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# JEDNOTNÝ POHĽAD NA OPTIMALIZÁCIU DOPRAVNEJ PONUKY

## UNIFIED VIEW OF OPTIMIZATION OF TRANSPORT SUPPLY

*Článok uvádza tzv. problém optimálnej dopravnej ponuky, ktorý možno všeobecne formulovať ako optimálne pokryť požiadavky. Tieto môžu mať veľmi odlišný charakter (napr. dodávanie tovaru, pokrývanie dopravných požiadaviek atď.). Napriek veľkej variabilite reálnych problémov optimálnej ponuky všetky tieto problémy majú niekoľko spoločných vlastností. Pre uvedený problém formuluje spojitý a diskrétny model, poukazuje na ich spoločnú štruktúru a ukazuje, ako riešiť príslušné matematické problémy.*

*The paper presents so called transport supply optimization problem, which can be generally formulated as to find an optimum covering of demand. The types of supply can be very different (as goods-supply, transport-supply etc). In spite of variability of real supply optimization problems they have several common properties. For the presented problem it formulates continuous and discrete model, shows that both models have similar structure and presents solutions of corresponding mathematical problems.*

### 1. Introduction

The supply optimization problems in transport management are very frequent. Somebody optimizes delivery of goods, another deals with public transport and a third considers the street network for cars. Once we speak about flows of demand elements, and at another time we see batches of them. Somewhere they optimize routes and stops (i.e. the space layout) and somewhere else, the time tables. The individual optimization problems usually differ very strongly one from to another. The main purpose of this paper is to present a unified form or structure of such a problem. Of course, it will not be universal, but there is a hope that it will cover a significant part of the transport-supply optimization problems.

However, a unified structure of problems does not imply unified methods of solution. The second purpose of this paper is to inspire colleagues to try to create new methods from the unified structure of the problem.

Transport supply is a response to the given transport demand. The demand can be modelled continuously or discretely.

1.1. For the given period, e.g. the morning peak, a *continuous demand* can be expressed by an O-D-matrix of *flows* $F = (f_{ij})$ where $f_{ij}$ represents the flow from the zone (or its centroid) $i$ to another one $j$. Suppose a *network* $G = (V, A, c, l)$ is given, where $V$ is the set of vertices, $A$ is the set of (oriented) arcs, $c(a)$ is the transition capacity of the arc $a$ and $l(a)$ is the length of $a$. An *assignment* of $F$ to $G$ (more precisely: to the set of all paths on the network $G$) can be obtained by some of well-known procedures (if it exists, of course). Then the continuous demand can be represented by a set of paths $P$ on $G$, each path $p$ having a size $f(p)$. To an O-D-pair $i,j$ there corresponds a set of paths $P_{ij}$ (containing one or more elements), which covers the demand $F$ in the sense that

$$f(a) = \sum_{a \in p} f(p) \le c(a) \qquad f_{ij} = \sum_{p \in P_{ij}} f(p)$$

These formulae mean that

- no arc capacity is exceeded
- each demand element belonging to the flow $f_{ij}$ passes from $i$ to $j$ through $G$ using exactly one path $p \in P$

We can look at the O-D-matrix $F$, the network $G$ and the set of paths $P$ from two points of view:

a) $P$ is the same demand as $F$, represented on the given network $G$

b) $G$ is the supply reflecting the demand $F$, $P$ represents the "portion" of $G$ assigned to $F$.

We meet the case b) observing flows of cars on roads: the road network $G$ is the supply corresponding to the demand $F$. In the case of flows of urban transport passengers, we meet a): the set $P$ on the network $G$ represents only another representation of the same demand. The form of the corresponding supply will be described in the sequel.

*Remark.* Till now we have mentioned two forms of demand models: O-D-matrices or sets of paths on the networks (derived from the previous one). In some cases we can proceed further by the partition-aggregation approach: 1. to part each $p = a_1, \ldots, a_k \in P$ into elementary paths $p_1$ on $a_1, \ldots, p_k$ on $a_k$, each one with the size $f(p)$ and 2. to aggregate all elementary paths on the same arc. Then we obtain a set $\overline{P}$ such that each $\overline{p} \in \overline{P}$ equals to some arc $a \in A$ and the following formula holds: $f(\overline{p}) = \sum_{\overline{p} \in p} f(p)$. In the sequel, we shall briefly say that $\overline{P}$ is another form of the same demand as $F$ or $P$.

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1.2. While continuous demand model deals with streams (flows) of transport elements (cars, passengers, etc), the *discrete demand* deals with *batches* (a batch = a group of transport elements moving together from the beginning of its trip to the end). A batch is represented by a quintuple $b = (o_b, d_b, t_b, a_b, n_b)$ where $o_b$ is the origin, $d_b$ the destination, $t_b$ the departure time from the origin, $a_b$ the arrival time to the destination and $n_b$ the number of elements of the batch $b$. A demand is determined by a set of batches $B$. Of course, we suppose $o_b, d_b \in V$ where $V$ is a given set of vertices of a given network $G$.

Similar to 1.1, we can suppose a network $G = (V, A, c, l)$ to be given, where $V$ is the set of vertices, A is the set of arcs, $c(a)$ is the transition capacity of the arc $a$ and $l(a)$ is the length of $a$. However, $G$ is not sufficient mathematical structure for the set $B$ to be assigned to. We have an important difference between the set of flows $F$ and the set of batches $B$. The flows are constant during the whole time period we consider and thus each one is determined by its origin, destination and size. On the contrary these data are not sufficient to determine a batch. We need to add the departure and arrival times, usually taken from the time set $T = \{0, 1, \ldots, 1339\}$, which represents each minute during 24 hours. Then the suitable structure for the assignment of $B$ will be the *time-space graph* $GT = (V \times T, A')$ where $((v_1, t_1), (v_2, t_2)) \in A'$ if $(v_1, v_2) \in A$ and $t_2 - t_1 \cong l(v_1, v_2)/s$, $s$ is a speed of a batch if transported from $v_1$ to $v_2$. An *assignment* of $B$ to $GT$ can be obtained by some of well-known procedures. A problem can occur in applying capacity constraints. For instance, it can be formulated as follows: For $a = (v, w)$ $c(a)$ means the number of unit elements, transportable through the vertex $v$ to $w$ during one hour (60 minutes). Thus if a batch $b$ containing $n_b$ elements is assigned to pass through $v$ into $a$ at the time $t$, the next batch is allowed to pass there no sooner than at time $t + 60n_b/c(a)$. After such an assignment a discrete demand can be represented by a set of paths $P$ on $GT$.

Similar to 1.1, we can look at the set $B$, the network $G$ and the set of paths $P$ from two points of view:

a) $P$ is the same demand as $B$, represented on the given network $GT$
b) $G$ is the supply reflecting the demand $B$, $P$ represents the "portion" of $GT$ assigned to $B$.

Compared with 1.1 our experience shows that the alternative b) is very rare in practice. Hence, if speaking about discrete demand, we shall suppose it is given either by a set of batches $B$, or by a set of paths $P$ on the space-time graph $GT$.

Both cases 1.1a) and 1.2a) deal with the situation where the demand can be characterized either by

(i) a set of demand elements $F$ resp. $B$ or
(ii) a set of paths $P$ on a graph $G$ resp. $GT$.

In regard to concerns about supply, we have already presented one form of its description in 1.1b) and 1.2b). The corresponding optimization problem can be formulated in the following way: To find "the cheapest" graph $G$ which enables the assignment of the given demand $F(B)$. Of course, the words "to find" and "the cheapest" ought to be said more precisely but we shall not do it. The problem is frequently studied in the traffic engineering bibliography. We shall concentrate our attention to the optimization of supply in the case a).

The application of the partition aggregation approach to the discrete demand needs some modification. One has to consider that the aggregation of two elementary paths on the same arc $((a_1, t_1), (a_2, t_2))$ is quite natural but what about the situation when one of them is on the arc $((a_1, t_1), (a_2, t_2))$ and the other one on $((a_1, t_1 + 1), (a_2, t_2 + 1))$? May we aggregate them? And what to do when 2 or $-1$ is instead of 1? What border $\epsilon$ to choose for such an $\epsilon$-aggregation? And what to do if an elementary path can be $((a_1, t_1), (a_2, t_2))$ $\epsilon$-aggregated to the left and to the right as well? In [3] one can see the complexity of such aggregation.

Both in 1.1 and 1.2 it is quite natural to adopt the following approach: If the demand is specified by a set $P$ resp. $\overline{P}$ of paths on the graph $G$ or $GT$ respectively, *the supply* ought to be described by the same way, i.e. as another set of paths $Q$ resp. $\overline{Q}$ on the same graph. How to formulate the optimization problem and how to solve it? The answers are in the next parts.

## 2. Optimization problem in the continuous model

Suppose the demand is described like in 1.1. Let $R$ be available rolling stock, i.e. let each $r \in R$ represent a vehicle with the capacity $c(r)$. Let $Q(R) = \{q(r): r \in R\}$ be a set of closed paths (=circles) on the network $G$. Then we can say $Q(R)$ a *continuous supply* generated by the rolling stock $R$ on the network $G$. We shall suppose the continuous supply to be realized in the following way:

We suppose a vehicle $r \in R$ moves periodically on the path $q(r)$ with the period $t(q(r))$ = the duration of the minimum operation cycle of the vehicle $r$ on $q(r)$ (in minutes), containing the necessary running and manipulation times. Doing this, the vehicle supplies the dynamic capacity (briefly d-capacity) $c(q(r)) = 60c(r)/t(q(r))$ on the path $q(r)$, i.e. it can (fully) satisfy an assigned demand not exceeding $c(q(r))$ on any arc belonging to $q(r)$. Regarding to it we can formulate the following problem:

2A) *The FQ-Continuous supply optimization problem.* Let $F$ be a demand, let $\boldsymbol{Q}(R)$ be a class of all possible sets $Q(R)$ on the network $G$ and let $w_F(Q)$ be an objective function on $\boldsymbol{Q}(R)$. The goal is to find a supply $Q(R) \in \boldsymbol{Q}(R)$ minimizing the value $w_F(Q(R))$.

In the prevalent cases a "man-machine" approach is adopted in the solution of this problem. The man chooses $Q(R)$ and the machine calculates the value $w_F(Q(R))$. One can ask why such a "primitive" approach is prevalent in solutions of the problems mentioned above. Why another more sophisticated approach is not used instead. The reason is in the fact that the "satisfaction level" of $F$ by $Q$ (or $G$) is calculated by means of the assignment and it is difficult to introduce it into some more sophisticated optimization model. But a way exists to overcome this obstacle, especially in the cases 1.1a), 1.2a):

I. Neglecting any constraint concerning limited rolling stock or finances to find an "ideal" assignment of the demand $F$ to the set of all paths on the network $G$. To denote the set of paths assigned to $F$ by $P$. To consider $P$ the new representation of the same demand is $F$.

II. To find the "best approximation" of the set $P$ in the class $Q(R)$.

From this moment we shall concentrate on step II., because step I is "classic" and well-known. The "best approximation" will be expressed by an objective function (a "distance" function).

2B) *The PQ-continuous supply optimization problem.* Let $m(P, Q)$ be a non-negative objective function on the pairs $P$, $Q$ of demand and supply, respectively. Then the problem can be formulated as follows:

Given network $G$ and demand $P$ on it. Given a rolling stock $R$. To find a supply $Q = Q(R) \in \boldsymbol{Q}(R)$ minimizing the value $m(P, Q)$.

We can see that no constraints are contained in this formulation. The idea is that a violation of a constraint can be expressed by an increase of the value of the objective function $m$.

Naturally, an alternative formulation can be reached using constrains separately, not included in $m$. Let the constraints be $m_i(P, Q)$, $i = 1, \ldots, n$. Then the problem can be formulated as follows:

Given a network $G$ and a demand $P$ on it. Given a rolling stock $R$. To find a supply $Q = Q(R) \in Q(R)$ fulfilling the constraints $m_i(P, Q)$, $i = 1, \ldots, n$ and minimizing the value $m(P, Q)$.

2C) *The $\overline{P}Q$-continuous supply optimization problem.* This problem can be formulated equally as in 2B. We only take into account that the "input" set $\overline{P}$ contains one-arc-path only.

*Example.* In practice, we can meet a continuous supply optimization problem, e.g. in urban bus transport. Bus routes correspond to paths on the urban street network. Usually, more than one vehicle operates on the same line, i.e. for the route $i$ there exists a set of buses $R_i \subset R$ assigned to the route $i$ and consequently $q(r) = i$ for each $r \in R_i$. Then we can define the d-capacity of the route $i$ as

$$c(i) = \sum_{r \in R_i} c(q(r)).$$

The "classic" constraint is that each passenger must have a possibility to be transported, i.e. for each $f_{ij} > 0$ there must exist a path $p = a_1, \ldots, a_k$ such that each arc $a_h$ belongs to some $q(r)$. Of course, we have to remember that the given rolling stock $R$ (i.e. the limited number of available buses) has another constraint as well.

The "classic" objective function is
- either the average travel speed of a passenger (to be maximized)
- or the maximum overloading of the bus (to be minimized).

The disadvantage of average travel speed consists of the fact that the solution of the problem has to determine the number of buses $x_q$ assigned to the route $q$. The constraint on the number of buses contains it in the linear form ("in the numerator"), but the speed of the (buses changing) passengers is derived from their travel times on individual routes and the expression of the travel time contains $x_q$ in the denominator. Hence one cannot avoid the use of non linear programming. On the other hand the maximum overloading can be expressed by the minimum ratio of supply and demand on the individual arcs and there $x_q$ is in the numerator and one can use linear programming which is much more simple then the non linear one.

## 3. Optimization problem in the discrete model

Suppose the demand is described like in 1.2. We shall proceed similarly as in 2. Let $R$ be a rolling stock, i.e. let each $r \in R$ represent a vehicle with the capacity $c(r)$. Let $Q(R) = \{q(r): r \in R\}$ be a set of paths (now we don't require that they are closed) on the time-space graph $GT$ and let $\boldsymbol{Q}(R)$ be the set of all possible sets $Q(R)$. Then we can call $Q(R)$ a *discrete supply* generated by the rolling stock $R$ on the network $G$. In contrast to the continuous one, the discrete supply consists of the paths $q(r)$ passed by a vehicle only once, without any repetition. Hence the concept of d-capacity is not necessary to be introduced. The capacity of $q(r)$ equals to the one of $r$ in this case and it must be considered in the assignment of the demand batches $b$ to the supply vehicle trips $q(r)$.

*The objective functions $w_B$ and $m$* can be introduced similarly as in 2 and *the discrete supply optimization problems* 3A, 3B and 3C can be formulated similarly as the continuous ones in 2 as well. Of course, $GT$ is put instead of $G$:

*Example.* In practice, we can meet this problem e.g. in regional (rural) bus transport. Bus journeys correspond to paths on the time-space graph, generated by a road network. A path determined by a concatenation of paths corresponding to individual journeys and paths corresponding to idle movements represents a bus daily duty.

## 4. Solutions of the optimization problems

Both continuous and discrete supply-optimization problems have similar mathematical structure: Let $G$ be a digraph, let $F(B)$, $P$ or $\overline{P}$ be a demand on it. Let $\boldsymbol{Q}$ be a class of admissible sets of paths on $G(GT)$ and let $w_F(w_B)$ or $m(P, Q)$ be a "distance" function. The problem is to find such $Q \in \boldsymbol{Q}$ that $w_F(Q)(_{wB(Q)})$ or $m(P, Q)$ is minimum.

Since the set $P(\overline{P})$ is given, we can denote $m_P(Q) = m(P, Q)$, more simply $m(Q) = m(P, Q)$. (or the same for $\overline{P}$).

The spectrum of different models and methods solving particular types of problems is very rich and it is impossible to mention them all. We have chosen some considering they can properly complete the content of the paper.

*4.1. Local optimization.* In fact, it is a modification of a man-machine evaluation of possible solutions. The difference is the replacement of a man by a machine. The computer starts with an initial solution, evaluates it and then looks for a better solution among the "neighbouring" solutions. Several metaheuristics are available for this process: tabu search, simulated annealing, genetic algorithms.

*Example 1. Multiple Travelling Salesman Problem.* This problem can be met in practice very often. We can mention bread or newspapers delivery, litter or fresh fruit collection, etc. If the problem has a low size, linear programming can solve it. If it is more extensive it needs to be solved by a heuristic method. Even the classic Clarke & Wright one is of the local-optimization type. It starts with the set of direct to-and-from paths and afterwards it looks for the best improvement of the topical solution by combining a pair of paths into a new one.

*Example 2. Bus daily duties optimization.* This problem is usually solved by other types of methods (see e. g. [1], [5]). But sometimes, the final result is not fully satisfactory, e.g. because of the fact that some constraints have not been introduced into the basic formulation because of the inability of used methods to take it into account. Then a local optimization can be adopted. It consists of the crossing of two duties. We can illustrate it graphically:

| | | | | | |
|---|---|---|---|---|---|
| Simple crossing | duty No. 1 | $j_{11}$ | $j_{12}$ | $j_{13}$ | $j_{14}$ |
| | duty No. 2 | $j_{21}$ | $j_{22}$ | $j_{23}$ | $j_{24}$ |

In the starting solution the duty No. 1 consists of the journeys $j_{11}$ the duty No. 1 consists of the journeys $j_{11}$ – the duty No. 1 consists of the journeys $j_{11}, j_{12}, j_{13}, j_{14}$, the duty No. 2 consists of the journeys $j_{21}, j_{22}, j_{23}, j_{24}$. After the simple crossing the new duty No. 1. consists of the journeys $j_{11}, j_{12}, j_{23}, j_{24}$ and the new duty No. 2. consists of the journeys $j_{21}, j_{22}, j_{13}, j_{14}$.

| | | | | | |
|---|---|---|---|---|---|
| Double crossing | duty No. 1 | $j_{11}$ | $j_{12}$ | $j_{13}$ | $j_{14}$ |
| | duty No. 2 | $j_{21}$ | $j_{22}$ | $j_{23}$ | $j_{24}$ |

After the double crossing the new duty No. 1. consists of the journeys $j_{11}, j_{22}, j_{23}, j_{14}$ and the new duty No. 2. consists of the journeys $j_{21}, j_{12}, j_{13}, j_{24}$.

*4.2. Cutting & Crossing.* This method is due to S. Palúch. He has informed me privately, no paper on it is known. The idea is the following:

| | | | | | |
|---|---|---|---|---|---|
| Cutting & crossing | duty No. 1 | $j_{11}$ | $j_{12}$ | $j_{13}$ | $j_{14}$ |
| | duty No. 2 | $j_{21}$ | $j_{22}$ | $j_{23}$ | $j_{24}$ |
| | duty No. 3 | $j_{31}$ | $j_{32}$ | $j_{33}$ | $j_{34}$ |
| | duty No. 4 | $j_{41}$ | $j_{42}$ | $j_{43}$ | $j_{44}$ |

the cut

The cut divided all (divisible in that time) duties into the first and the second parts (heads and tails). They form a bipartite graph. Then the cheapest matching problem is solved, and the heads are connected with the tails determined by the matching. In our example the new tail for the duty No. 1 is the fourth one, etc.

This method is applicable mainly in the case of the separability of objective functions. The function $m(Q)$ is said *separable* if

$m(Q) = \sum_{q \in Q} m(q)$ where $q$ denotes an individual duty. It is true e. g. if the value $m(q)$ expresses a combination of penalties for idle movements, minimum necessary size of vehicle, lack of driver's rest, etc.

*4.3. Choice from the wider set $Q_0$.* Suppose again a man-machine cooperation. A man creates a set, which contains many times more paths than is expected to be in the solution set $Q$. A computer chooses the paths q from $Q_0$ by a binary variable using linear or non-linear programming minimizing the value of the objective function. Then $Q = \{q \in Q_0: x_q = 1\}$.

*Example. Optimal bus routing and frequencing.* This approach is due to Erlander and Schéele [4]. They used travel speed as the objective function; therefore, they had to use non-linear programming. On the other hand, in [2] minimum supply-demand ratio objective function was adopted which enables use of linear programming.

*4.4. Hopes for the future.* The author hopes that new methods (heuristic and maybe even exact) could arise after a development of the theory of minimization of functions on the classes $Q$ of the sets $Q$ of paths on the networks.

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