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NÁVRH KOTVENIA CFRP LAMIEL

DESIGN OF ANCHORAGE OF CFRP STRIPS

Kotvenie externe lepenej výstuže je hlavným problémom pri zosilňovaní betónových konštrukcií pomocou CFRP lamiel (Štěpánek - Šustalová 2000, 2001). Tento článok popisuje spôsob skúšok a niektoré výsledky vyplývajúce z rôznych vlastností kotevných prvkov. Výsledky skúšok sú porovnávané s hodnotami získanými matematickou analýzou správania sa kotevnej oblasti a rozdelenia napätí pozdĺž kotevnej dĺžky.

Anchorage of externally bonded reinforcement is the main problem for strengthening concrete structures with CFRP strips (Štěpánek & Šustalová 2000, 2001). This paper describes a form of the tests and some results implicating from different properties of anchorage elements. The test results are compared with values obtained from mathematical analysis of anchorage zones behavior and normal stress distribution along the anchorage length.

Keywords

Anchorage of CFRP strip, bonded reinforcement, strengthening of concrete beam, analytical solution, experimental results

1. Theoretical basis of anchorage

Brosens & Van Gemert (1999) [1] published derivation of the fundamental equations for anchoring bonded non-prestressed CFRP strips at flexural stress specimen. The main assumptions in their derivation were:

- elastic behaviour of all materials used (concrete, steel, adhesive and CFRP);
- full composite action between a bonded strip and concrete;
- strains and stresses are uniformly distributed over the entire width of a cross section of the anchorage area.

Derivation of the fundamental equations for anchoring bonded non-prestressed CFRP strips on anchorage elements (Fig. 1) was published by Štěpánek & Šustalová [2].

Static equations of equilibrium and the elementary equations of elasticity were derived based on the fundamental equations of a concrete cross section strengthened with CFRP strips as follows

$$\frac{d^2\sigma_p(x)}{dx^2} - \frac{G_a\sigma_p(x)}{t_a t_p E_p} = \frac{G_a\sigma_c(x)}{t_a t_p E_c}, \tag{1}$$

$$\tau_p(x) = \frac{d\sigma_p(x)}{dx}\, t_a, \tag{2}$$

where: $\sigma_p(x)$ is normal stress in CFRP strip (N/mm^2),
$\sigma_c(x)$ is normal stress in the bottom of a concrete beam (N/mm^2),
$\tau_p(x)$ is shear stress in direction of axis x in adhesive (N/mm^2),
t_a is thickness of adhesive (mm),
t_p is thickness of CFRP plate (mm),
E_p is Young's modulus of CFRP plate (N/mm^2),
E_c is Young's modulus of concrete (N/mm^2),
G_a is shear modulus of adhesive (N/mm^2).

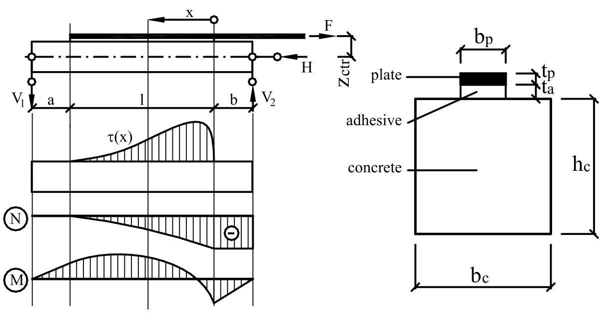

Fig. 1. Geometry details, distribution of internal forces

If we assume that

- the influence of the normal stress acting perpendicular to the concrete surface can be neglected,
- the distribution of internal forces of the tested specimen is known,

it is possible to write

$$M(x) = -F\, z_{ctr}\frac{x + a}{l + a + b} + b_p t_p \sigma_p(x)\, z_{ctr}, \qquad N(x) = b_p t_p \sigma_p(x). \tag{3}$$

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where $M(x)$ is the bending moment and $N(x)$ is the normal force (Fig. 1).

The normal stress distribution over the concrete section is given according to Navier's law

$$\sigma_c(x) = -\frac{b_p t_p}{A_{tr}}\,\sigma_p(x)\left[z_{ctr}^2\,\frac{A_{tr}}{J_{tr}} - 1\right] - F\,\frac{z_{ctr}^2}{J_{tr}}\,\frac{l - x + a}{l + a + b}, \quad (4)$$

where: A_{tr} is area (mm^2) of transformed cross section (transformed factor $m_a = \frac{E_a}{E_c}$ and $m_p = \frac{E_p}{E_c}$),

I_{tr} is moment of inertia of the transformed cross section (mm^4),

z_{ctr} is the distance of the loaded concrete fibres from to the centroid of the transformed cross section (mm).

The solution of non-homogeneous differential equations (1) and (2) can be found as follows

$$\sigma_p(x) = C_1 e^{Ax} + C_2 e^{-Ax} - \frac{A_c B_2}{A^2}\,(l - x + a), \quad (5)$$

$$\tau(x) = t_p\left[C_1 A e^{Ax} - C_2 A e^{-Ax} - \frac{A_c B_2}{A^2}\right], \quad (6)$$

with $A^2 = A_p + A_c B_1$, $A_p = \frac{G_a}{t_a t_p E_p}$, $A_c = \frac{G_a}{t_a t_p E_c}$,

$$B_1 = -\frac{b_p t_p}{A_{tr}}\left(\frac{z_{ctr}^2}{i_{tr}^2} - 1\right),\; B_2 = F\,\frac{z_{ctr}^2}{J_{tr}}\,\frac{1}{l + a + b}$$

The constants C_1 and C_2 are found out by using appropriate boundary conditions

$$\sigma_p\big|_{x=0} \text{ and } \sigma_p\big|_{x=1} = \frac{F}{A_p} \Rightarrow$$

$$C_1 = \frac{C_2 e^{-Al} + \frac{A_c B_2}{A^2}\,l - \frac{A_c B_2}{A^2}\,(l + a)}{e^{Al}}, \quad (7)$$

$$C_2 = \frac{-\left(\frac{F}{b_p t_p} - \frac{A_c B_2}{A^2}\,(a + l)\right)\cdot e^{Al} - \frac{A_c B_2}{A^2}\,(a + l)}{e^{Al} - e^{-Al}}.$$

The maximum shear stress is at the end of the plate

$$\tau_{max} = t_p\left(AC_1 - AC_2 - \frac{A_c B_2}{A}\right). \quad (8)$$

Fig. 2a shows the distribution of the normal stresses $\sigma_p(x)$, $\sigma_c(x)$ and the shear stress $\tau_p(x)$ along the x axis. In this solved example, the influence of anchorage by a distance of 35 mm from the end of plate can be neglected. The design details and material characteristics are: a beam 150 × 150 × 600 mm, strip SIKA CarboDur S512, $E_c = 27$ GPa, $E_p = 155$ GPa, $G_a = 5{,}33$ GPa. The scheme of loading is shown in Figure 1. Figure 2b shows the internal forces (normal force $N(x)$ and bending moment $M(x)$) in the concrete block along the x-axis.

The differential equation describing normal stress $\sigma_n(x)$ in the perpendicular direction to the lower surface of a concrete beam is given by

$$\frac{d^4\sigma_n(x)}{dx^4} + \frac{E_a b_p}{t_a E_p t_p}\,\sigma_n(x) = 0. \quad (9)$$

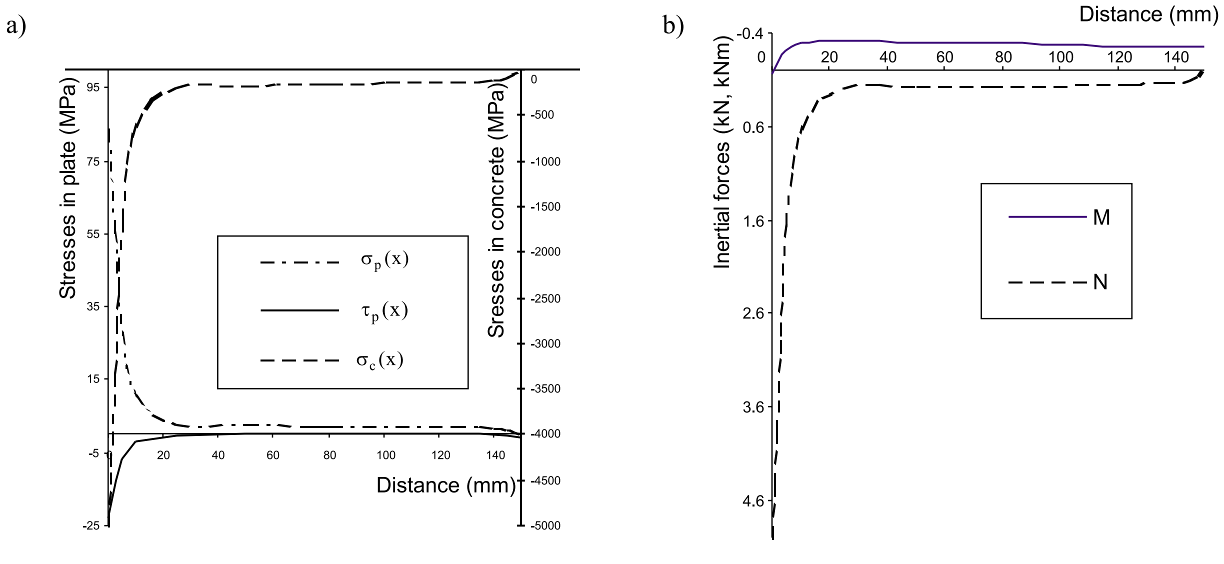

Fig. 2 Results of analytical solution of anchorage problem a) Stresses distribution along the anchorage zone $\sigma_p(x)$, $\tau_p(x)$, $\sigma_c(x)$ b) Dependence of internal forces in the concrete specimen on distance x

The solution of eq. (9) can be found as follows

$$\sigma_n(x) = e^{-\beta x}[D_1 \cos(\beta x) + D_2 \sin(\beta x)] \tag{10}$$

with $\beta = \sqrt[4]{\dfrac{E_a b_p}{4 t_a E_p I_p}}$.

The constants D_1, D_2 can be determined by using the proper boundary conditions:

$$\frac{d^2\sigma_n(x)}{dx^2} = \frac{E_a M_p(x)}{t_a E_p I_p} - \frac{E_a M_c(x)}{t_a E_c I_c}. \tag{11}$$

Then D_1 and D_2 can be calculated

$$D1 = \frac{\left[\dfrac{E_a}{t_a 2\beta^2 e^{-\beta l}}\left(\dfrac{M_p}{E_p I_p} - \dfrac{M_c}{E_c I_c}\right) + D_2 \cos(\beta l)\right]}{\sin(\beta l)}, \tag{12}$$

$$D_2 = -\frac{E_a V_1 a}{t_a 2\beta^2 E_c I_c}. \tag{13}$$

The distribution of the normal stress $\sigma_n(x)$ along the x-axis obtained from analytical solution for the identical example solved before is shown in Fig. 3.

2. Tests of anchorage blocks

Anchorage blocks of dimension 150/150/600 mm were prepared from concrete of class B15-B25 according to the Czech standard ČSN 73 1201 (1986). The producer and sponsor of the research work were Prefa Topos Tovačov and SIKA CZ.

The physical and mechanical properties of concrete specimens (Young modulus, tensile and stress strength) were determined by non-destructive methods before the beginning of the tests. The CFRP strips of a cross-sectional dimension 50/1.2 mm and Young's modulus 155 GPa was bonded to the prepared surface of different anchorage lengths – 150, 225 to 300 mm (see Fig. 4).

The aim of the tests was to determine the influence of anchorage length and size of the cross force (acting on the anchorage zone) on the ultimate axis force of the strip. The resistance tensionmeters were glued to the strips, the deformation along the length of anchorage was measured with a videoextensionmeter. The scheme

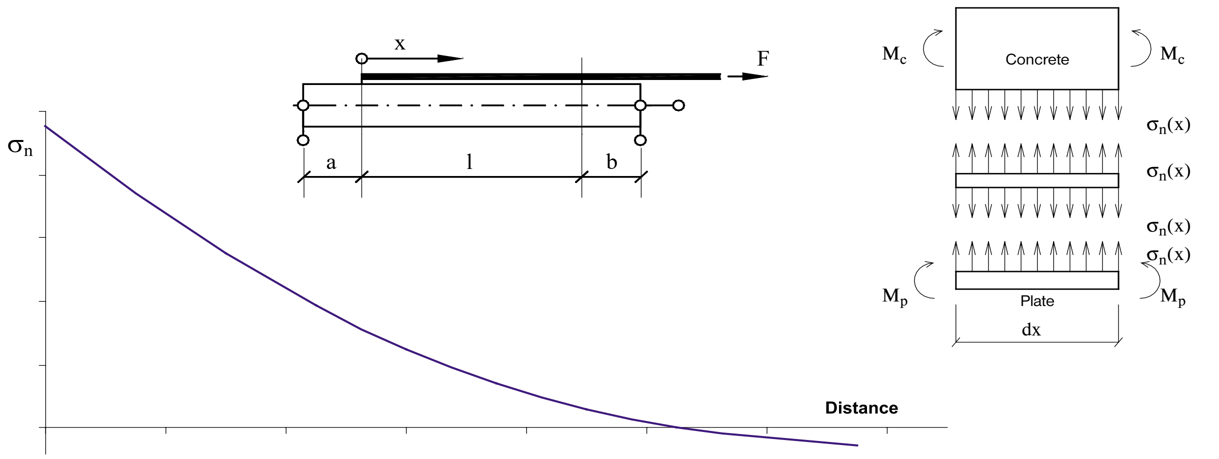

Fig. 3 Stress distribution along the anchorage zone $\sigma_n(x)$

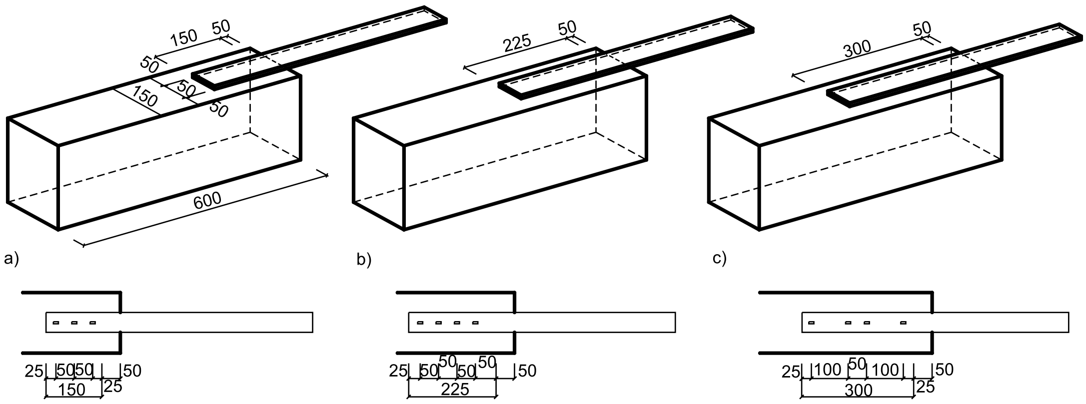

Fig. 4. The elevation of the anchorage zones of the specimen with strain gauges

of the anchorage zones and position of the measuring points are shown in Fig. 4.

Arrangement of the test (the test set-up and applied apparatus) is shown in Fig. 5. Fig. 5a represents anchoring without stirrup (without acting cross force on the anchorage area), Fig. 5b demonstrates anchoring with stirrup without prestressing and Fig. 5c illustrates anchoring with prestressed stirrups.

The results of the test without cross force acting on the bonded strip (alternative according Fig. 5a anchoring without stirrup) and with variable cross force acting of the glued length 150 mm (anchoring with stirrup – see Fig. 5b,c) is shown in Fig. 6.

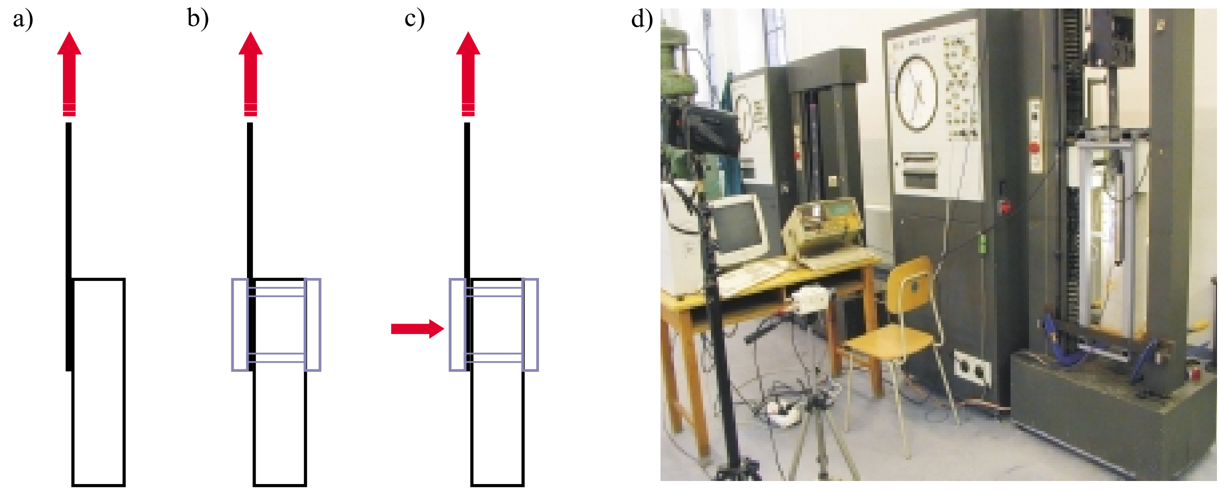

Fig. 5. The loading process and testing equipment
a, b, c) Scheme of tested alternatives of anchoring, d) Arrangement of test

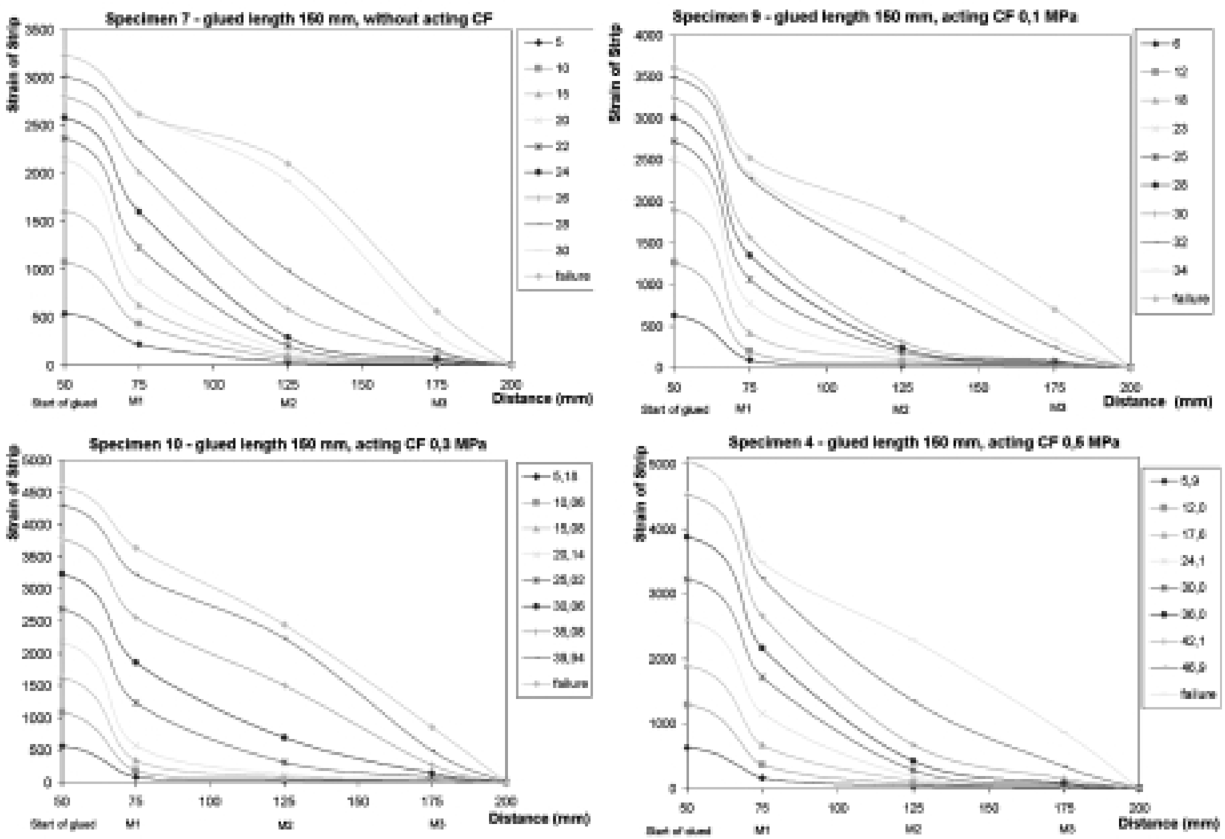

Fig. 6. Strain of CFRP strip with anchorage length 150 mm

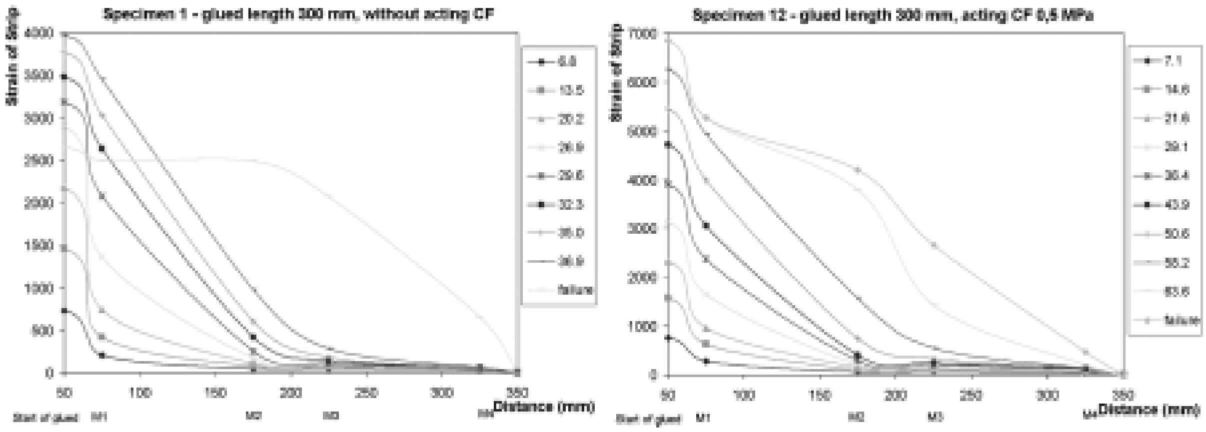

Fig. 7. Strain of CFRP strip with anchorage length 300 mm

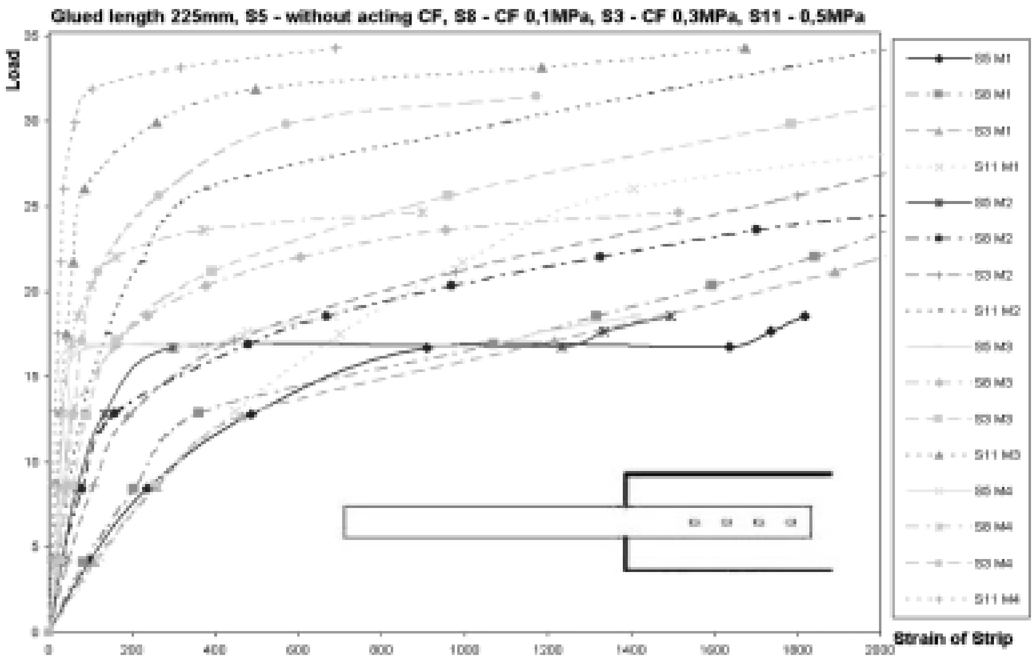

Fig. 8. Strain measured through resistance tensionmeter (M1, M2, M3, M4)

The actual length of the anchorage is in this case smaller than the necessary anchorage length. With a higher length of the anchorage it is possible to achieve a higher axial force in the strip (see Fig. 7).

Fig. 8 shows strain rise of CFRP strip at measuring points throughout the whole course loading on the specimens with glued length of 225 mm.

Comparison of the strip strain between analytical results and experimental tests at small tension normal force of the strip for the bonded length 150 mm is shown in Fig. 9.

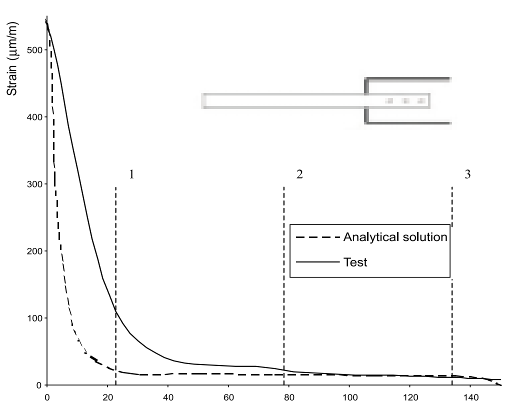

Fig. 9. Strain of CFRP strip along the x axis

3. Conclusions

The comparison of the theoretical and measured results of stress and strain at the anchorage areas demonstrates a good accordance at the linear area of behaviour. The theoretically derived design equations can be used for anchorage of non-prestressed and prestressed CFRP strips.

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