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METHODS FOR IDENTIFICATION OF COMPLEX INDUSTRIAL CONTROL OBJECTS ON THEIR ACCELERATING CHARACTERISTICS

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Resume

Theoretical identification methods for complex industrial control objects give very cumbersome and complex mathematical relations, the use of which for practical purposes is not constructive. In this regard, methods for obtaining mathematical models based on experimental data have now become the main focus of identification theory. In this paper is described the method of identification of industrial control objects developed according to their acceleration characteristics. The structure of the object under study is determined by the type of amplitude-phase frequency response and dynamic parameters are determined by experimental data. The high adequacy of the method is confirmed by similar studies on known (reference) models. The scientific novelty of the work consists in development of a new method for identifying complex industrial control objects by their acceleration characteristics.

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1 Introduction

Obtaining models based on observations and studying their properties is essentially the main content of science. These models may be more or less formalized, but they all have the main feature that they link observations into a general picture. The problem of obtaining adequate mathematical models of dynamical systems, based on observations of their behaviour is the subject of identification theory. The world around us consists entirely of dynamic systems, so knowledge of identification methods is crucial.

At present, with increasingly high requirements for management processes in various fields of engineering and technology, identification issues are becoming extremely important, since it is impossible to ensure the high-quality management of a system if its mathematical model was not known with sufficient accuracy.

Defining the system characteristics is dual to system management tasks since one cannot manage a system if its characteristics were unknown. Knowledge of the mathematical model before starting the management process significantly affects the effectiveness of its implementation [1-2].

This paper describes the developed methods for adequate identification of complex industrial controls s based on the results of an active experiment (acceleration characteristics).

2 The definition of objects equations by curves of acceleration

To obtain the equations of objects, an experiment is used, which consists in measuring and registering one or more transients. These processes correspond to particular solutions of the desired differential equation. Two types of experiments are most widely used in automation: removal of acceleration curves and removal of frequency characteristics. In the first case, the registered partial solution of the desired equation is the object's response to a standard step change in the input value, which is used to determine the object's equation. In the second case, not one particular solution is registered, but several. These solutions are steady fluctuations in the output value of the object, forced by artificial periodic fluctuations of the input value at various fixed frequencies. These particular solutions -

the frequency characteristics of the object - represent the initial material for the subsequent finding of the equation [3].

If an active experiment cannot be applied on an object, then the statistical dynamics methods are used without using artificial influences on the object [4].

For a linear operator, equation of the form

$$a_0 \cdot x^n + a_1 \cdot x^{n-1} + \ldots + x = b_0 \cdot x_0^m + b_1 \cdot x_0^{m-1} + \ldots + b_m \cdot x_0, \quad (1)$$

where $n > m$; x, x_0 - output and input of the object solution with zero initial conditions and abrupt change in input X_0, can be written analytically, as a function of time and coefficients:

$$x = x_{cm}(t, a_0, a_1, \ldots, a_{n-1}, b_0, b_1, \ldots, b_m); \quad (2)$$

and vice versa, if the order of the object Equation (1) is known and the reaction is obtained experimentally $x_{cm}(t)$ to a step change in input, then you can calculate $n + m + 1$ coefficients a_i, b_i (1). For this, the analytical solution is equated $x(t)$ in (2), in which the coefficients appear a_i and b_i and reaction $x^1_{cm}(t)$ at different times $t = t_i(i = 1,2,\ldots,m+n+1)$. The result is a system of $m + n + 1$ equation with unknown coefficients a_i, b_i:

$$x_{cm}(t_i, a_0, a_1, \ldots, a_{n-1}, b_0, b_1, \ldots, b_m) = x'_{cm}(t_i), \quad i = 1,2,\ldots,n+m+1, \quad (3)$$

from which the desired coefficients are calculated. In this way $n + m + 1$ discrete ordinates $x^1_{cm}(t)$ acceleration curves allow $m + n + 1$ unknown coefficients. Different variations of this idea are possible, up to the use of other types of reaction. One can take an excess number of ordinates and apply the least squares method.

Knowledge of the order of the object Equation (1) is essential here. The analytical form of solutions of Equation (2) is different for equations of different orders and if the a priori order n is less than the actual one, then this method of equating the ordinates of the analytical solution and the experimental solution; gives incorrect a_i and b_i. If n taken large, then this should not represent a problem, since the extra coefficients will turn to zero. A priori, the order of the object's equation is determined by the number of concentrated containers in the object [5-6].

2.1. Method for identifying the first-order object using an exponential acceleration curve

Figure 1 shows the acceleration curve $y(t)$ of a single-capacitive linear object. At a moment in time $t_0 = 0$ input quantity x changed jumped to a units.

It is necessary to determine numerically the mathematical model of the object.

The required equation has the form

$$T\frac{dy}{dt} + y = Kx \text{ or } \frac{Y(p)}{X(p)} = \frac{K}{Tp+1} \quad (4)$$

and you need to define constants T and K.

First, an analytical expression is found for the solution of the equation under the given conditions. This solution will contain constants T and K. The resulting acceleration curve is a graphical solution, then comparing the graph with its analytical expression, the constants of this analytical expression are determined. The general form of the solution for conditions $y = 0$ at $t = 0$ and $x = a$ at $t > 0$ is [7]:

$$y(t) = K \cdot a \cdot \left(1 + e^{-\frac{t}{T}}\right). \quad (5)$$

In principle, it is enough to take a couple of any points from the graph, substitute their coordinates into the solution and then from the two obtained equations calculate T and K. However, these equations are transcendental:

$$\left.\begin{aligned} y_1(t) K \cdot a \cdot \left(1 - e^{-\frac{t_1}{T}}\right) \\ y_2(t) K \cdot a \cdot \left(1 - e^{-\frac{t_2}{T}}\right) \end{aligned}\right\} \quad (6)$$

and to calculate their roots K and T is difficult.

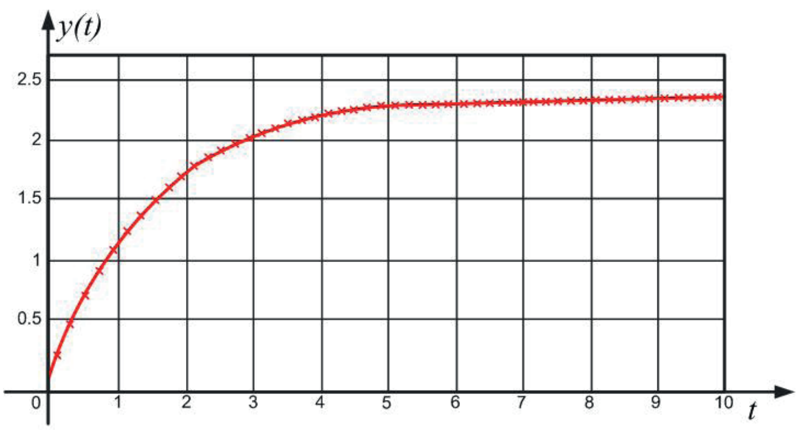

***Figure 1** For definition of the first-order object model along the acceleration curve*

Therefore, one can apply the following technique. In the steady state, is $y\ (t) = k \cdot a$, therefore, the ordinate of the asymptote tends to y, what makes it possible to determine K by simple division by a, i.e.

$$K = \frac{b}{a}. \tag{7}$$

To calculate T, the solution is differentiated with respect to time:

$$\frac{dy(t)}{dt} = K \cdot a \cdot \frac{1}{T} \cdot e^{-\frac{t}{T}} \tag{8}$$

and set t to zero

$$\lim_{t \to 0} \frac{dy}{dt} = K \cdot a \cdot \frac{1}{T} = \frac{b}{T} = tg\alpha, \tag{9}$$

where α is the angle of inclination of the tangent drawn to the graph $y(t)$ at $t = 0$. Therefore,

$$T = \frac{b}{tg\alpha}. \tag{10}$$

Thus, T is numerically equal to the length of the tangent within the range of the origin of coordinates to the point of its intersection with the mentioned asymptote.

This solution is the simplest, but not accurate, since it is difficult to indicate the ordinate of asymptote b. This solution uses only the beginning and end of the graph, while all the intermediate points are dropped from consideration, [8-9].

Now is considered a more accurate technique. The graph is broken equidistant to the interval Δt for ordinates y_0, y_1, y_2 etc. For these points, according to the solution of Equation (5) can be written

$$\left.\begin{aligned} y_0(t) &= K \cdot a \cdot \left(1 - e^{-\frac{0}{T}}\right); \\ y_1(t) &= K \cdot a \cdot \left(1 - e^{-\frac{\Delta t}{T}}\right); \\ y_2(t) &= K \cdot a \cdot \left(1 - e^{-\frac{2\Delta t}{T}}\right); \\ y_3(t) &= K \cdot a \cdot \left(1 - e^{-\frac{3\Delta t}{T}}\right); \end{aligned}\right\} \tag{11}$$

etc.

The previous equations are subtracted from the following ones in pairs:

$$\left.\begin{aligned} y_1 - y_0 &= K \cdot a - K \cdot a \cdot e^{-\frac{\Delta t}{T}}; \\ y_2 - y_1 &= K \cdot a \cdot e^{-\frac{\Delta t}{T}} - K \cdot a \cdot e^{-\frac{2\Delta t}{T}} \\ y_3 - y_2 &= K \cdot a \cdot e^{-\frac{2\Delta t}{T}} - K \cdot a \cdot e^{-\frac{3\Delta t}{T}}; \end{aligned}\right\} \tag{12}$$

etc.

For brevity, $e^{-\frac{\Delta t}{T}}$ is denoted as q, then, one can write:

$$\left.\begin{aligned} y_1 - y_0 &= K \cdot a \cdot (1 - q); \\ y_2 - y_1 &= K \cdot a \cdot q \cdot (1 - q); \\ y_3 - y_2 &= K \cdot a \cdot q^2 q \cdot (1 - q); \end{aligned}\right\} \tag{13}$$

etc.

Dividing each subsequent of these equalities by the previous one, one obtains a series of values for q:

$$\left.\begin{aligned} q_1 &= \frac{y_2 - y_1}{y_1 - y_0}; \\ q_2 &= \frac{y_3 - y_2}{y_2 - y_1}; \\ q_3 &= \frac{y_4 - y_3}{y_3 - y_2}; \end{aligned}\right\} \tag{14}$$

etc.

These numbers differ from one another due to experimental measurement and registration errors $y(t)$.

More accurate is the average value, which gives the arithmetic mean $\bar{q}$ from the calculated individual values q_i. Then the updated time constant $\bar{T}$ is determined from the expression

$$\bar{T} = -\frac{\Delta t}{\ln \bar{q}}. \tag{15}$$

Similarly, according to the known q_i, individual K_i are determined as:

$$\left.\begin{aligned} K_1 &= \frac{y_1 - y_0}{a \cdot (1 - q_1)}; \\ K_2 &= \frac{y_2 - y_1}{a \cdot q_2 \cdot (1 - q_2)}; \\ K_3 &= \frac{y_3 - y_2}{a \cdot q_3^2 \cdot (1 - q_3)}; \end{aligned}\right\} \tag{16}$$

Then the arithmetic means $\bar{K}$ of K_1, K_2, K_3 is K.

2.2 Method for identifying a second - order object with a monotone s-shaped acceleration curve

Figure 2 shows the acceleration curve of the object described by the second-order equation:

$$T_1 \cdot T_2 \cdot \frac{d^2 y}{dt} + (T_1 + T_2) \cdot \frac{dy}{dt} + y = K \cdot x. \tag{17}$$

Constants T_1, T_2 and K, can be calculated if it is known that the perturbation at the input was single $x = a = 1$ at $t > 0$.

As in the previous case, at the beginning, you should write the solution to the equation in general form [10].

Let the general form be, [4]

$$y_{\text{total}}(t) = C_1 \cdot e^{-\frac{t}{T_1}} + C_2 \cdot e^{-\frac{t}{T_2}} + K \cdot a. \tag{18}$$

In this case, it is necessary to determine five unknowns (K, T_1, T_2, C_1, C_2). One can reduce the number of unknowns to three (K, T_1, T_2,).

For this, from the initial conditions $y = 0$; $\frac{dy}{dt} = 0$ for $t = 0$ the arbitrary constants are defined:

$$\left.\begin{aligned} y_{total}(0) &= C_1 \cdot e^{-\frac{0}{T_1}} + C_2 \cdot e^{-\frac{0}{T_2}} + K \cdot a = 0; \\ y'_{total}(0) &= -C_1 \cdot \frac{1}{T_1} \cdot e^{-\frac{0}{T_1}} - C_2 \cdot \frac{1}{T_2} e^{-\frac{0}{T_2}} = 0; \end{aligned}\right\} \tag{19}$$

from

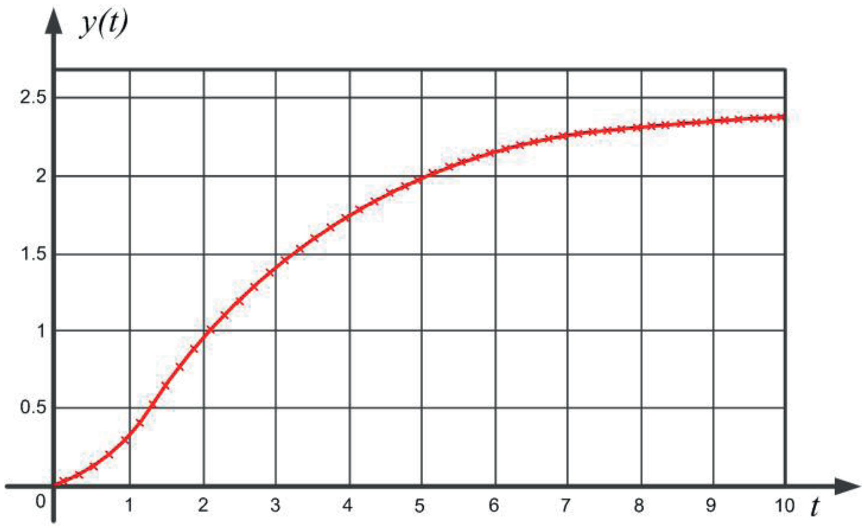

Figure 2 *For definition of the second-order object model with a monotonic acceleration curve S - figurative*

$$C_1 = \frac{k \cdot a \cdot T_1}{T_2 - T_1}; \quad C_2 = \frac{k \cdot a \cdot T_2}{T_1 - T_2}. \tag{20}$$

The desired particular solution is obtained in the form

$$y(t) = k \cdot a \cdot \left(1 + \frac{T_1}{T_2 - T_1} \cdot e^{-\frac{t}{T_1}} + \frac{T_2}{T_1 - T_2} \cdot e^{-\frac{t}{T_2}} \right). \tag{21}$$

Now, in principle, it is sufficient to take the coordinates y_i, t_i, three arbitrary points from a given graph, substitute them three times into the solution and from the three equations obtained in this way find the roots k, T_1, T_2. However, these equations are transcendental and the roots are difficult to calculate, so it is more convenient to use the following mathematical technique [11].

To do this, the graph *y(t)* is split equidistant to interval Δt for ordinates y_0, y_1, y_2 etc. and one then writes:

$$\left.\begin{aligned}
y_0 &= k \cdot a + \frac{k \cdot a \cdot T_1}{T_2 - T_1} + \frac{k \cdot a \cdot T_2}{T_1 - T_2};\\
y_1 &= k \cdot a \cdot \left(1 + \frac{T_1}{T_2 - T_1} \cdot e^{-\frac{\Delta t}{T_1}} + \frac{T_2}{T_1 - T_2} \cdot e^{-\frac{\Delta t}{T_2}} \right);\\
y_2 &= k \cdot a \cdot \left(1 + \frac{T_1}{T_2 - T_1} \cdot e^{-\frac{2\Delta t}{T_1}} + \frac{T_2}{T_1 - T_2} \cdot e^{-\frac{2\Delta t}{T_2}} \right);\\
y_3 &= k \cdot a \cdot \left(1 + \frac{T_1}{T_2 - T_1} \cdot e^{-\frac{3\Delta t}{T_1}} + \frac{T_2}{T_1 - T_2} \cdot e^{-\frac{3\Delta t}{T_2}} \right);
\end{aligned}\right\} \tag{22}$$

etc.

By designating $A_1 = K; A_2 = \frac{K \cdot T_1}{T_2 - T_1};$ $A_3 = \frac{K \cdot T_2}{T_1 - T_2}; p = e^{-\frac{\Delta t}{T_1}}; p = e^{-\frac{\Delta t}{T_2}}$ these equations are rewritten as

$$\begin{aligned}
y_0 &= A_1 + A_2 + A_3;\\
y_1 &= A_1 + A_2 \cdot p + A_3 \cdot q;\\
y_2 &= A_1 + A_2 \cdot p^2 + A_3 \cdot q^2;\\
y_3 &= A_1 + A_2 \cdot p^3 + A_3 \cdot q^3;\\
y_4 &= A_1 + A_2 \cdot p^4 + A_3 \cdot q^4;\\
y_5 &= A_1 + A_2 \cdot p^5 + A_3 \cdot q^5.
\end{aligned} \tag{23}$$

We will count the numbers 1, p and q roots of the cubic equation. Let the first line be multiplied by B_3, the second by - B_2, the third by - B_1, the fourth by - 1 and add them, then the right-hand sides add up to zero, so one gets, [6]:

$$\lambda^3 + B_1 \cdot \lambda^2 + B_2 \cdot \lambda + B_3 = 0\,. \tag{24}$$

$$y_0 \cdot B_3 + y_1 \cdot B_2 + y_2 \cdot B_1 = 0\,. \tag{25}$$

Then the same is done with the following four lines:

$$y_1 \cdot B_3 + y_2 \cdot B_2 + y_3 \cdot B_1 + y_4 = 0\,. \tag{26}$$

The next four lines will give

$$y_2 \cdot B_3 + y_3 \cdot B_2 + y_4 \cdot B_1 + y_5 = 0\,. \tag{27}$$

In these three equalities, the ordinates y_i, are known from the acceleration curve and the constants B_1, B_2, B_3 are the sought for ones. Having found them, one should then calculate the roots of the cubic equation:

$$\lambda_1 = 1; \lambda_2 = p = e^{-\frac{\Delta t}{T_1}}; \lambda_3 = q = e^{-\frac{\Delta t}{T_2}}, \tag{28}$$

i.e.

$$T_1 = -\frac{\Delta t}{\ln p} \text{ and } T_2 = -\frac{\Delta t}{\ln p}\,. \tag{29}$$

From any equation of system of Equations (23),

***Table 1** Experimental values*

t MIN	0	1	2	3	4	5
y(*t*)	0	0.31	0.80	1.21	1.5	1.7

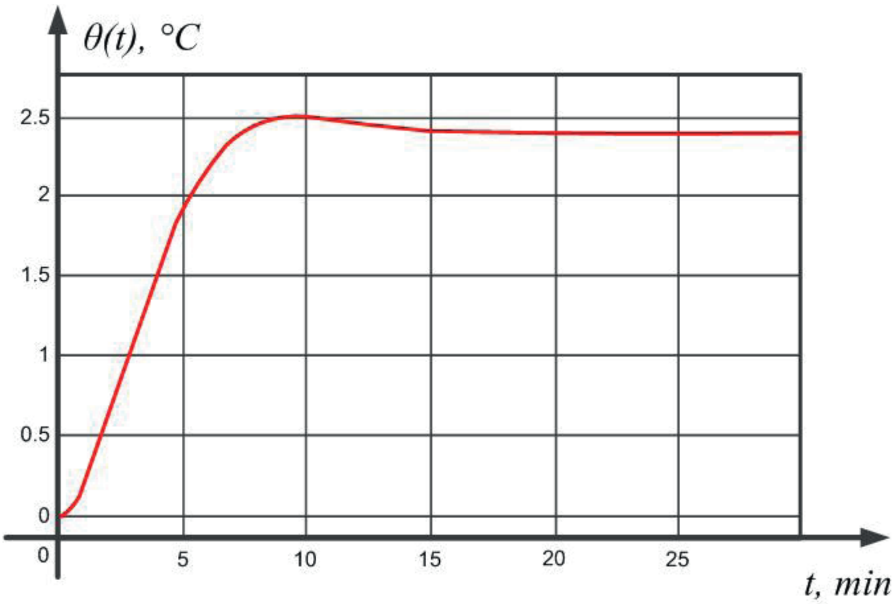

***Figure 3** For definition of the second-order object model with an oscillatory acceleration curve*

except for the first, it is necessary to calculate *K*.

To increase the accuracy, one cannot take only six initial ordinates, but more than average of the results.

In essence, this method is an approximation of a given graph by the sum of exponential terms [12].

Next is presented a numerical example. Taking = 1 min, six ordinates on the acceleration curve are measured (Figure 2) (Table 1).

From equations

$$0 \cdot B_3 + 0.31 \cdot B_2 + 0.8 \cdot B_1 + 1.21 = 0;$$

$$0.31 \cdot B_3 + 0.8 \cdot B_2 + 1.21 \cdot B_1 + 1.21 = 0;$$

$$0.8 \cdot B_3 + 1.21 \cdot B_2 + 1.5 \cdot B_1 + 1.7 = 0;$$

one calculates $B_1 = -1.97$; $B_2 = -1.19$; $B_3 = -0.222$.

Then, one can find the roots of the cubic equation: $\lambda_1 = 1; \lambda_2 = p = 0.37; \lambda_3 = q = 0.61$. This is relatively easy to do, since one root is known in advance. $(\lambda = 1)$. Then one finds:

$$T_1 = -\frac{\Delta t}{\ln p} = -\frac{1}{\ln 0.37} \approx 1\,\text{min},$$
$$T_2 = -\frac{\Delta t}{\ln p} = -\frac{1}{\ln 0.61} \approx 2\,\text{min}.$$

To calculate *K*, take one of the lines, except the first, of system in Equation (19), for example, the second:

$$y_1 = A_1 + A_2 \cdot p + A_3 \cdot q = K \cdot a \cdot \left(1 + \frac{K \cdot a \cdot T_1}{T_2 - T_1} \cdot p + \frac{K \cdot a \cdot T_2}{T_1 - T_2} \cdot q\right), \quad (30)$$

and substitute the numeric values

$$0.31 = k \cdot \left(1 + \frac{1}{2-1} \cdot 0.37 + \frac{2}{1-2} \cdot 0.61\right),$$

to get

$$k = 2.07.$$

2.3 Method for identifying the second-order object with an oscillatory acceleration curve

Equation of the object is defined according to the reaction of the output Δθ (t) (Figure 3) for the abrupt change in the input value *υ* to the value *υ* = *a*, m/min; Δθ (t) - temperature difference in the last suction chambers, *υ* - belt speed [7].

The acceleration curve has an oscillatory shape and it can be assumed that the sought equation is of the second order with complex roots [9]:

$$\frac{d^2\Delta\theta}{dt^2} - (\gamma_1 + \gamma_2) \cdot \frac{d\Delta\theta}{dt} + \gamma_1 \cdot \gamma_2 \cdot \Delta\theta = \gamma_1 \cdot \gamma_2 \cdot K \cdot \upsilon. \quad (31)$$

It is necessary to calculate constants $\gamma_1 \cdot \gamma_2 \cdot K$.

This equation is the same as in the previous case, only the designation of the constants is changed: $\gamma_1 = -\frac{1}{T_1}, \gamma_2 = -\frac{1}{T_2}$, since the time constant here, in the case of an oscillatory system, has no physical meaning. The equation of the oscillatory system is usually written in the form

$$\frac{d^2\Delta\theta}{dt^2} + 2\cdot\varepsilon\cdot\omega_0\cdot\frac{d\Delta\theta}{dt} + \omega_0^2\cdot\Delta\theta = K\cdot w\omega_0^2\cdot\upsilon\,, \quad (32)$$

where ε is the damping coefficient; ω_0 is the natural frequency of the system.

This shows the connection between the physical constants ε,ω_0 and numbers γ_1,γ_2. The latter were introduced for the convenience of calculations.

Using the results of the previous method, one can immediately write the solution to the equation:

$$\Delta\theta(t) = K\cdot a + \frac{K\cdot a\cdot\gamma_2}{\gamma_1-\gamma_2}\cdot e^{\gamma_1 t} + \frac{K\cdot a\cdot\gamma_2}{\gamma_1-\gamma_2}\cdot e^{\gamma_1 t}. \quad (33)$$

Since the control object has vibrational properties, the method of calculating constants used in the previous task will result in non-real numbers γ_1 and γ_2, i.e. complex

$$\gamma_1 = \alpha + j\beta;\ \gamma_2 = \alpha - j\beta. \quad (34)$$

Turning to the real quantities, the form of writing the solution must be accordingly transformed according to the Euler formula; it corresponds to the addition of two harmonics.

$$\begin{aligned}\Delta\theta(t) &= K\cdot a\cdot\left[\begin{matrix}1+\dfrac{\alpha-j\beta}{2j\beta}\cdot e^{(\alpha+j\beta)\cdot t}+\\ \dfrac{\alpha+j\beta}{-2j\beta}\cdot e^{(\alpha+j\beta)\cdot t}\end{matrix}\right]=\\ &K\cdot a\cdot\left\{\begin{matrix}1+e^{\alpha t}\cdot\\ \left[\begin{matrix}1+\dfrac{j\alpha+\beta}{-2\beta}\cdot(\cos\beta t + j\sin\beta t)+\\ \dfrac{j\alpha-\beta}{2\beta}\cdot(\cos\beta t - j\sin\beta t)\end{matrix}\right]\end{matrix}\right\}= \quad . \quad (35)\\ &K\cdot a\cdot\left[1+e^{\alpha t}\cdot\left(-\cos\beta t + \frac{\alpha}{\beta}\cdot\sin\beta t\right)\right]=\\ &K\cdot a\cdot\left[1+e^{\alpha t}\cdot\sqrt{1+\left(\frac{\alpha}{\beta}\right)^2}\cdot\sin\left(\beta t - arctg\frac{\beta}{\alpha}\right)\right].\end{aligned}$$

In principle, one could take the coordinates of the three arbitrary points of their graph $\Delta\theta(t)$, putting them in the solution, calculate the real roots from the three equations for *α, β* and *K*. However, since the equations turn out to be transcendental, it is very difficult to solve them and therefore it is more expedient to apply the technique considered in the previous problem [13].

For this, from the acceleration curve one takes six equidistant to the interval $\Delta t = 5\ min$, ordinate:

$$\Delta\theta_0 = 0; \Delta\theta_1 = 29.3; \Delta\theta_2 = 69; \Delta\theta_3 = 84.3;$$
$$\Delta\theta_4 = 79.8; \Delta\theta_5 = 71.1.$$

The system of equations is composed for determining the coefficients of the intermediate cubic Equation (21)

$$0\cdot B_3 + 29.3\cdot B_2 + 69\cdot B_1 + 84.3 = 0;$$
$$29.3\cdot B_3 + 69\cdot B_2 + 84.3\cdot B_1 + 79.8 = 0;$$
$$69\cdot B_3 + 84.3\cdot B_2 + 79.8\cdot B_1 + 71.1 = 0.$$

From that one calculates B_1 = -1.1654; B_2 = 1.12; B_3 = - 0.366 and gets the following cubic equation: $\lambda^3 - 1.654\,\lambda^2 + 1.12\,\lambda - 0.366 = 0$, the roots of which are then calculated (one root $\lambda_3 = 1$ is known in advance):

$$\lambda_1 = 0.327 + j\cdot 0.509; \lambda_2 = 0.327 - j\cdot 0.509; \lambda_3 = 1.$$

The complex roots λ_1 and λ_2 are represent ed in an exemplary form:

$$\lambda_1 = e^{-0.5+j\cdot 1.0}; \lambda_2 = e^{-0.5-j\cdot 1.0}.$$

Next, the constants γ_1 and γ_2 are determined as:

$$\left.\begin{aligned}&\gamma_1 = -\frac{1}{T_1} = \frac{\ln\lambda_1}{\Delta t} = \frac{-0.5+j\cdot 1.0}{5} =\\ &-0.1+j\cdot 0.2;\\ &\gamma_2 = -\frac{1}{T_2} = \frac{\ln\lambda_2}{\Delta t} = \frac{-0.5-j\cdot 1.0}{5} =\\ &-0.1+j\cdot 0.2;\\ &(\alpha = -0.1; \beta = 0.2).\end{aligned}\right\} \quad (36)$$

It remains to calculate the static transfer ratio *K*. This can be easily done by substituting the coordinates of an arbitrary point of the acceleration curve and the calculated constants α and β. Take for example the point $t = \Delta t = 5\ min,\ \Delta\theta(\Delta t) = \Delta\theta_1 = 29.3$:

$$\Delta\theta = K\cdot 1\cdot\left[1+e^{\alpha t}\cdot\left(-\cos\beta t + \frac{\alpha}{\beta}\cdot\sin\beta t\right)\right] \text{ or}$$

$$29.3 = K\cdot 1\cdot\left[1+e^{-0.1\cdot 5}\cdot\left(-\cos 0.2\cdot 5 + \frac{-0.1}{0.2}\cdot\sin 0.2\cdot 5\right)\right]$$

where one finds *K* = 70 *deg*/(*m*/*hour*).

The required numerical equation of the vibrational object will have the form:

$$\frac{d^2\Delta\theta}{dt^2} + 0.2\cdot\frac{d\Delta\theta}{dt} + 0.05\cdot\Delta\theta = 0.05\cdot 70\cdot\upsilon\,, \quad (37)$$

where *υ* - *m* / *hour*; $\Delta\theta$ — °C ; *[t] - min*.

The natural frequency of the object is

$$\omega_0 = \sqrt{\gamma_1\cdot\gamma_2} = 0.224\ rad/min;$$

And the damping factor is

$$\varepsilon = \frac{-\lambda_1-\lambda_2}{2\cdot\omega_0} = \frac{0.2}{2\cdot 0.234} = 0.45\,. \quad (38)$$

When this technique is extended to the higher-order systems, the general scheme of the method remains similar.

3 Conclusion

The article analyses the methods of determination for industrial control devices by the type of amplitude-phase frequency response and substantiates their complex mathematical relations ccording to the data of an active experiment with the means of obtaining acceleration characteristics.

In this paper are developed the methods for identification of industrial control objects. Using the method of statistical dynamics, the frequency characteristics of an industrial facility are determined.

Acceleration indices of the analyzed control object are converted to inertial link of the first order. The result is a mathematical model of an industrial control object in the form of a transfer function.

According to the data of an active experiment, acceleration characteristics are obtained (the object's response to a step input action), which, for the stable linear objects have one of the following types: exponential, S-shaped, or oscillatory. The shape of the acceleration curve determines the structure of the object and its dynamic parameters (transmission coefficients, time constants, delay time) are found by processing the acceleration curve using the special mathematical techniques. For example, a sequential connection of inertial links (an inertial link of the 2nd order) always gives an S-shaped Transition process. Therefore, if an unknown object of research gives an S-shaped acceleration curve, then its mathematical model can be identified as an inertial link of the 2nd order.

Similar conclusions can be drawn for objects with other types of acceleration characteristics.

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