducted in Slovak industry, contains chosen results for CONWIP control strategy [4].
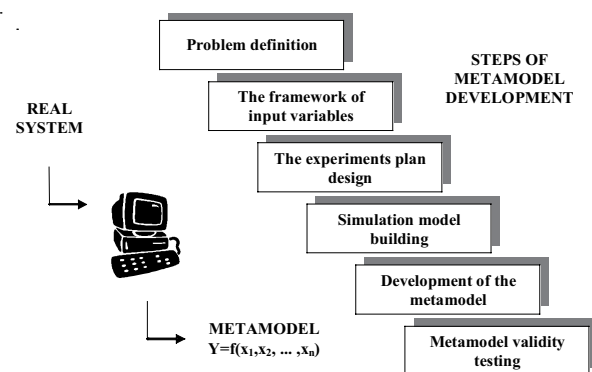
Fig. 6 Steps of metamodel development

Summarization of the simulation results   Table 1.

Experiment	E1	E2	E3	E4	E5	E6
No. of Conwip Cards	1	2	3	4	5	8
Avg. Time (min)	61.04	61.27	62.06	64.11	75.87	118.91
WIP (pc)	0.99	1.99	2.99	3.99	4.99	7.99
Production (pc)	16	32	47	60	63	63

Experiment	E7	E8	E9	E10	E11
No. of Conwip Cards	11	14	17	20	23
Avg. Time (min)	158.47	197.42	233.13	268.01	299.26
WIP (pc)	10.99	13.99	16.99	19.99	22.99
Production (pc)	64	64	64	63	64

The number of CONWIP cards directly determines the level of work-in-process inventories in the system and so influences the average throughput time of production orders and the total through-

put of the production system as well. The mathematical relationships and their graphical presentation are shown as follows.
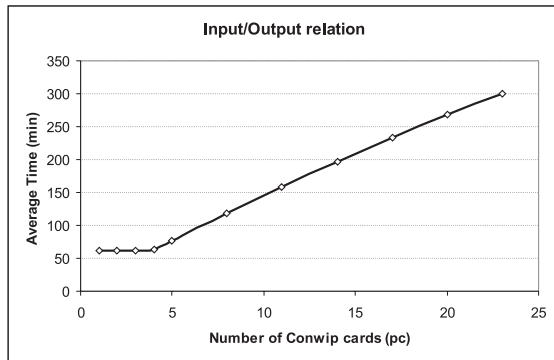
Fig. 7 Relationship between the Number of CONWIP Cards and
Average Throughput Time

The relationships among control factors and production parameters were described by using of regression analysis. The behaviour of complex manufacturing system using given control strategy was substituted by its simplified mathematical models (metamodells). The statistical validation (fitting of mathematical model to the simulation output data) was tested by R^2.

Results of regress analysis   Table 2.

Type of trend	Trend function	R^2
linear	$y = 11.734x + 30.209$	0.9899
logarithmic	$y = 79.865\ln(x) - 6.099$	0.8097
exponential	$y = 53.637e^{0,0825 \cdot x}$	0.9642
polynomial II.	$y = 0.0767x^2 + 9.9704x + 35.993$	0.9913
polynomial III.	$y = -0.0277x^3 + 1.0712x^2 + 0.5603x + 54.248$	0.9973
polynomial IV.	$y = 0.0028x^4 - 0.1603x^3 + 3.0816x^2 + 10.028x + 68.259$	0.9994
polynomial V.	$y = -0.0002x^5 + 0.014x^4 - 0.399x^3 + 5.189x^2 + 17.241x + 75.134$	0.9997

R^2 was calculated as follow:

$$R^2 = 1 - \frac{SSE}{SST}, 0 \le R^2 \le 1,$$

where $SSE = \sum(Y_i - \overline{Y_i})$ and $SST = \sum(Y_i^2) - \frac{(\sum Y_i)^2}{n}$.

Next Figure shows the comparison of several models with the original simulation data. It is evident that the trends with R^2 close to one give the best results.
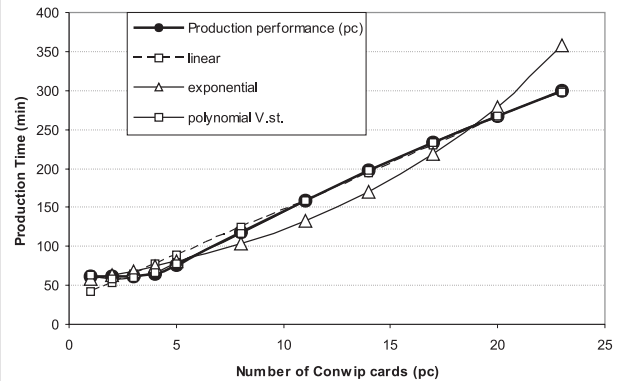
Fig. 8 Comparison of Chosen Models

The developed metamodel offers possibility to find out very quickly and without using simulation the production parameters (e.g. average throughput time, WIP, etc.). The mutual comparison of results from simulation and metamodelling shows insignificant difference (0.0319 min). The part of results of the metamodel validation are shown in the following table. The metamodel is valid on region from one to 22 cards.

Verification of the metamodel   Table 3.

Comparison of average production time	Number of Conwip cards						
	6	7	10	12	15	19	22
Simulation	89.52	104.62	145.84	173.09	210.14	256.89	290.20
Metamodelling	89.20	102.69	145.28	172.31	209.45	255.71	288.28

The following Figure shows the whole region of metamodel validity. The problem started by using the number of cards over 22. The deviation by 30 cards was significant and so it showed that it

Fig. 9 Progress of the Metamodel Equation Using
Polynomial Trend of 5-th Degree

was not appropriate to use polynomial equation of the 5-degree as a substitution of simulation data for these values.

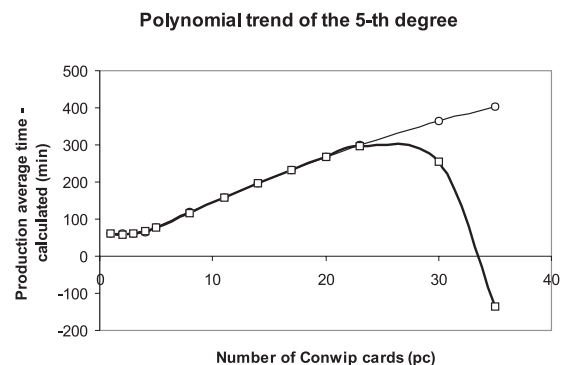The results of the study showed the possibility to simplify the decision making process by the control of complex production systems in the framework of Digital Factory environment.

6. Digital Factory for industry

The further development and prosperity of any country depends on quality of its engineers responsible for innovations. Investment into education brings almost 8 times higher increase of productivity than investment in capital assets.

The University of Zilina has conducted several research studies in industry. In the framework of co-operation with VW Slovakia the DMU model of a real gearbox was developed. The FMU of the whole assembly line for gearboxes assembly in VW was developed for training purposes. A new 3D digital model of production system was developed in co-operation with PSL, a.s. Povazska Bystrica.
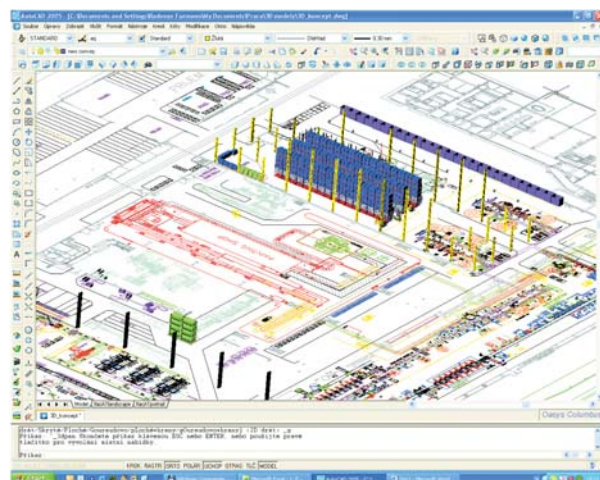
Fig. 10 Digital Model of Production System - Whirlpool Slovakia [3]

The models of chosen workplaces were developed in co-operation with Whirlpool Slovakia. Following figures show some chosen results.
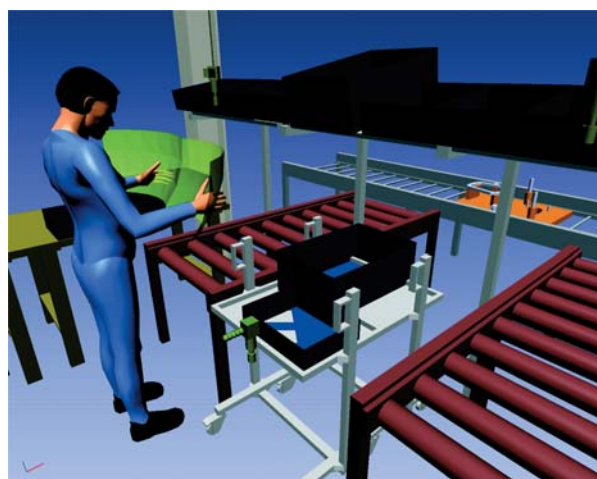
Fig. 12 VW Slovakia - Digital Model of Manual Workplace [3]

7. Conclusion

The future outlook shows that next generation products can benefit from digital manufacturing. Any type of process elements are stored so that as modifications are made at any stage of product development, they are made to the entire design and manufacturing process.

Current research requires huge investment. The governments support innovative research only partially. Our industry also requires Digital Factory solutions. Unfortunately till now it has not been woken up and apart from a few exceptions there is little willingness for investment into research and development.

Every university is obliged to educate students who will be able to design competitive products and production systems with

Fig. 11 VW Slovakia - Real Versus - 3D Digital Model [3]
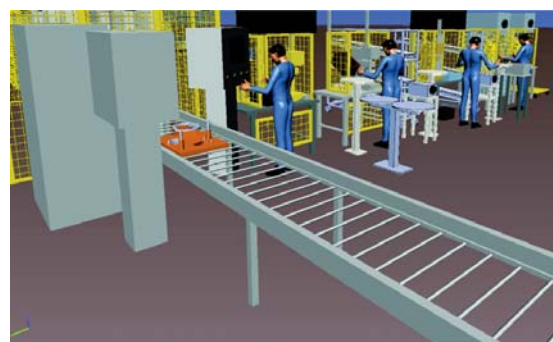
the help of advanced information technologies. The common intention of the University of Zilina is to establish a fully integrated system for the design of sophisticated production systems with its main focus on automotive and electronics industries. Such a system should enable to bring new technologies into industry as well as into education and simultaneously to support the education of future designers, designers of manufacturing systems, technologists and managers.

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Ladislav Simak *

INCREASING THE SECURITY LEVEL
IN THE SLOVAK REPUBLIC

Increasing the security level in every country is a complicated process that affects all spheres of social life and requires a longer period of time. The improvement of the security system in the Slovak Republic has been taking place continuously since the country joined the European Union and NATO and has not been finished so far.

Key words: security, security system, public administration, risks, crisis situations

1 Introduction

An ambitious objective of every government is to create optimal conditions for achievement of ever-increasing standard of living of all inhabitants. A necessary prerequisite to achieve the objective is not only a high level of national economy but also purposeful and efficient functioning of the whole system of public administration. Public administration has to be developed in harmony with changing external and internal conditions induced by new requirements. On the other hand, there are fewer and fewer available sources in the public sector, which brings pressures to reduce the state administration on central and local levels, its decentralization and transfer of competences to regional self-administration.

Present society cannot develop continuously in permanently changing conditions without complete provision of not only internal but also external security. Security has to be perceived as the state of social, natural, technical, technological system or any other system which in concrete internal and external conditions enables fulfillment of given functions and their development in compliance with interests of man and society. The security of Slovakia and its citizens is a complicated phenomenon consisting of a whole series of elements, internal and external phenomena as well as concrete processes and activities. It can hardly be secured without adopting specific legal regulations among which are laws, legal norms of lower legal validity but also international agreements and standards.

Since 1993 when we started to build a new legal environment in the Slovak Republic first laws and legal norms of lower legal validity have appeared and gradually formed a legal framework of crisis management. Its fundamental element is the Constitution of the Slovak Republic No. 460/1992 Coll. in the wording of further regulations, which provides basic human rights and freedoms and protects life as the basic value. An important role in this process was played by Law of the National Council of the Slovak Repub-

lic No. 42/1994 Coll. on civil protection of population in the wording of further regulations, which is, after a series of partial amendments, also at present the functional part of the legal environment of crisis management. The greatest number of laws was approved in the beginning of the 21st century when the Slovak Republic was getting ready for its accession to the EU and NATO. The extreme effort to meet all the requirements and standards of the European Union on the one hand, and a non-uniform attitude and split attitude of individual departments on the other hand, resulted in a series on nonsystematic steps having negative impact on the level of harmony in the crisis management legal environment.

As it comes from the audit of the state crisis management approved by the government of the Slovak Republic and also from the Complex Plan of building and further development of security system of the Slovak Republic and also from the Proposal of measures for the material "The objective of building and further development of the security system of the Slovak Republic up to the year 2010" the legal environment is not sufficiently interconnected and there are many unanswered questions in it. A significant shortcoming the crisis management has been facing since the setting up of the Slovak Republic has been an absence of a unified interpretation of basic terms used in the area of crisis management. The accession to the EU and NATO did not solve the problem and other new terms which are not correctly translated and understood in our mother tongue appear continuously.

The state security is the state which enables functioning, stability and development of the country, preserves peace, sovereignty, territorial wholeness and frontier indefatigability, internal order within the state, fundamental rights and freedoms of citizens and protection of lives and health of citizens, property and environment. The state security can be achieved through a wide range of tools among which is security policy of the state. It can be defined as a set of principles and procedures implemented to achieve a desired

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level security of the system. The security policy of the country is then a set of tools and procedures to achieve basic aims and objectives of the country in the field of defense, protection and internal order within the country and its citizens.

The priorities of security policy of the Slovak Republic were defined in the security strategy which expresses objectives and needs of citizens and the state and respects the EU security strategy. The present government has identified them in its Programme Proclamation and implements them in its activities both in domestic and foreign policies. To the basic objectives to achieve a higher level of security in Slovakia belong the following:

- To permanently take steps to provide security for citizens of the Slovak Republic and its territorial wholeness, sovereignty, independence and identity,
- To protect basic human rights and freedoms of citizens in every situation, including crisis situations,
- To develop democratic state arrangement, legal state and market economy in the Slovak Republic,
- To provide sustainable economic, social, environmental and cultural development of society and diversification of sources together with protection of critical infrastructure of the state,
- To participate in providing security and stability of EU member states NATO aliens,
- To foster good relations with other countries, to develop all kinds of mutually advantageous cooperation and to support partnerships and alliances with countries we share common principles and interests,
- To protect and foster international order, freedom, democracy, peace and stability in the world and uphold human rights,
- To foster transatlantic strategic partnership and support the extension of NATO and EU in the Euro-Atlantic space [1].

2 A Theoretical Treaty of the System of Crisis Management in Public Administration

To systematically solve the Security System of the Slovak Republic, a team of workers of the Crisis Management Department presented in 2003 a project draft for the VEGA draft agency under the title "A Model of Crisis Management in Public Administration". The project was approved under the project number 1/1259/04 and its solution began in 2004.

In the introductory stage of the solution a detail analysis of the contemporary situation of crisis management in the conditions of the Slovak Republic was carried out. The researchers outlined criteria by means of which they were able to assess the level of crisis management in different branches. They analyzed the task of each branch within the security system of the Slovak Republic and pointed out the structure of their mutual relations. Then they participated in the preparation of materials assessing the current situation in the building of the security system in the Slovak Republic and together with a commission of the deputy prime minister of the Slovak Republic they evaluated also the process of crisis management and its efficiency after the accession to the EU and NATO. In relation to the achieved results they prepared introduc-

tory materials which could have been used as amendments to the conception of the top level of security system of the Slovak Republic.

Within the process of theoretical and methodological unification of crisis management terminology the authors suggested to create a dictionary of crisis management terminology. On the basis of the decision of the cross-sectional commission almost one hundred basic and most frequently used terms were chosen. Within the period from September to October 2004 the team of researchers compiled *A Dictionary of Crisis Management Terms* in the volume agreed upon during the period of cross-sectional reviewing. The compiled dictionary was in the first stage published as study material for students of the faculty (ISBN 80-88829-75-5) and simultaneously, it was sent for cross-sectional reviewing. At the seminar held on 21 October 2004 in the hotel Bôrik in Bratislava it was declared that the expectations of prevailing majority of representatives of participating sectors were met. Practically all of them evaluated the presented material in a highly positive way agreed that after partial comments and completion of some terms the material could be administratively modified and presented at the session of the Security Council of the Slovak Republic and subsequently also of the Government of the Slovak Republic.

The Office of the Security Council of the Slovak Republic presented the modified dictionary to the Security Council of the Slovak Republic which, on its 10th session held on 14 June 2005 approved by its decree No 89 "Activities and tasks of the cross-sectional terminological commission". The commission will further continue in the process of completing and updating the terminological dictionary. The process of approving the dictionary was finished on 6 July 2005 when the government of the Slovak Republic discussed and by its decree No 523 approved the *"Terminological dictionary in the area of crisis management and principles of its usage"*. The Slovak Republic government recommended the terms to be used in legislative process in formation of new legal norms as well as in amendment of effective legal norms of crisis management. A shortcoming of the completed process was the formulation through which the government only recommended the usage of approved terms.

A range of possible effect of the state administration and self-administration during crisis solution in society was defined and consequences of social crisis and their influence on population were analyzed. The structure, tasks, internal and external relations of the integrated rescue system and its elements were assessed in detail. Consequently, the system of crisis management during crisis of non-military character of a great extent was optimized from the point of view of structure and competences. To support the suggested solution the research in compliance with outlined objectives at the level of central authorities of the state administration was carried out.

Further, a complex logical model of crisis management in public administration was completed and its chosen parts were optimized focusing on the solution of crisis phenomena at the level of central authorities of the state administration. Consequently, methodolo-

gies of crisis management were worked out and non-traditional methods of solution of chosen crisis phenomena were designed. Results achieved during the project solution were continuously presented in conferences and published in journals, mainly in Civil Defense. The conclusions were simultaneously used in the commission of vice-chairman of the security council of the Slovak Republic and conditions for the plan of gradual transformation of the security system of the Slovak Republic were created. On 10 October 2005 there was a meeting of the members of the research team with a working group of the deputy prime minister of the government of the Slovak Republic in the hotel Bôrik in Bratislava, during which the "Proposal of the change in the conception of security system of the Slovak Republic" was completed.

Another concrete output of the project was the *Analysis of impact of reorganization of public administration to the level of crisis management in authorities of local administration and territorial self-administration.* The analysis was carried out on the basis of the requirement delivered by the authorized representative of the government of the Slovak Republic responsible for reorganization of public administration. Within the analysis the transformation of public administration was assessed in all its complexity from the point of view of functionality of crisis management in public administration and capabilities of authorities of local state administration and self-administration to take preventive measures and at the same time to solve crisis situations. The emphasis was laid on the cooperation of the local state administration and self-administration and their mutual links in the area of crisis management. The SWOT analysis was done with the aim to define strong and weak points of transformation of the local state administration and also opportunities and threats produced by a wider safety environment. The analysis showed that the weakest point in the whole system of crisis management were villages, mainly those having a lower number of inhabitants. The best solution seems to be to connect such villages and create common offices in which crisis management could be performed in a way of transformation of state administration activities.

On the basis of the project results and mentioned complex assessments the staff engaged in the analysis preparation suggested the following:
• to modify the conception of the security system on the basis of the presented proposals,
• to make modifications of particular legal norms in the area of security with emphasis on:
 – the change in the responsibility of the regional security council from a decision-making body to advisory and to abolish the district security council,
 – transformation of the Central Crisis Headquarters and Main Post of Management to a Unified Crisis Headquarters of the Slovak Republic formed on the basis of the National Centre of Crisis Management which would be the executive body of the Slovak Republic government to solve crisis situations,
 – amendments to constitutional law No. 227/2002 Coll. on security of the country during war, state of war, martial law and state of emergency,

 – legislative amendment to the formation of the fifth crisis state under the name of emergency state,
 – working out a new law on crisis management of the country,
• to create the Situational and Assessment Centre of the Slovak Republic as a permanent above-sectional, professional body of the Slovak Republic government designed for continuous monitoring and analysis of security situation in the Slovak Republic and abroad,
• to specify conditions for gradual formation of the central body of the state administration in the area of crisis management,
• to create conditions for transformation of state administration in the area of crisis management to villages and for this purpose to create common offices within micro-regions or centralized villages,
• to abolish crisis management departments of district authorities and part of their staff employ in villages with extended legislature [8].

Presentation of the results was during educational courses for employees of sections of crisis management of central bodies of the state administration which took place in 2005 with the support of the European social fund (Unified programme document NUTS II – Bratislava, objective 3 – human sources development). More than 180 crisis managers from central bodies of the state administration as well as from the local state administration in Bratislava were trained in two three-month courses. In the subject Crisis management the participants got acquainted with the project results with special emphasis on proposals of changes in the security system of the Slovak Republic on the top level of the state management (creation of the National Centre of Crisis Management and Situational and Assessment Centre, transformation of the Central Crisis Headquarters, ...).

Apart from the above mentioned, the project results were also discussed in the section of Crisis Management on the 10th and 11th international scientific conferences "Solution of Crisis Situation in Specific Environment" organized by the Faculty of Special Engineering of Žilina University in Žilina in June 22 – 23, 2005 and June 29 – 30, 2006. Several members of the research team presented their contributions at the conferences. Their presentations were discussed and commented on by specialists from practical setting as well as representatives of research institutions and institutions of higher learning. Majority of reactions was positive and confirmed the correctness of the solution.

Last but not least, the project results were published in some articles in the journal Civil Defense – revue for civil protection of citizens. The mentioned articles have considerable importance for professional practical setting and shaping the opinion on the level of central bodies of the state administration, local state administration as well as executive bodies of crisis management.

3. Changes in opinions on the security system of the Slovak Republic and on the system of crisis management in public administration after the election in 2006

To solve crisis phenomena on the level of the Slovak Republic government it is necessary to create an efficient system of crises solution which will be a complex of subsystems of crisis management of individual branches. Their competences and possibilities as well as powers and tools should jointly provide new quality and thus also inevitable conditions for solution of any crises.

The increase of the security level of the Slovak Republic depends on a degree of harmonization of security needs with development of security environment as well as with economic, political, social, demographic and scientific and technical potential and capabilities of the state. The state has to provide the active security environment, act preventively and avoid stresses and crises. On the other hand, it has to be capable to solve – early, decisively and efficiently – all crisis situations by means of established forces, tools and sources.

In spite of the efforts of numerous specialists, the Security System of the Slovak Republic has not been completed. Although expert groups worked for several years, their results were not approved and accepted by the decision making bodies. All the efforts led only to partial success having mostly the character of legislative measures and other presented documents were put aside.

The present state administration has decided to prepare a new objective of building and further development of the security system of the Slovak Republic up to 2010. Crisis managers have again a challenge to pursue three basis steps which are mutually interrelated and represent institutional solutions of the basis problems:
• to optimize the security system of the Slovak Republic,
• to prepare the project of the national body of crisis management,
• to create a situational and analytical centre of the Slovak Republic.

The process of completion of the security system must be of a systematic character. It should, therefore, be based on a complex assessment of responsibilities, competences and tasks of all central bodies of the state administration, local state administration and territorial self-administration and relations among them in the area of crisis management. The transformation of institution of security system has to be carried out simultaneously with changes in executive bodies of crisis management among which are armed forces, armed security forces and other executive forces including

appointed legal and physical entities. These principles have not been met. Proclamations of top representatives of the Slovak Republic government about abolition of regional offices and reductions in the state administration were not preceded by any serious analyses and did not have any positive impact on the further development of the security system.

Within the framework of preparation of the principles to be included in the Objective of building and further development of security system of the Slovak Republic up to 2010 the emphasis has to be put on:
• the explicit definition of objectives which should provide a higher quality and level of crisis management,
• the unification of content of terms which are to be used in the Objective and have not been processed in the Terminological dictionary of crisis management,
• the harmonization of activities and processes which might be shared to provide external and internal security of the country (military and non-military risks and crisis situations resulting from them),
• the assessment of inevitability of above-sectional coordination in the area of crisis management,
• the assessment of level of sources to provide balance of needs of the security system,
• the creation of a unified system of crisis management planning and specification of principles concerning the implementation of the plans in the process of crisis situations,
• the preparation of proposals of legislative changes.

4 Conclusion

The process of providing a higher level of security in the Slovak Republic has its positive moments but also a number of faults and mistakes. In spite of the fact that several times it has been very near to qualitative changes, responsible representatives have not found enough courage to pursue the changes. Again, we are at the turning point from which there are several ways. It will be very important for those who are responsible for future development to choose the right way, or at least, to choose the direction which seems to be the most suitable in given conditions. The National Council approved the law through which it transformed the competences from the regional authority to the district authority in the seat of the region and the proposal of measures for preparation of the material "Plan of building and further development of security system of the Slovak Republic up to 2010" has been rejected. The security of Slovakia is an inevitable prerequisite for peaceful life of each citizen and therefore it has to receive due attention of all of us.

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COMMUNICATIONS

Miroslava Ruzickova *

DISCRETE AND DIFFERENTIAL EQUATIONS IN APPLIED MATHEMATICS

In the contribution we map investigations performed in mathematics at the Faculty of Science in 2003–2007. Main directions developed at the faculty are described and some of the latest achievements are presented. We illustrate some results from control theory for discrete delayed equations, perturbed linear problems for discrete equations, results concerning the structure of solutions of delayed differential equations, results of orthogonal polynomials, and a description of some numerical investigations in discrete least-squares and collocation methods for numerical solution of a boundary value and eigenvalue problems. Some of accompanying mathematical activities are mentioned as well.

1. Introductory Words about Mathematics at the Faculty

The last five years are distinguished by huge research in various mathematical disciplines in mathematical departments of the faculty. This research was also stimulated by intensive mathematical collaboration with mathematicians from outside and with participation of our colleagues in international conferences. We also organized several international meetings. We mention, at least, two successful meetings – the IMC conference (International Mathematical Conference) in 2003 in Zilina devoted to the 50th anniversary of Zilina University and 5th anniversary of the Faculty of Science, and the ICDDEA 2006 conference (International Conference on Differential and Difference Equations and Application) in Rajecke Teplice. The mathematical research in this period was supported by eight scientific projects VEGA (the Grant Agency of the Slovak Republic) as well as by two international scientific projects and three projects KEGA (the Cultural and Educational Grant Agency of the Slovak Republic). Recently, 6 members of the staff successfully defended their Ph.D. theses in applied mathematics and at present, two members of the staff are preparing for their habilitation and professor procedures in mathematics.

One of the traditional direction of research is the field of functional differential equations, difference and discrete equations and their applications. This tradition goes back to such outstanding authorities as prof. Pavol Marusiak (1935–2000) and prof. Jozef Moravcik (1934–2005). Another traditional direction of research is the field of orthogonal polynomials which was founded by prof. Josef Korous (1906–1981).

2. Main Mathematical Trends

We will give a short description of main mathematical directions pursued at the faculty together with formulations of selected important results.

2.1 Difference Equations

2.1.1 Controllability of Linear Discrete Equations

One of rapidly developed directions is investigation of controllability of linear discrete systems with delay. Such research is important due to numerous applications of mathematical models in natural sciences, economics and engineering. Let us consider a discrete system with pure delay

$$\Delta x(k) = Bx(k-m) + bu(k), \qquad (1)$$

where $m \geq 1$ is a fixed integer, $k \in Z_0^\infty$, $Z_s^q := [s, s+1, ..., q]$, B is a constant $n \times n$ matrix,

$$\Delta x(k) = x(k+1) - x(k),$$

$x : Z_{-m}^\infty \to R^n$ is unknown solution, $b \in R^n$ is a given nonzero vector and $u : Z_0^\infty \to R$ is the input scalar function. Together with (1) we consider an initial (Cauchy) problem

$$x(k) = \varphi(k) \qquad (2)$$

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with a given initial function $\varphi : Z_{-m}^0 \to R^n$.

System (1) is called relatively controllable, if for any initial function $\varphi : Z_{-m}^0 \to R^n$, any finite terminal state $x = x^* \in R^n$, and any finite terminal point k_1 equal or greater than a fixed integer $k^* \in Z_1^\infty$ there exists a discrete function $u^* : Z_0^{k_1-1} \to R$ such that the system (1) with the input $u = u^*$ has a solution $x^* : Z_{-m}^{k_1} \to R^n$ such that $x^*(k_1) = x^*$ and $x^*(k) = \varphi(k)$ if $k \in Z_{-m}^0$.

The crucial role is, in several investigations of this type, played by so called discrete matrix delayed exponential (see [8]) defined as follows:

$$
e_m^{Bk} \overset{def}{=} \begin{cases} \Theta \text{ if } k \in Z_{-\infty}^{-m-1}, \\ I \text{ if } k \in Z_{-m}^0, \\ I + B \cdot \binom{k}{1} \text{ if } k \in Z_1^{m+1}, \\ \ldots \\ I + B \cdot \binom{k}{1} + B^2 \cdot \binom{k-m}{2} \\ + \ldots + B^\ell \cdot \binom{k-(\ell-1)m}{\ell} \\ \text{ if } k \in Z_{(\ell-1)(m+1)+1}^{\ell(m+1)}, \\ \ell = 0, 1, 2, \ldots \end{cases}
$$

where Θ is $n \times n$ null matrix.

The discrete matrix delayed exponential can be successfully used for representing the solutions of discrete systems. In the case of the problem (1), (2) we have:

$$
x(k) = e_m^{BK}\varphi(-m) + \sum_{j=-m+1}^0 e_m^{B(k-m-j)}\Delta\varphi(j-1) \\ + \sum_{j=1}^k e_m^{B(k-m-j)}bu(j-1)
$$

where $k \in Z_1^\infty$.

Let us define an auxiliary $n \times n$ matrix

$$
S \overset{def}{=} (b, Bb, B^2b, \ldots, B^{n+1}b)
$$

and the vector

$$
\xi \overset{def}{=} x^* - e_m^{Bk_1}\varphi(-m) - \sum_{j=-m+1}^0 e_m^{B(k_1-m-j)}\Delta\varphi(j-1)
$$

In [9] there are given conditions for relative controllability and the control function $u = u^*(k)$, $k \in Z_0^{k_1-1}$ is found for the problem

$$
\Delta x(k) = Bx(k-m) + bu(k), k \in Z_0^{k_1-1}, \tag{3}
$$

$$
x(k) = \varphi(k), k \in Z_{-m}^0, \tag{4}
$$

$$
x(k_1) = x^*. \tag{5}
$$

Theorem 1 *Problem (3)-(5) is relatively controllable if and only if assumptions*

$$
\text{rank } S = n \tag{6}
$$

and

$$
k_1 \geq (n-1)(m+1) + 1 \tag{7}
$$

hold simultaneously.

Theorem 2 *Let the conditions of relative controllability (6), (7) be valid. Then a control function $u = u^*$ for the problem (3)-(5) can be expressed in the form*

$$
u^*(k) = b^T \left(e_m^{B(k_1-m-k-1)}\right)^T G^{-1}\xi
$$

where the matrix

$$
G \overset{def}{=} \sum_{j=1}^{k_1} e_m^{B(k_1-m-j)} bb^T \left(e_m^{B(k_1-m-j)}\right)^T
$$

is nonsingular.

2.1.2 Perturbed Linear Problems, Asymptotic Decomposition of their Solutions

Let us consider the perturbed non-homogeneous linear problem in the form

$$
x(k+1) = A(k)x(k) + f(k) + \varepsilon A_1(k)x(k) \tag{8}
$$

where $A, A_1, f \in B(Z)$, $B(Z)$ is the Banach space of vector-valued functions $x : Z \to R^n$ bounded on Z with the $\|x\| = \sup_{k \in Z}|x(k)|$, $|x(k)| := \|x\|_{R^n}$ and Z denotes the set of all integers. Let us assume that the generating problem

$$
x(k+1) = A(k)x(k) + f(k) \tag{9}
$$

has no solutions bounded on the whole line Z for arbitrary $f \in B(Z)$ and system

$$
x(k+1) = A(k)x(k) \tag{10}
$$

has a dichotomy on Z_+ and Z_-. Here symbols Z_+ or Z_- denote the sets of all non-negative or non-positive integers.

A typical solved problem is: *Is it possible to make the problem (9) solvable by means of linear perturbations and, if it is possible, then what kind of the perturbation A_1 should be for the problem (8) to become solvable everywhere?*

This question can be answered with the help of $d \times r$ matrix

$$
B_0 = \sum_{k=-\infty}^{+\infty} H_d^*(k+1)A_1(k)X_T(k),
$$

the construction of which involves the perturbation term of problem (8). Using the method of generalized inverse operators [6] we can find bifurcation conditions, when solutions bounded on the whole line Z of problem (8) appear in the form of a part of Laurent series in powers of a small parameter ε. Results obtained can be illustrated by one theorem from [5]:

Theorem 3 *Consider the problem (8) and assume that for arbitrary $f \in B(Z)$ generating problem (9) has no solutions bounded on Z and (10) has a dichotomy on Z_+ and Z_- with projectors P and Q, respectively. If rank $B_0 = d$ then, for sufficiently small $\varepsilon \in \left(0, \varepsilon_0\right]$:*

1) *The operator $L_\varepsilon : B(Z) \to B(Z)$*

$$(L_\varepsilon x)(k) \overset{def}{=} z(k + 1) - A(k)x(k) - \varepsilon A_1(k)x(k) \qquad (11)$$

is a Fredholm one with

$$\operatorname{ind} L_\varepsilon = \dim \ker L_\varepsilon - \dim \ker L_\varepsilon^* = \rho = r - d$$

where the operator L_ε^* is the adjoint one to L_ε

$$\left(\operatorname{ind} L_0 = \rho = r - d, \dim \ker L_0 = r, \dim \ker L_0^* = d\right).$$

2) *The homogeneous problem (10) has a ρ-parametric family of solutions*

$$x_0\left(k, \varepsilon, c_\rho\right) = \sum_{i = -1}^{\infty} \varepsilon^i \overline{X}_i(k) P_{B_\rho c_\rho}$$

where $c_\rho \in R^\rho$ is arbitrary and $\rho = \dim \ker Ł_\varepsilon$ with the properties:

$$x_0\left(\cdot, \varepsilon, c_\rho\right) \in B(Z), x_0\left(k, \cdot, c_\rho\right) \in C\left(0, \varepsilon_0\right].$$

The terms $\overline{X}_i(k)$ can be determined.

3) *The problem adjoint to (11) has only trivial bounded solution*

(dim ker $L_\varepsilon^* = 0, \varepsilon \in C\left(0, \varepsilon_0\right]$) .

4) *The problem (8) has, for arbitrary $f \in B(Z)$, a ρ-parametric set of solutions*

$$x(k, \varepsilon) = z(k, \varepsilon, c_\rho) : x(\cdot, \varepsilon, c_\rho) \in B(Z),$$

$$x(k, \cdot, c_\rho) \in C\left(0, \varepsilon_0\right]$$

in the form of the series

$$x(k, \varepsilon, c_\rho) = \sum_{i = -1}^{\infty} \varepsilon^i \left[\overline{x}_i(k, \overline{c}_i) + \overline{X}_i(k) P_{B_\rho c_\rho}\right]$$

convergent for sufficiently small $\varepsilon \in \left(0, \varepsilon_0\right]$; where

$$\overline{x}_i(k, \overline{c}_i), \overline{c}_i \text{ and } \overline{X}_i(k)$$

can be determined.

Similarly, conditions for the existence of solutions bounded on the whole line Z, which turn into one of generating solutions of system (8), for nonlinear difference system

$$x(k + 1) = A(k)x(k) + f(k) + \varepsilon Z(x, k, \varepsilon)$$

were derived. The proposed approach to the analysis of boundary-value problems for systems of ordinary differential equations can be also applied (with relevant modifications) to systems with delayed argument [3] and to finding conditions for the bifurcation of a weak T-periodic solution of the linear abstract wave equation and suggest an algorithm for constructing of such solution [4]. Weak solutions were studied and the Vishik-Lyusternik method has permitted one to find the desired weak T-periodic solution in the form of part of the Laurent series. Boundary value problems for systems of second order linear and nonlinear ordinary differential equations and linear impulsive systems are studied using the Vishik-Lyusternik method and the theory of pseudoinverse matrices (in the sense of Moore-Penrose) in [28, 29].

2.1.3 Bounded Solutions

A lot of effort was devoted to investigation of existence of bounded solutions of different solutions via different methods that are used in the previous part. With so called retract type technique and Lyapunov technique, new methods and results were published, e.g., in [14, 20, 21, 35]. Part of results obtained was extended to some special and arbitrary time-scales.

2.2 Differential Equations

2.2.1 Oscillatory and Asymptotic Properties of Solutions, Periodic Solutions, Stability of Solutions

The subject of research was the investigation of oscillatory and asymptotic properties of nonlinear functional differential equations and systems with retarded arguments [22, 23, 31]-[33]. For example, a nonlinear delay differential equation of the form

$$\dot{x}(t) + p(t)x(t) = q(t)f\left(x(\tau(t))\right) \qquad (12)$$

was investigated for $t \geq t_0$, where

$$p, q : \left[t_0, \infty\right) \to R, f : R \to R$$

are continuous functions and $\tau : \left[t_0, \infty\right) \to (0, \infty)$ is a continuous increasing function, $\lim_{t \to \infty} \tau(t) = \infty$. Together with (12) we consider an initial problem

$$x(t) = \varphi(t), t \in \left[t_{-1}, t_0\right], t_{-1} = \tau(t_0) \qquad (13)$$

with a continuous initial function φ. Let us denote the solution of (12), (13) as $x_\varphi(t)$. The next stability theorem illustrates the character of results obtained.

Theorem 4 *Suppose that an interval $I = [0,b]$, $b > 0$, is mapped by f into itself, $([t_{-1}, t_0], I)$, $0 < q(t) \le p(t)$ for $t \in [t_0, \infty)$,*

$$\int_{t_0}^{\infty} p(s)ds = \infty$$

and

$$\lim_{t \to \infty} \int_{t_0}^{t} q(s) \exp\left(-\int_{s}^{t} p(u)du\right)ds = 0.$$

Then $\lim_{t \to \infty} x_{\varphi}(t) = 0$.

The cognate equations have recently found a variety of applications in several fields of natural sciences. Especially the nonlinear differential equations with time delays model processes in radiophysics, optics, neural interactions, etc. This is due to influence of the past history of the processes on their evolution.

Research was also focused on the behavior of linear or nonlinear integro-differential equations with distributed delays or discrete delays [1, 2]. Such equations recently appeared in the theory of circulating fuel nuclear reactor and also can be a good model in one dimensional viscoelasticity.

In other articles some economic models are treated (e.g. [24]). For example, the stability of neoclassical growth model or the dynamics of basic macroeconomical model of national income is observed.

At the present time we are interested in existence of positive periodic solutions of functional differential equations. Periodic processes can occur in some population models [25].

In applications it is important to know if all solutions of given equation are for $t \to +\infty$ convergent to a finite limit or if there are divergent solutions. Such problems are solved in [16, 18] for equations of the form

$$\dot{y}(t) = \beta(t)[y(t - \delta) - y(t - \tau)]$$

with arbitrary positive and continuous function β and with positive constants $\delta, \tau, \tau > \delta$. New results on existence of solutions of singular initial problems of systems of differential equations were derived. The main results are given in [17, 19].

2.2.2 Structure of Solutions, Existence of Positive Solutions

As was noted above, equations with time lag are used in many mathematical models of different phenomena. A general qualitative result about the structure of solutions of linear differential equations with delay was proved in [15]. Let us consider a linear equation

$$\dot{x}(t) = -\sum_{i=1}^{m} c_i(t)x(t - r_i(t)) \tag{14}$$

with $c_i : [t^*, \infty) \to R$, $i = 1, ..., m$ and $r_i : [t^*, \infty) \to (0, r]$

Theorem 5 *Let us suppose the existence of a positive solution $\tilde{x}$ of (14) on $[t^* - r, \infty)$. Then either every solution x of (14) on $[t^* - r, \infty)$ is represented in a unique way by the formula*

$$x(t) = \tilde{x}(t)(K + \varphi(t)),$$

where $K \in R$ depends on x, and φ is a continuous function dependent on x and satisfying $\varphi(+\infty) = 0$, or every solution x of (14) on $[t^* - r, \infty)$ is represented in a unique way by the formula

$$x(t) = \tilde{x}(t)(KY(t) + \delta(t)),$$

where Y is a continuous, increasing function which is the same for each x, satisfies $Y(+\infty) = \infty$, $K \in R$ depends on x, and δ is a bounded continuous function dependent on x.

Theorem 6 *Suppose the existence of a positive solution of (14) on $[t^* - r, \infty)$. Then there exist two positive solutions x_1 and x_2 of (14) on $[t^* - r, \infty)$ satisfying the relation*

$$\lim_{t \to \infty} \frac{x_2(t)}{x1(t)} = 0, \tag{15}$$

such that every solution $x = x(t)$ of (14) on $[t^* - r, \infty)$ can be represented by the formula

$$x(t) = Kx_1(t) + O(x_2(t)), \tag{16}$$

where the constant K depends on x.

In [13] (see [10]–[12] as well) a linear equation

$$\dot{x} = -a(t)x(t - r) \tag{17}$$

being a particular case of (14) is considered, where $a : [t_0, \infty) \to R^+ = (0, \infty)$ and $r \in R^+$ and assume $0 < A \le a(t) \le B < 1/(r e)$ on $[t_0, \infty)$ and $A < B$ are constants. It can be verified that the equation for $\lambda : \lambda = Ae^{\lambda r}$ has just two different positive roots $\lambda_A^*, \lambda_A^{**}, \lambda_A^* < \lambda_A^{**}$ and the equation $\lambda : \lambda = >Be^{\lambda r}$ has just two different positive roots $\lambda_B^*, \lambda_B^{**}, \lambda_B^* < \lambda_B^{**}$ such that $\lambda_A^* < \lambda_B^*$ and $\lambda_A^{**} > \lambda_B^{**}$. Under some additional assumptions it is proved that there are positive solutions $x = x_1$ and $x = x_2$ of (17) such that

$$e^{-\lambda_B^*(t - t_0 + r)} \le x_1(t) \le e^{-\lambda_A^*(t - t_0 + r)},$$

$$e^{-\lambda_A^{**}}t^p < x_2(t) \le e^{-\lambda_B^{**}}t^q,$$

where $p < 0$ and $q > 0$ are fixed numbers. Solutions $x_1(t), x_2(t)$ satisfy (15) and every solution of (17) can be represented by formula (16). A part of results remain valid for a general functional equation $\dot{x}(t) = -f(t, x_t)$ under the main assumption

$$A\varphi(-r) \le f(t, \varphi) \le B\varphi(-r).$$

2.3 Numerical Mathematics

In the latest period a deep research concerning some numerical aspects in discrete least-squares and collocation methods for numerical solution of boundary value and eigenvalue problems were performed.

Although the least-squares and collocation methods belong to the classical tools in the numerical solution of partial differential equations, there is still enough space for their investigation with respect to efficiency, stability and applicability in important real-world problems. During the years of research and numerical experiments the scientists and engineers have revealed that the crucial point of successful application of the mentioned methods is based on the optimal choice of matching points ensuring the highest possible accuracy allowed by the used trial functions and as small as possible condition number of the resulting coefficient matrices.

One of the possible sets of suitable matching points could be taken the Lebesgue points which are optimal interpolation points minimizing the Lebesgue constant Λ_n. This constant plays central role in the Lagrange interpolation process according to the inequality

$$\|u - L_n\| \le (1 + \Lambda_n)\|u - p_n\|,$$

where L_n is the Lagrange interpolation polynomial of the function u, i.e., $L_n(x_i) = u(x_i)$, $i = 0, 1, 2, ..., n$ and p_n is the best polynomial approximation of u. Unfortunately, as written in [34]: *"Almost nothing seems to be known about Lebesque points in more than one dimension. Nor are we aware of a feasible method for computing them numerically."*

Another set of suitable matching points can be considered the Fekete points for given domain which maximize the corresponding Vandermonde determinant. These points are known for rectangular and triangular domains, however, the computation of the Fekete points for general two-dimensional domains with complicated boundary shape or multiple connected domains is still an open problem. To avoid this terra incognita we concentrated our attention to the most important requirement in the matrix computations expressed in the following inequality

$$\frac{\|x - x_a\|}{\|x\|} \le C \cdot \mathrm{cond}(A) \times \left(\frac{\|\Delta A\|}{\|A\|} + \frac{\|\Delta b\|}{\|b\|} \right).$$

This inequality describes how much the roundoff error propagation may destroy accuracy of the approximate solution x_a of the system of linear equation $Ax = b$ using LU-decomposition. It is evident that small condition number $(\mathrm{cond}(A))$ of the coefficient matrix A guarantees small error of the approximate solution x_a. Consequently, this dependence can be used to reveal a new way of computing another set of points which could be suitable for our purposes. As a new possibility we tried to find suitable matching points which minimize condition number of the coefficient matrix A. The results concerning discrete least-squares solution of the Poisson problem

$$-\Delta u(x) = f(x), \quad x \in \Omega$$

and the eigenvalue problem

$$-\Delta u(x) = \lambda g(x) u(x), \quad x \in \Omega$$

subject to the homogeneous Dirichlet boundary conditions

$$u(x) = 0, \quad x \in \partial\Omega$$

for rectangular, circular and triangular domain Ω are presented [36, 37]. The eigenvalue approximations computed for these domains show that at least 10 digits of accuracy for the first 10 eigenvalues may be obtained using less than 200 basis functions. Moreover, the conformal mapping technique can be used for computing the eigenvalue approximations for domains with exotic shapes generated by conformal mapping of rectangular, circular and triangular domains.

2.4 Orthogonal Polynomials and their Applications

In [30] several theorems are proved expressing integrals involving polynomials orthogonal in a finite symmetric interval by means of their coefficients. In the proofs for the polynomials $\left\{P_n(x)\right\}_{n=0}^{\infty}$ orthonormal in a finite symmetric interval I associated with even weight function $\upsilon(x)$ it is used the well-known following property: $P_n(-x) = (-1)^n P_n(x)$. Denote

$$P_n(x) = \sum_{k=0}^{n} p_k^{(n)} x^{n-k}, \quad p_0^{(n)} > 0$$

and

$$p_0 = 0, \quad p_n = \frac{p_0^{(n-1)}}{p_0^{(n)}}$$

for natural n. The following results are valid:

$$\int_I x^2 P_n^2(x) \upsilon(x) dx = p_{n+1}^2 + p_n^2,$$

$$\int_I x^3 P_n(x) P_{n-1}(x) \upsilon(x) dx = p_n \left(p_{n+1}^2 + p_n^2 + p_{n-1}^2 \right),$$

and for a positive integer $k < n$ we proved:

$$\int_I x^k P_n(x) P_{n-k}(x) \upsilon(x) dx = \prod_{i=0}^{k-1} p_{n-i}.$$

In [7] we studied the classical Jacobi orthogonal polynomials $\left\{ P_n(x); \alpha, \beta \right\}_{n=0}^{\infty}$ which are orthogonal in the interval $I = [-1, 1]$ with respect to the weight function

$$J(x) = (1 - x)^\alpha (1 + x)^\beta,$$

where $\alpha > -1$, $\beta > -1$. Especially their special cases – ultraspherical polynomials (when $\alpha = \beta$ in $J(x)$), Chebyshev polynomials of the first kind ($\alpha = \beta = -1/2$ in $J(x)$), Chebyshev polynomials of the second kind ($\alpha = \beta = 1/2$ in $J(x)$) and Legendre polynomials ($\alpha = \beta = 0$ in $J(x)$) are often used in the approximation theory

and they appear as the solutions of some problems of mathematical physics. For certain integrals of classical Jacobi orthonormal polynomials there are derived formulas interconnecting them with cyclometric functions. It holds:

Theorem 7 For $x \in [-1, 1]$

$$\int_x^1 P_n^2(t; \alpha, \beta) J(t) dt = \frac{1}{\pi} \arccos x + g_2^{(n)}(x)$$

and

$$\int_x^1 t\, P_n^2(t; \alpha, \beta) J(t) dt = \frac{1}{\pi} \sqrt{1 - x^2} + g_3^{(n)}(x)$$

where for $i = 2, 3 : x \in [-1, 1] \Longrightarrow \left| g_i^{(n)}(x) \right| < c_i n^{-1}$.

Another view of our research on orthogonal polynomials could be taken through the works [26, 27] dealing with generalized function spaces. Especially the work [27] (cited 95 times in SCI) can be estimated as a pioneering work in this direction.

3 Mathematical Perspectives

The youngest faculty celebrates 10 years of its existence. Nevertheless, this short period showed that applied mathematics successfully developed in both national and international contexts and has a promising perspective in the future. Presently, the mathematical groups are supported by four grant projects of the Grant Agency of Slovak Republic (VEGA), one international project and several institutional projects. The international collaboration is large and results mainly in publishing common research papers. In spite of it we will try to upgrade these contacts and create international research groups supported by scientific grants. The forthcoming stimulating mathematical event, organized by the faculty will be the ICDDEA 2008 conference (International Conference on Differential and Difference Equations and Application) in Strecno.

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