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GEOPOTENCIÁLNY MODEL ZEME – APROXIMÁCIA TVARU ZEME, METÓDY TESTOVANIA GEOPOTENCIÁLNYCH MODELOV

GEOPOTENTIAL MODEL OF EARTH – APPROXIMATION OF EARTH SHAPE, GEOPOTENTIAL MODEL TESTING METHODS

Stokesové geodynamické koeficienty v rozvoji tiažového potenciálu Zeme slúžia na matematicko-fyzikálny popis vonkajšieho tiažového poľa, tvaru a rozmeru Zeme. V článku sú uvedené výsledky testovania presnosti geopotenciálneho modelu EGM96 na území Slovenska.

Stokes' geodynamic parameters in the expansion of the gravity potential of the Earth serve for a mathematical and physical description of an outer gravity field of the Earth, the shape and the size of the Earth. In this article, the results from testing of an accuracy of the geopotential model EGM 96 at the territory of Slovakia are published.

1. Úvod – historické pozadie určovania tvaru Zeme

Meškanie kyvadlových hodín astronóma Richera pri premiestnení z Paríža do Cayenne (pri rovníku) vysvetlil I. Newton zmenšením gravitačnej sily v dôsledku sploštenia Zeme na póloch. Na základe všeobecnej teórie gravitácie (Philosophiae naturalis principia mathematica, 1687) za predpokladu, že Zem je homogénne teleso, určil Newton jej sploštenie $f = \frac{a-b}{a} = \frac{1}{230}$, kde a, b sú veľkosti rovníkovej a pólovej poloosi.

Neskôr Ch. Huygens predpokladal, že každá častica Zeme je priťahovaná len do jej stredu (celá hmotnosť je sústredená v ťažisku). Na základe svojho modelu vypočítal sploštenie $f = 1/578$.

Tento nesúhlas medzi oboma výsledkami objasnil A. Clairaut (Théorie de la figure de la Terre, 1743). Uvažoval sféroid, zložený z koncentrických vrstiev, ktorých hustota rastie ku stredu Zeme a odvodil vzťahy známe ako Clairautov teorém, vyjadrujúci vzájomný vzťah medzi geometrickými a fyzikálnymi parametrami Zeme

$$f = \frac{5}{2}q - \beta = \frac{5}{2}\frac{a^2\omega^2}{GM\,a^{-1}} - \frac{g_p - g_e}{g_e} = \frac{5}{2}\frac{a^3\omega^2}{GM} - \frac{g_p - g_e}{g_e} = \frac{5}{2}\frac{a\omega^2}{g_e} - \frac{g_p - g_e}{g_e},\; q = \frac{a^3\omega^2}{GM},$$

kde g_p a g_e sú tiažové zrýchlenie na póloch a rovníku, ω je uhlová rýchlosť rotácie Zeme.

Na základe spracovania údajov o dráhach družíc, altimetrických údajov topografie povrchu svetového oceánu a údajov z tiažových meraní je možné v súčasnosti pomerne podrobne defi-

1. Introduction – historical background of estimation of Earth's shape

The delay of astronomer Richer's pendulum clock during the relocation from Paris to Cayenne (near the equator) was explained by I. Newton as a decrease in the gravitational force resulting from flattening of the Earth at the poles. Based on the common theory of gravitation (Philosophiae naturalis principia mathematica, 1687) and providing the Earth to be a homogeneous body, Newton estimated the Earth's flattening as $f = \frac{a-b}{a} = \frac{1}{230}$, where a, b are the sizes of the equatorial axis and the pole axis, respectively.

Later on, Ch. Huygens assumed every Earth's element to be forced just into the earth's center (entire mass is centered inside the center of gravity). Based on his model, he calculated the flattening as $f = 1/578$.

The contradiction between both findings was explained by A. Clairaut (Théorie de la figure de la Terre, 1743). He considered a spheroid composed of concentric layers whose density increased towards the Earth's center, and has derived forms known as Clairaut's theorem, expressing the correlation between the geometric and the physical parameters of the Earth

where g_p, g_e are the gravity acceleration on the poles and on the equator, ω is the angular velocity of the Earth's rotation.

Nowadays, it is possible to define the shape and the size of the Earth and the gravity field outer the Earth with quite high accuracy based on processing of the satellite tracking data, the altime-

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novať tvar a rozmer Zeme, ako aj popísať tiažové pole v okolí Zeme.

V spolupráci NIMA (National Imagery and Mapping Agency) a NASA GSFC (National Aeronautics and Space Administration Goddard Space Flight Center) bol vytvorený Gravitačný model Zeme EGM 96 (Earth's gravity model 1996) [10].

Pracovná skupina WG GGT (Working Group Global Geodesy Topics) spolupracuje na testovaní presnosti určenia EGM 96, pričom pre tento účel bola vytvorená metodológia testovania [1] a vytvorená testovacia sieť na území oceánov a kontinentov [2], [3], [4].

ter data of ocean surface topography and data obtained from the Earth's gravity measurements.

NIMA (National Imagery and Mapping Agency) and NASA GSFC (National Aeronautics and Space Administration Goddard Space Flight Center) have participated to develop the gravity model of the Earth EGM 96 (Earth's gravity model 1996) [10].

Working group WG GGT (Working Group Global Geodesy Topics) collaborates on testing the accuracy estimation of EGM 96. For this purpose the testing methodology [1] and the testing network over oceans and continents [2], [3], [4] have been created.

2. Aproximácia tvaru Zeme – normálny hladinový sféroid a elipsoid

Tiažový potenciál $W(\rho, \Phi, \Lambda)$ vyjadruje jeho rozvoj do radu sférických funkcií ($n, k \to \infty$)

2. Approximation of Earth's shape – normal level spheroid and ellipsoid

The gravity potential $W(\rho, \Phi, \Lambda)$ expresses its expansion to the series of spherical functions ($n, k \to \infty$)

$$W(\rho, \Phi, \Lambda) = \frac{GM}{\rho}\left[1 + \sum_{n=2}^{\infty}\sum_{k=0}^{n}\left(\frac{a_o}{\rho}\right)^n (J_n^{(k)} cosk\Lambda + S_n^{(k)} sink\Lambda)P_n^{(k)}(sin\Phi)\right] + \frac{1}{2}\rho^2\omega^2\cos^2\Phi =$$

$$= \frac{GM}{\rho}\left\{1 + \sum_{n=2}^{\infty}\sum_{k=0}^{n}\left(\frac{a_o}{\rho}\right)^n (J_n^{(k)} cosk\Lambda + S_n^{(k)} sink\Lambda)P_n^{(k)}(sin\Phi) + \frac{1}{3}\frac{\rho^3\omega^2}{GM}[1 - P_n^{(0)}(sin\Phi)]\right\}, \quad (2.1)$$

kde ρ je geocentrický rádius vektor bodu P; Φ, Λ sú jeho geocentrické súradnice, a_o je ľubovoľný dĺžkový parameter (spravidla veľká poloos hladinového rotačného elipsoidu), $J_n^{(k)}$, $S_n^{(k)}$ sú Stokesove geodynamické parametre stupňa n a rádu k [5]

where ρ is the geocentric radius-vector of the point P; Φ, Λ are its geocentric coordinates, a_o is an arbitrary parameter of length (usually the semi-major axis of the level rotation ellipsoid), $J_n^{(k)}$, $S_n^{(k)}$ are Stokes' geodynamic parameters of degree n and order k [5]

$$\left.\begin{matrix} J_n^{(k)} \\ S_n^{(k)} \end{matrix}\right\} = \frac{(2 - \delta_{0,k})}{M a_0^n}\frac{(n-k)!}{(n+k)!}\int_M \rho'^n P_n^{(k)}(sin\Phi')\left\{\begin{matrix} cosk\Lambda' \\ sink\Lambda' \end{matrix}\right\}dm, \quad \delta_{0,k} = \begin{cases} 1, & k = 0, \\ 0, & k \neq 0, \end{cases} \quad (2.2)$$

M je hmotnosť Zeme, $dm(\rho',\Phi',\Lambda')$ je hmotný element Zeme, $P_n^{(k)}(sin\Phi)$ sú Legendreove pridružené funkcie [9], [5]

M is the weight of the Earth, $dm(\rho',\Phi',\Lambda')$ is the mass element of the Earth, $P_n^{(k)}(sin\Phi)$ are Legendre's associated functions [9], [5]

$$P_n^{(k)}(sin\Phi) = \frac{1}{2^n n!}cos^k\Phi\frac{d^{n+k}}{d(sin\Phi)^{n+k}}[-cos^{2n}\Phi] = \frac{(2n)!}{2^n n!(n-k)!}cos^k\Phi\left[sin^{n-k}\Phi - \frac{(n-k)(n-k-1)}{2(2n-1)}sin^{n-k-2}\Phi + \right.$$

$$\left. + \frac{(n-k)(n-k-1)(n-k-2)(n-k-3)}{2\cdot 4(2n-1)(2n-3)}sin^{n-k-4}\Phi - \ldots\right]. \quad (2.3)$$

Obmedzenie stupňa rozvoja tiažového potenciálu na určitý počet členov popisuje normálny tiažový potenciál a hladinová plocha ním definovaná je normálny hladinový sféroid. Pri splnení podmienok $J_n^{(0)} = \frac{A - C}{M a_o^2}$, $J_1^{(0)} = J_1^{(1)} = S_1^{(1)} = 0$ a rozvojom stupňa $n = \bar{n} = 2$, $k = 0$, je definovaný tiažový potenciál Clairautovho sféroidu

The limitation of a degree of the expansion of the gravity potential to certain number of coefficients is described by the normal gravity potential and the level surface defined by it is the normal leveling spheroid. Under the condition $J_n^{(0)} = \frac{A - C}{M a_o^2}$, $J_1^{(0)} = J_1^{(1)} = S_1^{(1)} = 0$ and by setting the degree of expansion $n = \bar{n} = 2$, $k = 0$, the geopotential of Clairaut's spheroid is defined as

$$U = \frac{GM}{\rho}\left\{1 + \left(\frac{a_o}{\rho}\right)^2 J_2^{(0)}P_2^{(0)}(sin\Phi) + \frac{1}{3}q\left(\frac{a_o}{\rho}\right)^{-3}[1 - P_2^{(0)}(sin\Phi)]\right\}. \quad (2.4)$$

Geoid a normálny hladinový sféroid sú idealizované fyzikálne telesá nevhodné na matematické vyjadrenie priestorových vzťahov (Clairautov sféroid je plocha 14. stupňa [8]). Túto požiadavku spĺňa až zemský rotačný elipsoid, ktorý je matematickou aproximáciou tvaru Zeme. Plocha normálneho hladinového sféroidu je blízka ploche zemského referenčného elipsoidu, preto je možné sféroid nahradiť elipsoidom. Z rovnice rotačného elipsoidu vyplýva pre geocentrický rádius vektor ρ približný vzťah

Geoid and the normal level spheroid are only idealized physical bodies, which are not suitable for mathematical expression of spatial relations (Clairaut's spheroid is an surface of 14th degree [8]). Just using the earth's rotation ellipsoid, which is the mathematical approximation of the Earth's shape, can satisfy this condition. The surface of the normal level spheroid is similar to the surface of the earth's reference ellipsoid therefore the spheroid can be replaced by the ellipsoid. For the geocentric radius-vector ρ following form can be derived from an equation for the rotation ellipsoid

$$(X^2 + Y^2)a^{-2} + Z^2 a^{-2}(1 - e^2)^{-1} - 1 = a^{-2}[\rho^2 cos^2\Phi cos^2\Lambda + \rho^2 cos^2\Phi sin^2\Lambda] + a^{-2}\rho^2 sin^2\Phi(1 - e^2)^{-1} - 1 =$$

$$= \rho^2 cos^2\Phi(1 - e^2) + \rho^2 sin^2\Phi - a^2(1 - e^2) \Rightarrow \rho = \frac{a\sqrt{1 - e^2}}{\sqrt{1 - e^2 cos^2\Phi}} \cong a\sqrt{1 - e^2}\left(1 + \frac{1}{2}e^2 cos^2\Phi\right) =$$

$$= a\left\{1 - \frac{1}{2}e^2 + \frac{1}{2}e^2\left[\frac{2}{3} - \frac{2}{3}P_2^{(0)}(sin\Phi)\right]\right\} = a\left[1 - \frac{1}{6}e^2 - \frac{1}{3}P_2^{(0)}(sin\Phi)\right], \qquad (2.5)$$

a z rozvoja normálneho tiažového potenciálu Clairautovho sféroidu vyplýva pre jeho rádius vektor

The radius vector can be derived from an expansion of the normal geopotential of Clairaut's spheroid

$$\rho = R_o\left\{1 + \frac{1}{3}q\left(\frac{a_o}{\rho}\right)^{-3} + \left[\left(\frac{a_o}{\rho}\right)^2 J_2^{(0)} - \frac{1}{3}q\left(\frac{a_o}{\rho}\right)^{-3}\right]P_2^{(0)}(sin\Phi)\right\}, \quad R_o = \frac{GM}{U_o}, \qquad (2.6)$$

kde R_o je dĺžkový rozmerový faktor.

where R_o is the size factor of length.

Z porovnania rovníc rádius vektorov elipsoidu (2.4) a sféroidu (2.5) vyplýva Clairautova rovnica

The comparison between the equations for radius vector of ellipsoid (2.4) and spheroid (2.5) yields the Clairaut's equation

$$R_o\left(J_2^{(0)} - \frac{1}{3}q\right) = \frac{a\,e^2}{3} \Rightarrow J_2^{(0)} - \frac{1}{3}q = \frac{a\,e^2}{3R_o} \Rightarrow J_2^{(0)} - \frac{1}{3}q = \frac{2}{3}f \Rightarrow f = -\frac{3}{2}J_2^{(0)} + \frac{1}{2}q, \qquad (2.7)$$

vyjadrujúca vzťah medzi geometrickou veličinou f a fyzikálnymi veličinami sféroidu $J_2^{(0)}$, q s ohľadom na prijaté zjednodušenia $R_o \equiv a, f = \frac{a - b}{a} = 1 - \sqrt{1 - e^2} \cong \frac{1}{2}e^2, \rho = a_o$.

which expresses the relation between the geometric parameter f and the physical parameters of spheroid $J_2^{(0)}$, q, considering adopted simplifications $R_o \equiv a, f = \frac{a - b}{a} = 1 - \sqrt{1 - e^2} \cong \frac{1}{2}e^2, \rho = a_o$.

Poznámka: Helmert doplnil rozvoj potenciálu hladinového sféroidu zonálnym členom stupňa n = 4 a odvodil tiažové zrýchlenie na povrchu normálneho hladinového elipsoidu (aproximujúceho sféroid), ktoré sa používa napriek jeho zastaranosti aj v súčasnosti v prípadoch geodetických základov na Krasovského referenčnom elipsoide, pretože oba elipsoidy majú prakticky rovnaké hodnoty sploštenia.

Note: Helmert added a zonal member of degree n = 4 into expansion of the potential level spheroid and derived a gravity acceleration on a surface of the normal level ellipsoid (that approximates spheroid), which is still used in causes of geodetic bases on Krasovsky's reference ellipsoid, as both ellipsoids have practically equal value of flattening.

Normálny tiažový potenciál U a zrýchlenie γ normálneho hladinového rotačného elipsoidu vyjadrujú rovnice [8]

The normal gravity potential U and the acceleration γ of the normal level rotation ellipsoid are expressed by equations [8]

$$U = \frac{GM}{ae} arctan\, e' + \frac{1}{3}\omega^2 a^2, \qquad (2.8)$$

$$\gamma = \frac{GM}{a\sqrt{a^2 sin^2\beta + b^2 cos^2\beta}}\left[1 + \frac{\omega^2 a^3 e}{GM}\frac{p'_o}{p_o}\left(\frac{1}{2}sin^2\beta - \frac{1}{6}\right) - \frac{\omega^2 a^2 b}{GM}cos^2\beta, \qquad (2.9)$$

kde

where

$$p_o' = 3(1 + e'^{-2})(1 - e'^{-1} \arctan e') - 1\,, \quad p_o = \frac{1}{2}\left[\left(1 + \frac{1}{3}e'^{-2}\right)\arctan e' - 3e'^{-1}\right], \; \tan\beta = \frac{b}{a}\tan B\,.$$

3. Teoretický princíp testovania geopotenciálnych modelov

Na určenie presnosti geopotenciálneho modelu je potrebné poznať geocentrické súradnice a Molodenského normálne výšky h na bodoch testovacej siete. Použitie ortometrických výšok h' a geoidu v oblastiach kontinentov nie je vhodné z dôvodu použitia hypotézy o rozložení hustoty hmoty medzi geoidom a zemským povrchom. Naproti tomu teória Molodenského normálnych výšok nepredpokladá nijaké hypotézy [12].

Teoreticky je skutočný tiažový potenciál $W(\rho, \Phi, \Lambda)$ v bode P na zemskom povrchu rovný normálnemu tiažovému potenciálu v bode P_o na teluroide (obr. 1)

$$W(\rho, \Phi, \Lambda) = U(\Phi_o, h)\,. \quad (3.1)$$

Vzdialenosť medzi teluroidom a elipsoidom je normálna výška h a vzdialenosť medzi zemským povrchom a teluroidom je výšková anomália ζ.

Normálny tiažový potenciál $U(\Phi_o, h, GM, a_o, \omega, f)$ na teluroide je definovaný ako funkcia geocentrickej gravitačnej konštanty GM, strednej uhlovej rýchlosti rotácie Zeme ω, veľkosti hlavnej poloosi a_o a sploštenia f hladinového rotačného elipsoidu vzťahom [8]

3. Theoretic principle of geopotential model testing

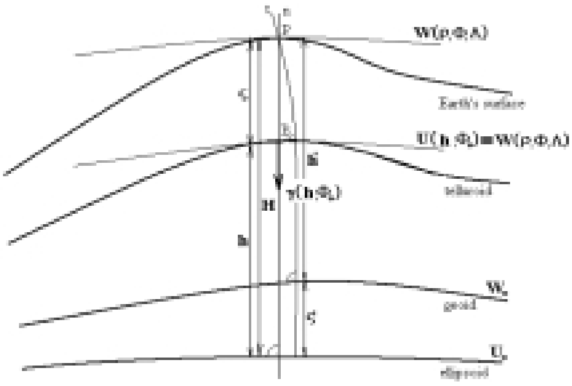
The accuracy estimate requires the geocentric coordinates and Molodensky's normal heights h at the points of testing network. The orthometric heights h' and geoid over continents are not suitable for using because of hypothesis about distribution of density of mass between the geoid and the earth's surface. On the other hand, the Molodensky's normal heights theory does not assume any hypothesis [12].

Theoretically, the actual gravity potential $W(\rho, \Phi, \Lambda)$ at the point P on the earth's surface is equal to the normal potential at the point P_o on the telluroid (Fig. 1)

$$W(\rho, \Phi, \Lambda) = U(\Phi_o, h)\,. \quad (3.1)$$

The distance between the telluroid and ellipsoid is the normal height h and the distance between the earth's surface and the telluroid is the height anomaly ζ.

The normal gravity potential $U(\Phi_o, h, GM, a_o, \omega, f)$ on the telluroid is defined as a function of the geocentric gravitational constant GM, the mean angular velocity of the Earth's rotation ω, the size of the semi-major axis a_o and the flattening f of the level rotation ellipsoid [8]



Obr. 1. Teória výšok, normálna výška a ortometrická výška h'
Fig. 1, Theory of heights, normal height and orthometric height h'

$$U(\rho_o, \Phi_o) = \frac{GM}{\rho_o}\left[1 - \sum_{n=1}^{\infty}(-1)^{n+1}\frac{3e^{2n}}{(2n+1)(2n+3)}\left(1 - n + 5n\frac{J_2^{(0)}}{e^2}\right)\left(\frac{a_o}{\rho_o}\right)^{2n} P_{2n}^{(0)}(\sin\Phi_o)\right] + \frac{1}{2}\omega^2\rho_o^2\cos^2\Phi_o =$$

$$= \frac{GM}{\rho_o}\left[1 - \sum_{n=1}^{\infty}\left(\frac{a_o}{\rho_o}\right)^{2n}\frac{\sin\Phi_o}{n!}\frac{d^n}{d(\sin^2\Phi_o)^n}[-\sin^{2n-1}\Phi_o\cos^{2n}\Phi_o]\,P_2^{(0)}(\sin\Phi_o)\right] + \frac{1}{2}\omega^2\rho_o^2\cos^2\Phi_o\,, \quad (3.2)$$

alebo v uzavretom tvare pomocou krivočiarych súradníc u, v, w [7]

or in closed form using curvilinear coordinates u, v, w [7]

$$U(\Phi_o, h) = U(u, v, w) = \frac{GM}{ae}\,\mathrm{arcotan}^{-1}\sinh w + \frac{1}{3}\omega^2 a^2 e^2\cosh^2 w + P_2^{(0)}(\cos u)\,\times$$

$$\times\left\{\frac{1}{3}\frac{\omega^2 a^2}{P}[(3\sinh^2 w + 1)\,\mathrm{arcotan}^{-1}\sinh w - 3\sinh w] - \frac{1}{3}\omega^2 a^2 e^2\cosh^2 w\right\} =$$

$$= \frac{GM}{ae}\left\{\mathrm{arcotan}^{-1}\sinh w + \frac{1}{3}qe\,[(3\sinh^2 w + 1)\,\mathrm{arcotan}^{-1}\sinh w - 3\sinh w]\,\times\right.$$

$$\left.\times\left[\frac{3 - 2e^2}{e^2}\arctan\frac{e}{\sqrt{1 - e^2}} - 3\frac{\sqrt{1 - e^2}}{e}\right]^{-1} eP_2^{(0)}(\cos u) + \frac{1}{3}qe^3\cosh^2 w\,[1 - P_2^{(0)}(\cos u)]\right\}, \quad (3.3)$$

kde | where

$$P = \frac{3-2e^2}{e^2}\, arctan\, \frac{e}{1-f} - 3\,\frac{1-f}{e}\,,\ P_2^{(0)}(cos\, u) = \frac{3}{2}\, cos^2\, u - \frac{1}{2}\,,\ e^2 = 2f - f^2 = \frac{a^2-b^2}{a^2}\,,\ f = \frac{a-b}{a}\,,\ q = \frac{\omega^2 a_o^3}{GM}\,. \quad (3.4)$$

Vzájomný vzťah medzi u, v, w a X, Y, Z, resp. geodetickými zemepisnými súradnicami B, L, H vyjadrujú rovnice [6], [5]

The relation between u, v, w and X, Y, Z, or geodetic coordinates B, L, H can be expressed as follows[6], [5]

$$\begin{aligned} X &= ae\, cos h\, w\, sin u\, cos v = (N + H)\, cos B\, cos L\,, \\ Y &= ae\, cos h\, w\, sin u\, sin v = (N + H)\, cos B\, sin L\,, \\ Z &= ae\, sin h\, w\, cos u = [N(1 - e^2 + H)]\, sin B\,, \end{aligned} \quad (3.5)$$

$$u = arccos\, \frac{Z}{ae\, sin h\, w}\,,\ v = arctan\, \frac{Y}{X}\,,\ w = \log(cos h\, w + sin h\, w)\,,\ cos h^2\, w = \sqrt{1 + sin h^2\, w}\,,$$

$$sin h\, w = \frac{\sqrt{\rho^2 - a^2e^2 + \sqrt{(\rho^2 - a^2e^2)^2 + 4a^2e^2Z^2}}}{2a^2e^2}\,,\ \rho^2 = X^2 + Y^2 + Z^2\,,\ N = \frac{a}{\sqrt{1 - e^2\, sin^2\, B}}\,,$$

$$tan\,\Phi = (1 - e^2)\, tan B = \frac{b^2}{a^2}\, tan B\,. \quad (3.6)$$

Na testovanie geopotenciálneho modelu na území Slovenska je potrebné vyjadriť tiažový potenciál $W(\rho, \Phi, \Lambda)$ v slapovom systéme MEAN [13]

The testing of the geopotential model over the territory of Slovakia requires expressing the gravity potential $W(\rho, \Phi, \Lambda)$ in the mean tidal reference system MEAN [13]

$$\begin{aligned} W = \frac{GM}{\rho} \Bigg\{ 1 &+ \sum_{n=2}^{\bar{n}} \sum_{k=0}^{n} \left(\frac{a_o}{\rho}\right)^n (J_n^{(k)} cos\, k\Lambda + S_n^{(k)} sin\, k\Lambda)\, P_n^{(k)}(sin\,\Phi) + \frac{1}{3}\left(\frac{a_o}{\rho}\right)^{-3} q[1 - P_2^{(0)}(sin\,\Phi)] + \\ &+ (1 + k_s - h_s)\left[\frac{GM_L}{GM}\left(\frac{\rho}{\rho_L}\right)^3 P_2^{(0)}(sin\,\delta_L)\, P_2^{(0)}(sin\,\Phi) + \frac{GM_S}{GM}\left(\frac{\rho}{\rho_S}\right)^3 P_2^{(0)}(sin\,\delta_S)\, P_2^{(0)}(sin\,\Phi)\right]\Bigg\}\,, \end{aligned} \quad (3.7)$$

kde δ_S, δ_L, ρ_S, ρ_L sú geocentrické deklinácie a geocentrické rádius vektory slapotvorných telies (Slnko a Mesiac); GM_L, GM_S sú selenocentrická a heliocentrická gravitačná konštanta, k_s, h_s sú sekulárne Loveho čísla [11], ktoré vyjadrujú plastickú reakciu pružnej Zeme na gravitačné pôsobenie Mesiaca a Slnka, $\bar{n}$ je maximálny stupeň rozvoja potenciálu.

Presnosť určenia geopotenciálneho modelu je možné vyjadriť ako funkciu rozdielu medzi anomáliou výšky vypočítanou z modelu a anomáliou výšky určenej z GPS a nivelácie

$$\delta_\zeta = H - h - \left[\frac{W(\rho, \Phi, \Lambda) - U(\rho, \Phi)}{\gamma(\rho, \Phi)}\right], \quad (3.8)$$

kde H je elipsoidická výška bodu P; $U(\rho, \Phi)$, $\gamma(\rho, \Phi)$ sú normálny potenciál a normálne tiažové zrýchlenie v bode P na zemskom povrchu v tvare [8]

where δ_S, δ_L, ρ_S, ρ_L are the geocentric declinations and the geocentric radius-vectors of the Sun and the Moon; GM_L, GM_S are the selenocentric gravitational constant and the heliocentric gravitational constant, respectively, k_s, h_s are the secular Love numbers [11], which describe the elastic response of the Earth to the gravitational effect of the Sun and the Moon, $\bar{n}$ is the maximum degree of expansion of the potential.

The accuracy estimation of the geopotential model can be expressed as a function of difference between the height anomaly computed from the geopotential model and the height anomaly determined from GPS and leveling.

$$\delta_\zeta = H - h - \left[\frac{W(\rho, \Phi, \Lambda) - U(\rho, \Phi)}{\gamma(\rho, \Phi)}\right], \quad (3.8)$$

where H is the ellipsoidal height of the point P; $U(\rho, \Phi)$, $\gamma(\rho, \Phi)$ are the normal potential and the normal gravity acceleration at point P on the earth's surface in form of [8]

$$\gamma(\rho, \Phi) = \sqrt{\left[\frac{\partial U(\rho, \Phi)}{\partial \rho}\right]^2 + \frac{1}{\rho^2}\left[\frac{\partial U(\rho, \Phi)}{\partial \Phi}\right]^2} = \frac{GM}{\rho^2}\sum_{i=0}^{\infty}(2n+1)\left(\frac{a_o}{\rho}\right)^{2n} J_{2n}^{(0)} P_{2n}^{(0)}(sin\,\Phi)\,, \quad (3.9)$$

alebo [6] | or [6]

$$\gamma(u, w) = \sqrt{\left(-\frac{1}{h_w}\frac{\partial U}{\partial w}\right)^2 + \left(\frac{1}{h_u}\frac{\partial U}{\partial u}\right)^2}\,, \quad (3.10)$$

$$\frac{\partial U}{\partial u} = \frac{1}{2}\,\omega^2 a^2 e^2 \cos h^2 w \sin^2 u - \frac{1}{2}\,\frac{\omega^2 a^2}{P}\,[(3 \sin h^2 w + 1) \arctan \sin h\, w - 3 \sin h\, w] \sin^2 u\,,$$

$$\frac{\partial U}{\partial v} = 0\,,$$

$$\frac{\partial U}{\partial w} = -\frac{GM}{ae \cos h\, w} + \frac{1}{2}\,\omega^2 a^2 e^2 \sin^2 u \sin h^2 w + \frac{1}{3}\,\frac{\omega^2 a^2}{P}\,[3 \sin h^2 w \arctan \sin h\, w - 2\,\frac{\sin h^2 w}{\cos h\, w} - 4 \cos h\, w]\, P_2^{(0)}(\cos u)\,,$$

$$h_u^2 = h_w^2 = a^2 e^2 (\cos h^2 w - \sin^2 u)\,,\ P = \frac{3 - 2e^2}{e^2}\, arctg\, \frac{e}{1-f} - 3\,\frac{1-f}{e}\,,$$

$$\gamma(u, w) = \sqrt{\frac{1}{a^2 e^2 (\cos h^2 w - \sin h^2 u)}}\, \left\{\left(\frac{GM}{ae \cos h\, w}\right)^2 + \frac{1}{4}\,\omega^4 a^4 e^4 (\sin^4 u \sin h^2 2w + \sin^2 2u \cos h^4 w) + \right.$$

$$+ \frac{1}{9}\,\frac{\omega^4 a^4}{P^2}\left[3 \sin h\, 2w \arctan \sin h\, w - 2\,\frac{\sin h^2 w}{\cos h\, w} - 4 \cos h\, w\right]^2 [P_2^{(0)}(\cos u)] +$$

$$\left. + \frac{1}{4}\,\frac{\omega^4 a^4}{P^2}\,[(3 \sin h^2 w + 1) \arctan \sin h\, w - 3 \sin h\, w] 2 \sin^2 2u \right\}^{\frac{1}{2}}. \qquad (3.11)$$

4. Presnosť určenia geopotenciálneho modelu EGM 96 na území Slovenska

Presnosť určenia geopotenciálneho modelu EGM 96 bola testovaná na 55 bodoch, ktorých predbežné výsledky sú zo spoločného spracovania kampaní SGN 93, 95, 98, 99, CEGRN 94, 95, 96, 97 a TATRY 98, 99 [11] a pripojenia do ITRF97 postupom uvedeným v [10].

Tiažový potenciál $W(\rho, \Phi, \Lambda)$ na zemskom povrchu je rovný normálnemu potenciálu $U(\Phi_o, h)$ na teluroide iba teoreticky.

Rozdiel

$$\delta W = W(\rho, \Phi, \Lambda) - U(\Phi_o, h) \qquad (4.1)$$

definuje nepresnosť určenia geopotenciálneho modelu za predpokladu, že chyby v určení výšok môžeme zanedbať.

Obr. 2 vyjadruje rozdiely $\delta W = W_{EGM\,96} - U(P_o)$ medzi tiažovým potenciálom $W_{EGM\,96}$ vypočítanom z modelu EGM 96 a normálnym potenciálom na teluroide $U(P_o)$ získaného z GPS a nivelácie.

V obr. 5 sú vyjadrené rozdiely medzi anomáliami výšok $\zeta_{EGM\,96}$ vypočítaných z geopotenciálneho modelu EGM 96 a anomáliami výšok ζ_{H-h} určených z GPS a nivelácie.

4. Accuracy estimation of geopotential model EGM 96 at territory of Slovakia

The accuracy estimation of the geopotential model EGM 96 was tested at 55 points. The estimations can be obtained from a common processing of the campaigns SGN 93, 95, 98, 99, CEGRN 94, 95, 96, 97 and TATRY 98, 99 [11] and connection into ITRF97 according to the technique described in [10].

The gravity potential $W(\rho, \Phi, \Lambda)$ on the earth's surface is equal to the normal potential $U(\Phi_o, h)$ on the telluroid only theoretically.

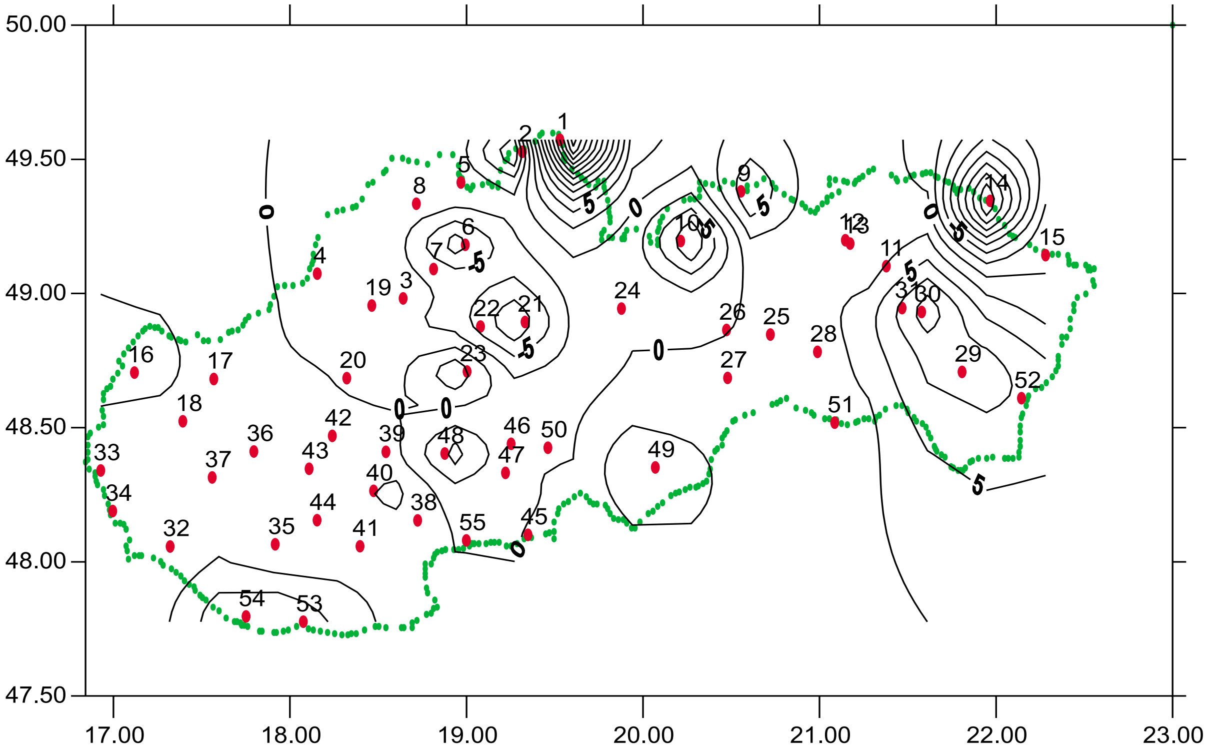
The difference

$$\delta W = W(\rho, \Phi, \Lambda) - U(\Phi_o, h) \qquad (4.1)$$

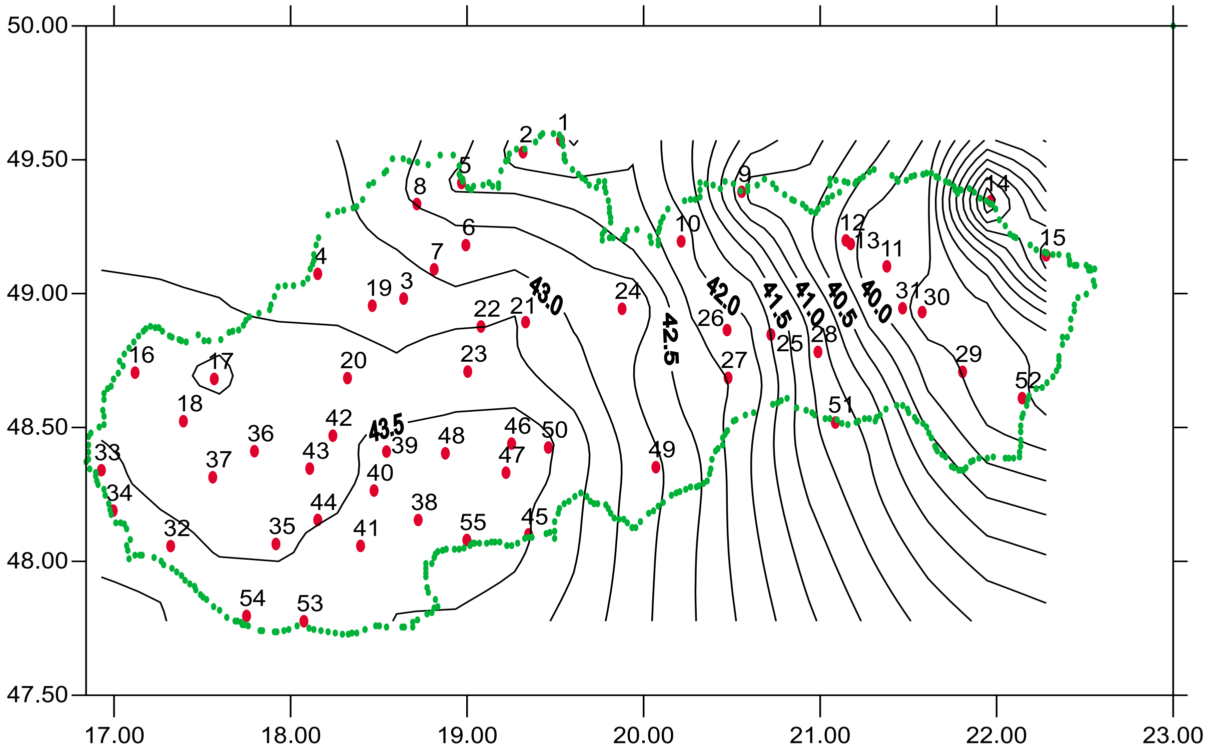
defines the inaccuracy in the geopotential model under assumption that errors of heights estimation can be neglected.

Figure 2 shows the differences $\delta W = W_{EGM\ 96} - U(P_o)$ between the gravity potential $W_{EGM\,96}$ computed from EGM 96 model and the normal potential on the telluroid $U(P_o)$ obtained from GPS and leveling.

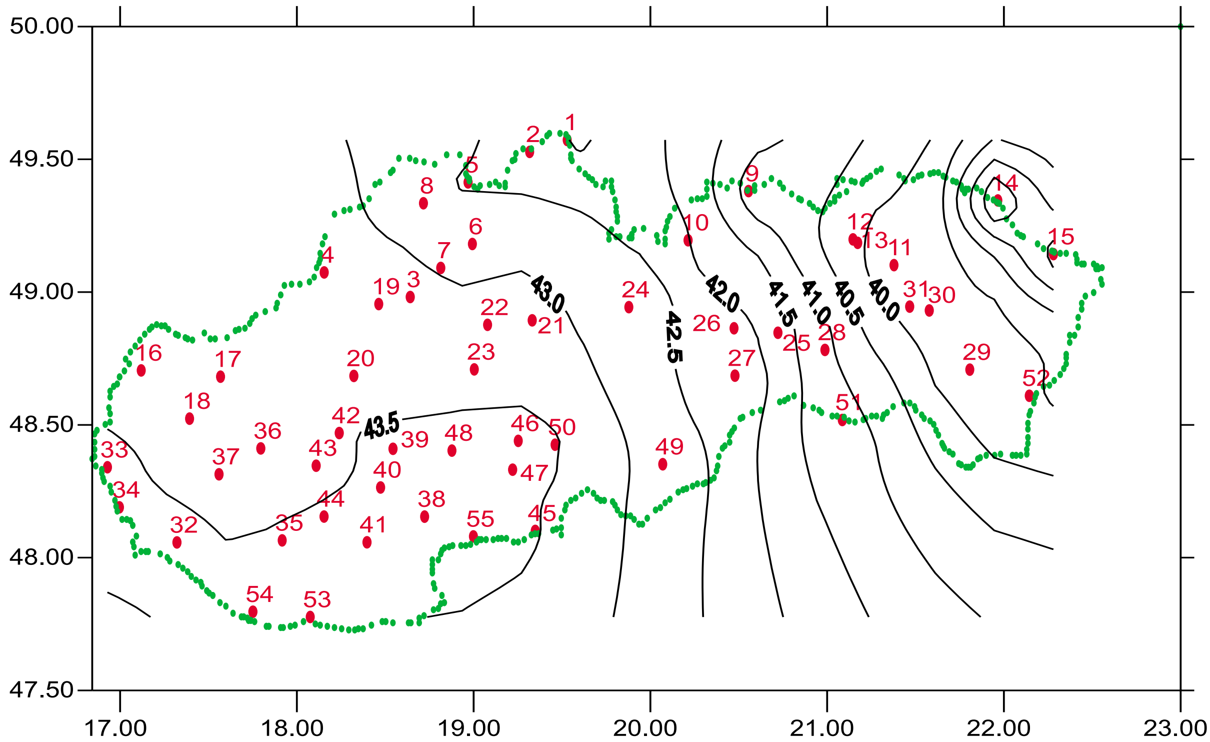
Figure 5 shows the differences between height anomalies $\zeta_{EGM\ 96}$ computed from EGM 96 model and height anomalies ζ_{H-h} determined from GPS and leveling.



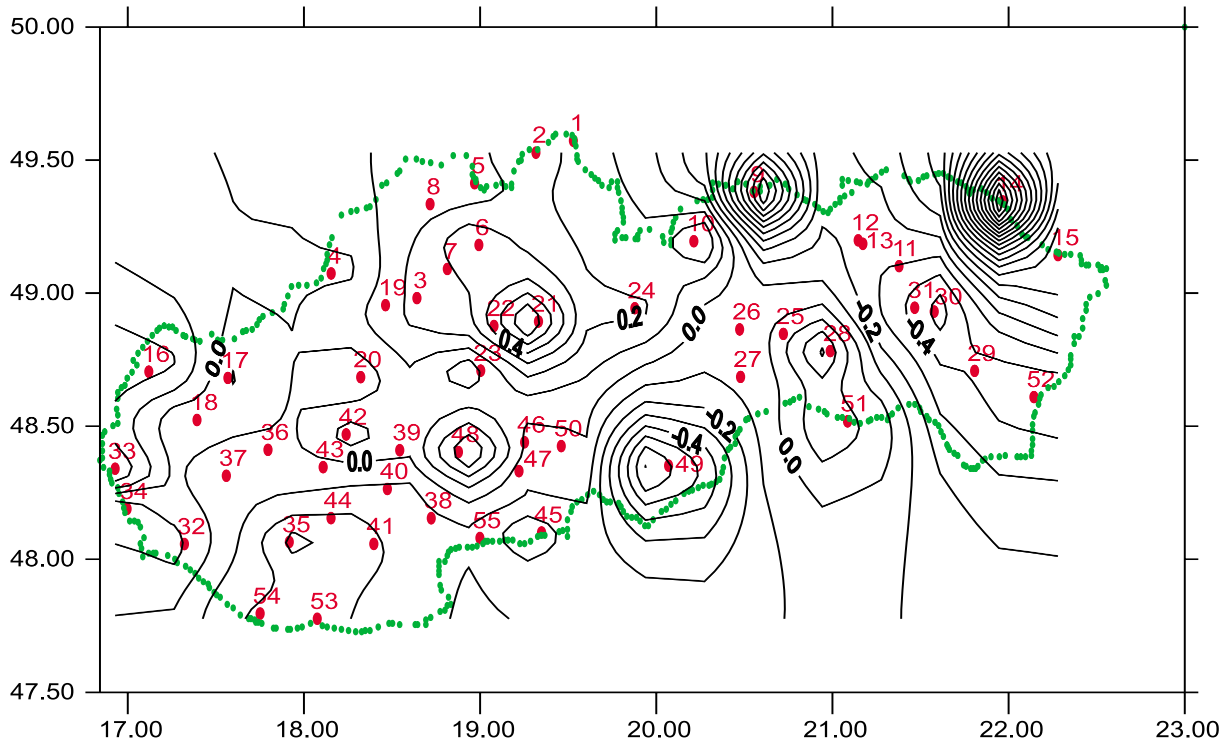
Obr. 2. Rozdiely medzi tiažovým potenciálom $W_{EGM\ 96}$ vypočítaným z EGM 96 a normálnym potenciálom $U(P_o)$ určeným z GPS a nivelácie $[m^2.s^{-2}]$
Fig. 2. Differences between gravity potential $W_{EGM\ 96}$ computed from EGM 96 and normal potential $U(P_o)$ obtained from GPS and leveling $[m^2.s^{-2}]$



Obr. 3. Anomálie výšok ζ_{H-h} určené z GPS a nivelácie [m]
Fig. 3. Height anomalies ζ_{H-h} determined from GPS and leveling [m]



Obr. 4. Anomálie výšok $\zeta_{EGM\ 96}$ vypočítané z EGM 96 [m]
Fig. 4. Height anomalies $\zeta_{EGM\ 96}$ computed from EGM 96 [m]



Obr. 5. Rozdiely $\delta\zeta = \zeta_{H-h} - \zeta_{EGM\ 96}$ medzi anomáliami výšok vypočítaných z EGM 96 a určených z GPS a nivelácie [m]
Fig. 5. Differences $\delta\zeta = \zeta_{H-h} - \zeta_{EGM\ 96}$ between height anomalies computed from EGM 96 and obtained from GPS and [m]

5. Záver

Nepresnosť určenia geopotenciálneho modelu EGM 96 na území Slovenska vyjadrujú maximálne rozdiely tiažového potenciálu $|\delta W| = 20\ \mathrm{m^2s^{-2}}$, maximálne rozdiely anomálie výšky $|\delta\zeta| = 1{,}50$ m, stredná chyba tiažového potenciálu $\sigma_{W_{EGM\,96}}$ a stredná chyba anomálie výšky $\sigma_{\zeta_{EGM\,96}}$:

5. Conclusion

The inaccuracies in the EGM 96 model estimation at territory of Slovakia are expressed by the maximal differences of the gravity potential $|\delta W| = 20\ \mathrm{m^2s^{-2}}$, the maximal difference of height anomalies $|\delta\zeta| = 1.50$ m, the mean error of gravity potential $\sigma_{W_{EGM\,96}}$ and the mean error of height anomaly $\sigma_{\zeta_{EGM\,96}}$:

$$\sigma_{W_{EGM\,96}} = \sqrt{\frac{\sum_{i=1}^{n} \delta W_i^2}{n-1}} = \pm 6.8\ \mathrm{m^2s^{-2}}\ ,\quad \sigma_{\zeta_{EGM\,96}} = \sqrt{\frac{\sum_{i=1}^{n} \delta\zeta_i^2}{n-1}} = 0.46\ m\ ,\ i \in \langle 1, n\rangle\ ,$$

kde n je počet bodov.

where n is the number of points.

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