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# A DESIGN OF AN INDUCTION METHOD DEVICE WITH RESPECT TO THE SIGNAL AND NOISE SENSITIVITY

*An objective of this article is the relationship between the signal sensitivity, the noise sensitivity and the distribution of turns of winding on any chosen axis-symmetric core.*

*The analytical study originates from Faraday induction law and Nyquist theorem. Different windings were compared with respect to their physical properties.*

## 1. Introduction

Induction method is often used for the measuring of low frequency magnetic field. The optimization of equipment in Fig. 1 consists of looking for the best combination of material of the core, its shape and winding. The relation between the material susceptibility, the core shape and the resultant sensitivity of the equipment was analyzed in [1]. An objective of this article is to find the relationship between the signal sensitivity, the noise sensitivity and the distribution of turns of winding on any chosen axis-symmetric core.

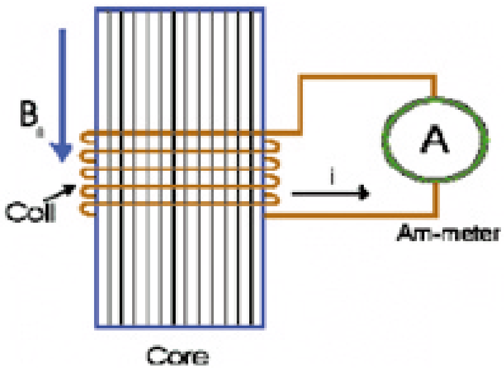


*Fig. 1. The scheme of the studied equipment*

First, the analytical study based on Faraday induction law is done. In this step the impact of the amount of turns and the distribution of turns on the core on the sensitivity are separated. Investigation of thermal noise originates from Nyquist theorem [2]. As a result of the theoretical part one has optimization criteria for searching the windings.

The relevant physical properties of each winding are self-inductance and mutual inductance of the turn and applied magnetic field. These properties are calculated by the finite element method. Next, a computer program for finding the best winding is used. Different windings were investigated on the cylindrical core having the length of 1 m, diameter of 0.084 m and susceptibility of 5000 inserted into a quasi stationary homogeneous magnetic field.

## 2. An induction method

Magnetic flux through the winding is proportional to a flux density of the applied magnetic field $B$ and the value of electric current I in the circuit

$$\Theta = M\,B + L\,I, \tag{1}$$

where self-induction $L$ and mutual inductance $M$ are functions of the shape, material of the core and functions of distribution of turns on the core.

Periodical change of flux density with angular speed $\omega$ and amplitude $B_o$ causes current with the amplitude

$$Io = \frac{\omega M}{\sqrt{R^2 + \omega^2 L^2}} B_o, \tag{2}$$

where $R$ is the circuit resistance.

The normally used windings have so many turns $n$ that it is possible to use integration through the density of turns

$$\rho(x) = \frac{1}{n}\frac{dn}{dx}. \tag{3}$$

The density of turns $\rho$ is normalized function $\int_0^l \rho(x)dx = 1$. Various windings differ by the number of turns and by the distribution of turns on the core.

The coefficients $L$, $M$ in (1) can be determined only by a computer program calculating the distribution of magnetic field

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in the space. Using (3), it is possible to separate the whole numbers of turns $n$ from their distribution $\rho$. The result of modification is

$$M = \int_0^l n(x)\, M(x)\, dx = n \int_0^l \rho(x)\, M(x)\, dx = n\, M_1, \qquad (4a)$$

$$L = \int_0^l n(x)\, L(x, y)\, n(y) dxdy =$$

$$= n^2 \int_0^l \int_0^l \rho(x)\, L(x, y)\, \rho(y) dxdy = n^2\, L_1, \qquad (4b)$$

where the operator $M(x)$ (Fig 2a) and operator of self-induction $L(x, y)$ (Fig. 2b) are determined numerically for each core by the finite element method. The coefficients $L_1$, $M_1$ are self-induction and mutual induction of the winding standardized per one turn.

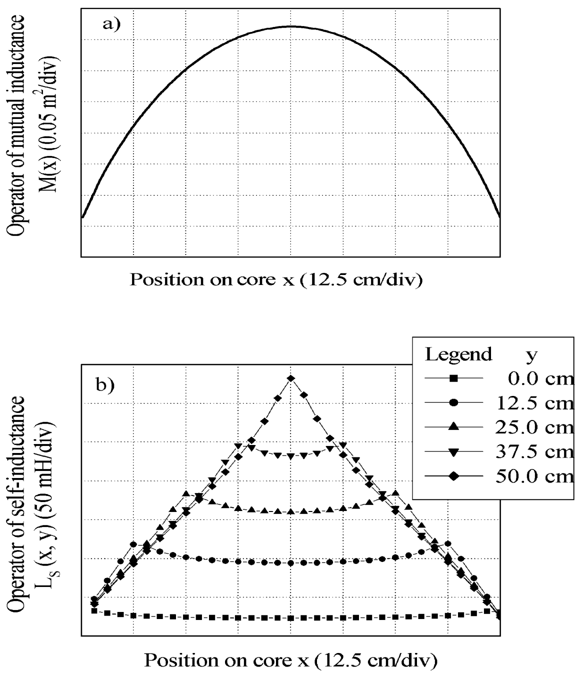


*Fig. 2. a) Development of the operator of the mutual inductance M(x) of the winding and applied homogeneous magnetic field $B_o$. Variable x goes through the length of core. b) Development of the symmetrised operator of self - inductance $L_S(x, y) = L(x, y) + L(x, l - y)$. Developments were calculated by Finite element method.*

The resulting circuit resistance consists of an internal resistance of the measuring apparatus $R_A$ and winding resistance $R_W$. The winding resistance is proportional to the number of turns $n$, so one can define $R_W = n\, R_1$. The total circuit resistance is

$$R = R_A + n\, R_1, \qquad (4c)$$

Adding (4a, b, c) into (2), one gets the signal sensitivity of equipment

$$k_S = \frac{I_o}{B_o} = \frac{n\, M_1\, \omega}{\sqrt{(R_A + n\, R_1)^2 + \omega^2\, n^4\, L_1^2}}\, . \qquad (5)$$

## 3. Thermal noise

Each electric circuit exhibits fluctuations in thermal equilibrium. These fluctuations are superposed to the measured signal as noise. The important parts of noise in studied equipment are noise of magnetic core (i.e. Barkhausen`s jumps) and Nyquist thermal noise. The effective value of thermal noise current is

$$I_{ef}^2 = \frac{k_B\, T}{2\pi} \frac{R}{|Z|^2} \Delta\omega = \frac{k_B\, T}{2\pi} \frac{R}{R^2 + \omega^2\, L^2} \Delta\omega, \qquad (6)$$

where $k_B$ is Boltzman constant, $Z$ is impedance of circuit, $R$ (4c) is total circuit resistance and $\Delta\omega$ is bandwidth of the signal. The noise sensitivity is

$$k_N = \frac{I_o}{I_{ef}} = \sqrt{\frac{2\pi}{k_B\, T}} \frac{\omega B_o}{\sqrt{\Delta\omega}} \frac{M}{\sqrt{R}}\, . \qquad (7)$$

Temperature $T$, amplitude of measured flux density $B_o$, frequency of measured signal $\omega$ and the bandwidth $\Delta\omega$ of amplifier have nothing in common with the winding, so they are suspect to be external quantities to the winding. Noise purity of the signal is the best if the winding with the maximal portion $M/\sqrt{R}$ is used. Adding (4a, c) one gets:

$$k_N \equiv \frac{M}{\sqrt{R}} = \frac{n\, M_1}{\sqrt{R_A + n\, R_1}}\, . \qquad (8)$$

Noise sensitivity (8) is a monotonic increasing function of number of turns $n$. In terms of the signal purity, the optimal number of turns does not exist.

So, one needs another approach. The winding resistance can be expressed as a function of the electrical conductivity of used material $\sigma$, the cross area of used conductor $S$ and its length $d$. The optimal winding can be found under the condition that the volume of material used for wire $V = d\, S$, is a given constant parameter. Then, the winding resistance is

$$R_W = \frac{1}{\sigma} \frac{d}{S} = \frac{1}{\sigma} \frac{d^2}{V} = \frac{d_1^2}{\sigma V} n^2 = \alpha\, n^2, \qquad (9)$$

where $d_1$ is a length of one turn and $\alpha$ is appropriate substitution. The total circuit resistance is $R = R_A + \alpha\, n^2$ and

$$k_N \equiv \frac{n\, M_1}{\sqrt{R_A + \alpha\, n^2}}\, . \qquad (10).$$

The noise sensitivity (10) is a monotonic increasing function of $n$ and converges the value $M_1/\sqrt{\alpha}$ to its limit. It is suitable to express the number of turns $n$ as a function of new parameter $\epsilon$, $\epsilon \in \langle 0, 1 \rangle$. The implicit form of this function is $k_N(n_\epsilon) \equiv \epsilon\, M_1/\sqrt{\alpha}$. Using (10) one gets the explicit form

$$n_\epsilon = \sqrt{\frac{\epsilon^2}{1 - \epsilon^2} \frac{R_A}{\alpha}}\, . \qquad (11)$$

Then, the winding resistance and circuit resistance are

$$R_W(n_\epsilon) = \frac{\epsilon^2}{1 - \epsilon^2} \text{ and } R(n_\epsilon) = \frac{1}{1 - \epsilon^2} R_A\, , \qquad (12)$$

respectively.

Substituting $p = \alpha(\omega L_1)^{-1}$ one gets a relation for noise sensitivity:

$$k_N = \sqrt{\frac{2\,\pi\,\omega}{k_B\,T\,\Delta\omega}\,\frac{M_1\,B_o}{\sqrt{L_1}}\,\underbrace{\frac{\epsilon}{\sqrt{p}}}_{x}}\,. \qquad (13)$$

and adding (14), (15) into (5) one gets :

$$k_S = \frac{1}{n_\epsilon}\,\frac{M_1}{\sqrt{L_1^2 + \left(\frac{\alpha}{\epsilon^2\,\omega}\right)^2}} = \frac{1}{R_A}\,\frac{M_1\sqrt{\omega}}{\sqrt{L_1}}\,\underbrace{\sqrt{\frac{p\,\epsilon^2\,(1-\epsilon^2)}{\epsilon^4 + p^2}}}_{y}\,. \qquad (14)$$

for final sensitivity.

The most important result of this article is the relation between the noise sensitivity and the signal sensitivity which can be expressed in the form

$$y = \sqrt{\frac{x^2 - p\,x^4}{1 + x^4}}\,. \qquad (15)$$

This relation (15) is demonstrated for different values of the parameter $p$ in Fig. 3. It can be seen that the decrease of the parameter $p$ (i.e. using more material with better conductance) leads to the increase of both the signal and the noise sensitivity. This effect is considerable until p > 0.01.

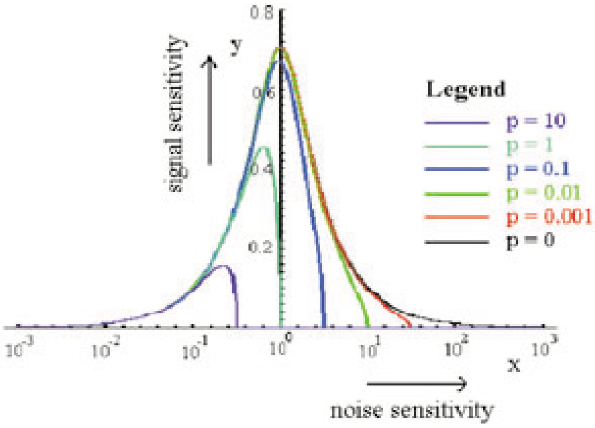


*Fig. 3. Relation between noise sensitivity and signal sensitivity as a function of parameter p.*

## 4. Optimal distribution of turns in winding

Another result comes from the comparison of the relations (13) and (14). The appropriateness of windings have to be measured by the ratio $M_1/\sqrt{L_1}$.

On the core there can be an infinite number of different windings. In real time one can search for only a finite amount of possibilities. So, the development of the density of turns $\rho(x)$ was tabulated in 17 incremental points on the whole length of core. Because of the mirror symmetry of the system the symmetrical windings $\rho(x) = \rho(l - x)$ were investigated only. In the process of searching, 23 million windings were compared.

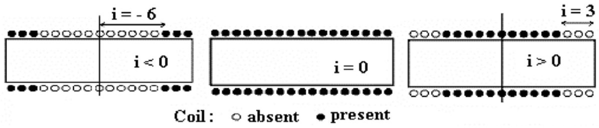


*Fig. 4. Distribution of the turns ρ(x) on the core, for different values of index i.*

Processes of optimization were done for many different situations. In spite of a huge set of possibilities, the results belong to a small set of windings. Therefore, it may be useful to know their physical properties, see Fig. 5. One can index these windings by one parameter, for example $i$, which describes the distribution of turns on the core, see Fig. 4. If $i = 0$ the turns are homogeneously distributed along the core. For $i < 0$ the windings are concentrated on the edges of core and for $i > 0$ the turns are close to the centre.

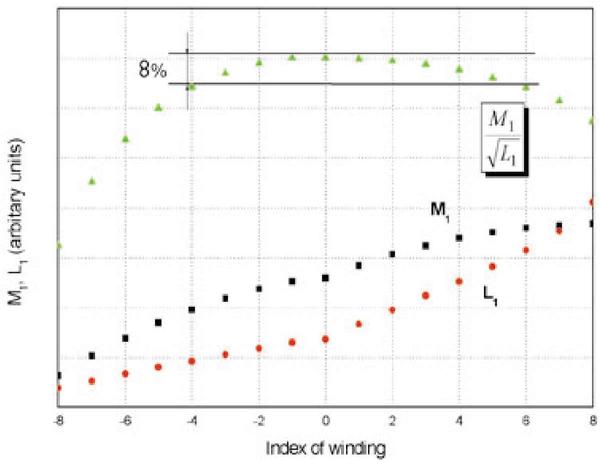


*Fig. 5. Physical properties for special windings. Self-inductance $L_1$ and mutual $M_1$ inductance are standardized per 1 turn. Signal sensitivity $k_S$ is proportional to $M_1/\sqrt{L_1}$.*

In the case of cylindrical core there are many windings with almost the same appropriateness (Fig. 5). This fact enables a designer to satisfy other criteria in a process of construction. In any case the optimal number of turns should be.

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