THE STABILITY INDICATORS OF THE SECTION ARTICULATED BUSES

Vladimir Sakhno [1], Juraj Gerlici [2], Viktor Polyakov [1], Anatolii Korpach [1], Oleksii Korpach [1], Kateryna Kravchenko [2,*]

[1]National Transport University M. Omelianovicha-Pavlenka, Kiev, Ukraine
[2]University of Zilina, Zilina, Slovak Republic

*E-mail of corresponding author: kateryna.kravchenko@fstroj.uniza.sk

Resume

The mathematical model of a multi-section articulated bus has been improved in the article, which allowed to determine the stability indicators of the two and three-section articulated buses for the Metrobus system. The critical speed for the three-section articulated bus was 28.06 m/s and for the two-section articulated bus - 30.89 m/s. Since the motion of multi-section articulated buses will be carried out on separate lanes with a speed of 25 - 28 m/s, one can assume that the stability of rectilinear traffic of both two and three-section buses is provided. In non-stationary modes of motion with two-section articulated buses, the lateral speed of the first bus decreases by almost 8.5% compared to the three-section and the second trailer-bus increases by almost 9.5% compared to the three-section. In this case, the motion stability of multi-section articulated buses can be considered satisfactory, as the lateral accelerations in the center of mass of all sections do not exceed 0.45 g.

Article info

Received 12 May 2022
Accepted 13 September 2022
Online 29 September 2022

Keywords:

two and three-section
articulated bus
stability
trailer
acceleration

Available online: https://doi.org/10.26552/com.C.2022.4.B301-B309

ISSN 1335-4205 (print version)
ISSN 2585-7878 (online version)

1 Formulation of the problem

One of the main problems of the modern big city is the global crisis of normal functioning of the urban environment due to the structural growth of the motorization level, oversaturation of the road network with traffic flows. This leads to a sharp deterioration in transport services, traffic jams, rising noise and air pollution, a practical drop in speed, rising energy costs, increasing the victim's number of road accidents [1-9].

Recently, many cities around the world have introduced so-called BRT (Bus Rapid Transit) systems [2-4]. These systems have become a cheaper alternative to the subway and other rail transport, including trams. The BRT transport now operates in more than 200 cities around the world. The advent of Metrobus system will help evolve "minibuses" from cities and move to a more progressive model of urban transport.

The main economic advantage of a high-speed bus over a regular one is much lower fuel consumption per passenger [1-9]. This is achieved through rational, specially designed driving modes. On the BRT line, as a rule, multi-section articulated bus run especially large capacity (18 or 22, 24, 25 m), but the main difference from the usual city routes is that they run on a separate (dedicated) lane at short intervals, for example, 1 minute. Along with the undeniable advantages of bi-articulated buses and trolleybuses, they also have disadvantages - worse manoeuvrability and stability. Therefore, solving issues related to improving the manoeuvrability and stability of multi-section articulated buses is an urgent task.

2 Analysis of scientific publications

Stability of motion belongs to the properties of motor vehicles that do not have a strict certainty in terminology, requirements, indicators and methods of assessment. In practice, they use experimental characteristics that determine the stability of vehicles during movement and in theory, direct and indirect indicators and their dependences, among which the main ones are the critical speed of movement and lateral accelerations acting in the center of mass of individual sections. At present, the problem of determining the stability conditions for the semi-trailer trucks has been sufficiently studied. Thus, in [10-11], a simplified analysis of the manoeuvrability and stability of vehicle combinations, such as a tractor in combination with one or two semitrailers or a truck and a full trailer, was carried out. Vehicle combinations are considered linear dynamic systems with two degrees of freedom for each unit. The motion equations are derived taking into account the effect of braking and acceleration and the characteristic equation for motion with constant speed is obtained. In [12], the three-dimensional dynamic models of a truck and a trailer were developed, based on which a dynamic model of the semi-trailer truck was constructed. Based on the first-order approximation theory of ordinary differential equations and the theory of Hopf bifurcation, the linear and nonlinear stability of each element and the semi-trailer truck as a whole under rectilinear motion is studied. Numerical results show that for the nonlinear and linear models the critical velocities differ little from each other. In [13], the equations of vertical and lateral dynamics of a semi-trailer truck with 6 degrees of freedom are reduced to a matrix form. The motion of such a vehicle in the vertical and lateral planes has been investigated. It is shown that the developed method can be applied to analyze the stability of the motion, in particular, of passenger trains. In [14], a multivariate extension of the D2-IBC (Data Driven - Inversion Based Control) method is considered and its application to control the stability of the motion of the semi-trailer trucks is discussed in detail. In [15], a model of the semi-trailer truck with 31 freedom degrees was constructed using the AutoSum package and the directions of improving the semi-trailer truck stability are shown. At the same time, it is shown that its stability can be significantly improved by using an inerter, which is considered effective for increasing the stability and performance of multi-section road trains. However, as practice shows, determining the system's behavior nature in the field of instability and identifying the causes of their occurrence has not lost its relevance.

The characteristics of the maneuverability and stability of the vehicle motion, as is known, are determined by a combination of the operational, mass-geometric and design parameters of its modules (vehicle for the Metrobus system) and their control systems. In general, the desired combinations of these parameters from the point of view of stability, even for the same vehicle in the range of operational loads and run speeds, are different. As a consequence, it is difficult to obtain accurate design parameters and quantitative indicators according to the stability criteria of its motion at the early stages of the design of vehicles. Success in solving such problems depends on how well the mathematical model is chosen and its essential parameters that describe the dynamic system behavior in different modes of motion. Therefore, the aim of the work is a comparative assessment of the two and three-section articulated buses in both stationary and non-stationary modes.

Figure 1 shows the general view (a) and the calculation scheme of the three-section articulated bus (b). Metrobus consists of a driving link, which has front steering wheels and can have rear driving wheels, both swivel and non-swivel. Both the first and the second trailing single-axle link can have both rotating and non-rotating wheels. At the point O_0', the first towing link rests on the leading one, at the second point O_1', the towing link rests on the first. Interaction forces between the links of the road train arise at the coupling points O_0' and O_1'.

3 Comparative assessment of the motion stability of the two and three-section articulated buses

In [16], a system of differential equations was obtained by the sections method, which describes the plane-parallel motion of the sections of a three-section articulated bus, Figure 1 (b). This system of equations is written in the form:

$$\begin{aligned}
&m(\dot{v} - u\omega) = -X_2 + XA - XB\cos\gamma_1 + \\
&+ YB\sin\gamma_1; \\
&m(\dot{u} + v\omega)Y_2 + YA - YB\cos\gamma_1 + XB\sin\gamma_1; \\
&J\dot{\omega} = aYA - bY_2 - c(YB\cos\gamma_1 + XB\sin\gamma_1) + \\
&+ M_1 + M_2; \\
&J_1\dot{\omega}_1 = -YA\lambda\cos\gamma_0 + XA\lambda\sin\theta - M_1 = 0; \\
&J_2\dot{\omega}_2 = d_1YB - b_1Y_3 + \\
&+ c_1(YC\cos\gamma_2 + XC\sin\gamma_2) - M_3; \\
&J_3\dot{\omega}_3 = d_2YC - b_2Y_4 + M_2 - M_3.
\end{aligned} \tag{1}$$

The reactions at the connection points of the three-section articulated bus sections are defined as follows:

$$\begin{aligned}
&XC = m_3\dot{v}_3 - m_3\omega_3u_3 + X_4; \\
&YC = m_3\dot{u}_3 + m_3\omega_3v_3 - Y_4; \\
&XB = m_2\dot{v}_2 - m_2\omega_2u_2 + m_3\dot{v}_3\cos\gamma_2 - \\
&- m_3\omega_3u_3\cos\gamma_2 - X_4\cos\gamma_2 + X_3 - m_3\dot{u}_3\sin\gamma_2 - \\
&- m_3\omega_3v_3\sin\gamma_2 + Y_4\sin\gamma_2; \\
&XA = m_1\dot{u}_1\sin\theta + m_1\omega_1v_1\sin\theta - Y_1\sin\theta - \\
&- m_1\dot{v}_1\cos\theta + m_1\omega_1u_1\cos\theta - X_1\cos\theta; \\
&YA = -m_1\dot{u}_1\cos\theta - m_1\omega_1v_1\cos\theta + Y_1\cos\theta - \\
&- m_1\dot{v}_1\sin\theta + m_1\omega_1u_1\sin\theta - X_1\sin\theta.
\end{aligned} \tag{2}$$

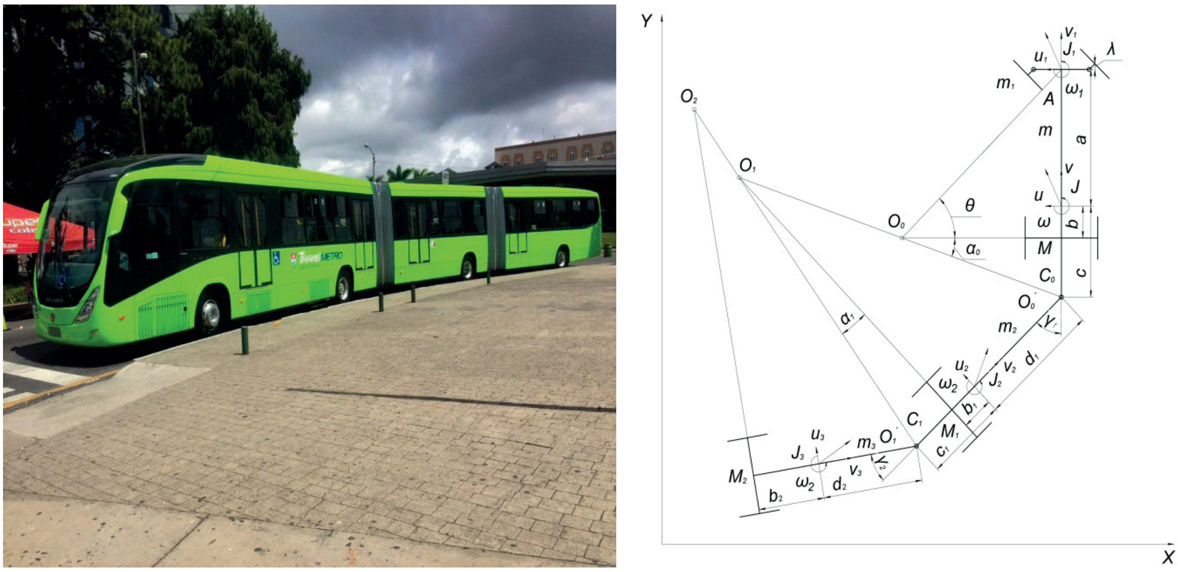
In the systems of Equations in (1) and (2), the following designations are accepted:

a – the distance from the front axle to the center of the bus mass;

b – the distance from the rear axle to the center of the bus mass;

c – the distance from the center of the bus mass to the coupling point with the first trailer;

λ – the side shift of the front controlled wheels of the bus, due to the longitudinal inclination of the pivot axis;



a) b)

Figure 1 *Van Hool ExquiCity Metrobus (a); the estimated scheme of the three-section articulated bus for Metrobus system*

θ – the rotation angle of the front axle wheels of the bus;
γ_1, γ_2 – the first and second joint angles;
b_1 – the distance from the mass center of the first trailer to its axis;
c_1 – the distance from the mass center of the first trailer to the coupling point with the second trailer;
d_1 – the distance from the mass center of the first trailer to the coupling point with the bus;
b_2 – the distance from the mass center of the second trailer to its axis;
d_2 – the distance from the mass center of the second trailer to the coupling point with the first trailer;
m, m_1, m_2, m_3 – the mass of the controlled module of the bus, first and second trailer, respectively;
J – the central moment of the bus;
v, u – the longitudinal and cross projections of the velocity vector of the mass center on the axes associated with the bus, respectively;
ω – the bus angular velocity along the vertical axis;
J_1 – the central moment of inertia of the wheel control module of the bus;
v_1, u_1 – the longitudinal and cross projections of the velocity vector of the mass center of the control wheel module of the bus, respectively;
ω_1 – the angular velocity of the control wheel module of the bus;
J_2 – the central moment of first trailer inertia;
v_2, u_2 – the longitudinal and cross projections of the velocity vector of the mass center of the first trailer, respectively;
ω_3 – the angular velocity of the first trailer;
J_3 – the central moment of second trailer inertia;
v_3, u_3 – the longitudinal and cross projections of the velocity vector of the mass center of the second trailer, respectively;
ω_3 – the angular velocity of the second trailer.

The system of equations in (1) includes lateral forces acting on the wheels of the axles of the multi-section articulated bus. Today, there are several analytical approximations of the dependence of the lateral reaction applied in the wheel contact patch on the side-slip angle, but the most widespread is the following formula [17]:

$$Y_i = \frac{k_i \delta_i}{\sqrt{1 + k_i \delta_i^2 (\varphi^2 G_i^2)^{-1}}}, \quad (3)$$

where
δ_i, Y_i – the side-slip angles of the bus axles wheels and lateral force, respectively;
j – the coefficient of lateral adhesion between the wheel tire and the supporting surface;
k_i – the side-slip resistance coefficient.

The need to take into account the nonlinearity is explained by the fact that only in a very narrow range the dependence between the forces acting on the axle and the axle retraction angles is close to linear, while for other values of the side-slip angles this dependence is nonlinear and the lateral force cannot exceed the adhesion forces Y^*. As the lateral force approaches its maximum value, partial slip in the lateral direction begins and then full slip. In accordance with this, the maximum value of the lateral force $Y = Y^*$ can be found based on the fact that

$$\lim_{\delta \to +\infty} Y(\delta) = \frac{k}{\chi} = Y^*, \Rightarrow \chi = \frac{k}{Y^*}, Y = \varphi G, \quad (4)$$

where
φ – the lateral adhesion coefficient between the tire and the ground;

G – the vertical wheel load.

If one denotes the resistance coefficient to the side-slip in the absence of longitudinal forces acting on the wheel through k_0, then the value of k will be determined as:

$$k = k_o \cdot \frac{\sqrt{1 - \left(\frac{X}{\varphi G}\right)^2}}{1 + 0.375\frac{X}{G}}, \tag{5}$$

where

X – the longitudinal force acting on the wheel, the value of which is determined by the ratio

$$X = \frac{M}{r},\ if\ \frac{M}{r} < \varphi G, \quad X = \varphi G,\ if\ \frac{M}{r} \geq \varphi G, \tag{6}$$

where

M – the traction (braking) moment that is applied to the wheel.

In the absence of traction (braking) moment on the wheels of the bus axles, the lateral forces on the wheels of its axles are determined by relationships, respectively:

$$Y_i = \frac{k_i \delta_i}{\sqrt{1 + \chi_i^2 \delta_i^2}},\ \chi_i = \frac{k_i}{\varphi Z_i}, \tag{7}$$

where

Z_i – the normal reaction of the supporting surface to the i-th axle of the bus.

The wheel side-slip angles of the multi-section articulated bus axles are defined as

$$\begin{aligned}
&\delta_1 = -arctg\frac{u_1}{\nu_1} - arctg \times \\
&\frac{\times \nu \sin\theta + (u + a\omega)\cos\theta - -(\omega + \dot{\theta})\lambda + b_1(\omega + \dot{\gamma}_1)}{\nu\cos\theta + (u + a\omega)\sin\theta}, \\
&\delta_2 = arctg[(-u + b\omega)/\nu], \\
&\delta_3 = arctg(-u_2 + b_1\omega/\nu_2) = arctg \times \\
&\frac{\nu\sin\gamma_1 - (u - c\omega)\cos\gamma_1 + +(\omega + \dot{\gamma}_1)d_1 + b_1(\omega + \dot{\gamma}_1)}{\nu\cos\gamma_1 + (u - c\omega)\sin\gamma_1}, \\
&\delta_4 = [(-u_3 + b_2\omega_3)/\nu_3] = arctg \times \\
&\times\{[\nu\cos\gamma_1 + (u - c\omega)\sin\gamma_1]\sin\gamma_2 - \\
&-[\nu\sin\gamma_1 + (u - c\omega)\cos\gamma_1 - (\omega + \dot{\gamma}_1)] \times \\
&\times\cos\gamma_2 + (\omega + \dot{\gamma}_1 + \dot{\gamma}_2)d_2\}/ \\
&/\{[\nu\cos\gamma_1 + (u - c\omega)\sin\gamma_1]\cos\gamma_2 + \\
&+\begin{bmatrix} -\nu\sin\gamma_1 + (u - c\omega)\cos\gamma_1 - \\ -(\omega + \dot{\gamma}_1)d_1 - c_1(\omega + \dot{\gamma}_1) \end{bmatrix}\sin\gamma_2\}.
\end{aligned} \tag{8}$$

The moment of resistance to turning the wheels of the controlled module of the bus is proportional to the angles of its turning:

$$M_{h1} = h_1 \cdot \dot{\theta}, \tag{9}$$

where

h_1 – the coefficient of viscous friction in the steering parts.

The resistance moments of turning of the bus sections are defined as

$$M_{0i} = \frac{2}{3}Z_{0i}\mu\frac{R_i^2 - r_i^2}{R_i^2 + r_i^2}, \tag{10}$$

where

Z_{0i} – the vertical load in the coupling device;

μ – the friction coefficient in the coupling device;

R_i, r_i – the larger and smaller radius of the coupling device, respectively.

The longitudinal forces on the wheels of the axles of the bus are determined by the known dependence

$$X_i = f \cdot Z_i, \tag{11}$$

where

f – the rolling resistance coefficient of bus wheels.

The obtained system of equations allows to study the behavior of a three-section articulated bus in both stationary and non-stationary mode motions, as well as to determine the critical speed v_{cr} of the bus, which has two approaches to its quantitative estimation [16]: the first is associated with the study of characteristic equations (the 1st Lyapunov's method) or Lyapunov's functions (the 2nd Lyapunov's method), the second - with the output of the motion parameters for the allowable range. The critical speed v_{cr}, as a stability criterion, connects the design and operational parameters of the multi-section articulated bus and its speed and allows you to find its maximum value, exceeding which leads to the loss of the motion stability.

Analytical expressions for variables $U(\gamma_i), \omega(\gamma_i)$ corresponding to steady mode motions $(\dot{U} = 0, \dot{\omega} = 0, \dot{\gamma}_i = 0, \ddot{\gamma}_i = 0 (i = 1,2))$ can be obtained from the system of equations in (1) on a circular trajectory of sufficiently large radius, provided that $v = const$:

$$\begin{cases}
(m + m_1 + m_2 + m_3)\omega\nu = Y_1 + Y_2 + Y_3 + Y_4; \\
-c(m + m_2 + m_3)\omega\nu = \\
aY_1 - bY_2 - c(Y_3 + Y_4) + M_{h1}; \\
[mc + m_2 d_1 + m_3 d_2]\omega\nu = d_1 Y_3 + d_2 Y_4 + \\
+ M_{h1} - M_{01}; \\
m_2 d_2 \omega\nu = Y_3(d_2 + b_2) + M_{02}; \\
m_3 d_3 \omega\nu = I_3 Y_4 + M_{03} - M_{02};
\end{cases} \tag{12}$$

It is impossible to solve the system of equations in (12) in general form and analyze the dependences of the lateral and angular velocities $(U\ and\ \omega)$ of the bus and the joint angles γ_i of the articulated bus on its design parameters due to the complexity of the disclosure of the sixth-order determinant and the cumbersomeness of the expressions themselves. Therefore, to calculate the determinants of the system in (12), it is necessary to use numerical methods, for example, the Maple software. Then the variables obtained using the Maple software will make it possible to determine the influence of various design and operational factors on the stability indicators of the articulated bus.

According to the linearity of the lateral slip forces Y_{ij}, as a function of the side-slip angle δ_i, the solutions of the equations system in (12) will be the values of the variables appropriating to stationary modes, namely:

$$U=\frac{\Delta_U}{\Delta};\omega=\frac{\Delta_\omega}{\Delta};\gamma_1=\frac{\Delta_{\gamma 1}}{\Delta};\gamma_2=\frac{\Delta_{\gamma 2}}{\Delta}, \tag{13}$$

where

Δ – the main determinant of the system;

$\Delta_U, \Delta_\omega, \Delta_{\gamma 1}, \Delta_{\gamma 2}$ – determinants of the system to find the appropriate variables.

The roots of characteristic equations can be determined by numerical methods. Note that the description of the articulated bus motion, which is actually a nonlinear object, linear equations are a replacement for one problem with another, with which the first may have nothing to do (due to nonlinearity of assignments and terms of the equations of motion higher than the first order).

Hence the following task: to establish the necessary and sufficient conditions for stability on the first approximation. According to Lyapunov's theorem on the stability of a steady mode motion by the first approximation [14], if all the characteristic equation roots of the first approximation system of the perturbed motion equations have negative real parts, then the undisturbed motion is stable and asymptotically stable, no matter what the higher orders in the differential equations of perturbed motion.

The conditions under which all the roots have negative real parts are determined by the Liénard-Shipard criterion: for the characteristic equation to have all the roots with negative real parts, it is necessary and sufficient that:

1) all the characteristic equation coefficients were positive;
2) the main diagonal minors of the Hurwitz matrix compiled for the given characteristic equation were positive. Those conditions are fulfilled if all the denominators of Equation (13) - the main determinants of the system, are positive. What it looks like:

$$\nu<\nu_{cr}=\frac{\beta}{-\alpha}, \tag{14}$$

where α and β in Equation (14) will be defined as

$$\beta=\|\beta_{ij}\|_1^4;\ \alpha=\|\alpha_{ij}\|_1^4. \tag{15}$$

Taking into account the fact that the stationary mode motions of the articulated bus are not only rectilinear modes, but the circular modes, as well, then for the realizability of such a motion it is necessary to fulfill the condition $R > 0$. The turning radius $R=\nu/\omega$ in the theory of wheeled vehicles is usually called the curvature radius of the trajectory point of the longitudinal axis of the driving link, the speed of which is directed along the axis [16]. For the radius R one has:

$$R=\frac{\begin{array}{l}-\left\{\begin{array}{l}m(k_3d_1+M_{o2})(k_4d_2+M_{o3})\times\\ \times(M_{h1}-M_{o2})+[c(k_1+k_2)+\\ +M_{h1}-M_{o1}[m_1M_{02}(k_4d_2+M_{03})-\\ -m_2M_{03}(k_3c_1-M_{02})]\end{array}\right\}\times\\ \times\nu^2+[c(k_1+k_2)+M_{h1}-M_{o1}]\times\\ \times\left\{\begin{array}{l}(k_4d_2+M_{03})[M_{o2}(M_{o2}+M_{o3})-\\ -k_1(\mu_1+c_1M_{h1})]+k_4\\ \times(k_3c_1-M_{02})\times d_2M_{o3}\end{array}\right\}+\\ +(k_3d_1+M_{o2})(k_4d_2+M_{o3})[(k_1+k_2)\mu_1-\\ -(M_{h1}-M_{01})^2\end{array}}{\begin{array}{l}(k_1+k_2)G_2-(M_{h1}-M_{o1})\gamma_{10}]\times\\ \times(k_3d_1+M_{o2})(k_4d_2+M_{03})-\\ -[c(k_1+k_2)+M_{h1}-M_{01}]\times\\ \times[(k_4d_2+M_{03})k_3G_2)]\end{array}} \tag{16}$$

where

k_i – the sum of the resistance wheels coefficients of the i-th axis;

$\gamma_{10}=k_{1i}\gamma_0; M_1=k_{1i}a; M_2=k_{2ii}b; M_3=k_{3i}d_1;$
$M_4=k_{4i}d_2; \mu_1=k_{1i}a^2; \mu_2=k_{2i}b^2; \mu_3=k_{3i}d_1^2;$
$\mu_4=k_{4i}d_2^2; G_2=\sum_i k_{1i}a_i\gamma_i.$

As follows from Equation (16), the turning radius of the three-section articulated bus depends on the mass and geometric parameters of its sections, as well as the resistance coefficients of the wheels of the axles of the first k_3 and the second trailer-bus section k_4.

Equation (16) is presented in the form:

$$R=\frac{l}{\gamma_0}-\nu^2\varphi\left(\begin{array}{l}k_{1i},k_{2i},k_{3i},k_{4i},\gamma_i,m,m_1,m_2,\\ a,b,c,c_1,d_1,d_2\end{array}\right), \tag{17}$$

Since $\varphi(\infty,\infty,\infty,\infty,\gamma_i,m,m_1,m_2,a,b,c,c_1,d_1,d_2)=0$, then Equation (17) includes, as a special case, a linearized expression $R=l/\gamma_0$ for the radius of trajectory curvature of the bus rear axle on wheels rigid in the lateral direction. If $\varphi(\infty,\infty,\infty,\infty,\gamma_i,m,m_1,m_2,a,b,c,c_1,d_1,d_2)< or >0$, then R will be greater or less than 0 and the three-section articulated bus will have insufficient or excessive agility. If $\varphi(\infty,\infty,\infty,\infty,\gamma_i,m,m_1,m_2,a,b,c,c_1,d_1,d_2)=0$ the three-section articulated bus is neutral with respect to understeer: its turning radius is the same as in the multi-articulated bus on rigid lateral wheels.

For $\gamma_i>0$, the denominator of Equation (16) will be greater than 0, respectively, the condition of the feasibility of circular motion and the stability of rectilinear motion is satisfied only if $\nu<\nu_{cr}$. From Equation (15) one obtains:

$$\xi\nu^2 + \eta > 0; \eta > -\xi\nu^2; \nu^2 < \frac{\eta}{-\xi} \Rightarrow \nu_{cr}^2 = \frac{\eta}{-\xi},$$

where

$$\nu_{cr}^2 = \frac{\begin{gathered}[c(k_1+k_2)+M_{h1}-M_{o1}\{(k_4d_2+M_{o3})\times \\ \times[M_{o2}(M_{o2}+M_{o3})-k_3(\mu_3+c_1M_{o3})]+ \\ +k_4(k_3c_1-M_{o2})(d_2M_{o3}+\mu_4)\}+ \\ +(k_3d_1+M_{o2})(k_4d_2+M_{o3})[(k_1+k_2)\times \\ \times(\mu_1+\mu_2)-(M_{h1}-M_{o1})^2\end{gathered}}{\begin{gathered}m(k_3d_1+M_{o2})(k_4d_2+M_{o3})(M_{h1}-M_{o1})+ \\ +[c(k_1+k_2)+M_{h1}-M_2][m_1M_{o2}\times \\ \times(k_4d_2+M_{o3})-m_2M_{o3}(k_3c_1-M_{o2})\end{gathered}} \quad (18)$$

According to Equation (18), calculations of the critical speed of motion of the three and two-section articulated buses are performed. The calculations were performed on the following initial data:

$v = 5\text{m/s}, \lambda = -0.023m, a = 3.68\,m; b = 2.32\,m;$
$c = 8.71\,m; c_1 = 3.7\,m; d_1 = 4.17\,m; d_2 = 4.17\,m;$
$R = 1.1\,m; r = 0.2\,m; m = 18000\,kg; J = 38500\,kg\cdot m^2;$
$m_1 = 400\,kg; J_1 = 18.5\,kg\cdot m^2; m_2 = 9500\,kg\cdot m^2;$
$J_2 = 31200\,kg\cdot m^2; m_3 = 9500\,kg\cdot m^2; J_3 = 31200\,kg\cdot m^2;$
$k_f = 0; k_1 = 160000\,\text{N/rad}; k_2 = 320000\,\text{N/rad};$
$k_3 = 180000\,\text{N/rad}; k_4 = 320000\,\text{N/rad}; h = 30,$
$\varphi = 0.8; \gamma_0 = 0; \theta = \theta_0 + k_\theta \cdot n; k_\theta = 0.05;$
$n = 1, 2\ldots10; \nu := 0; X1 = X2 = X3 = X4 = 0.$

In the case of the two-section articulated bus, should be accepted:
$c_1 = 0, b_2 = 0, d_2 = 0, m_3 = 0, J_3 = 0, k_4 = 0.$

For the multi-section articulated bus selected for calculation, the critical speed was:

- for the three-section articulated bus - 28.06 m/s;
- for the two-section articulated bus - 30.89 m/s.

Despite the fact that the motion of the multi-articulated bus by the Metrobus system is carried out

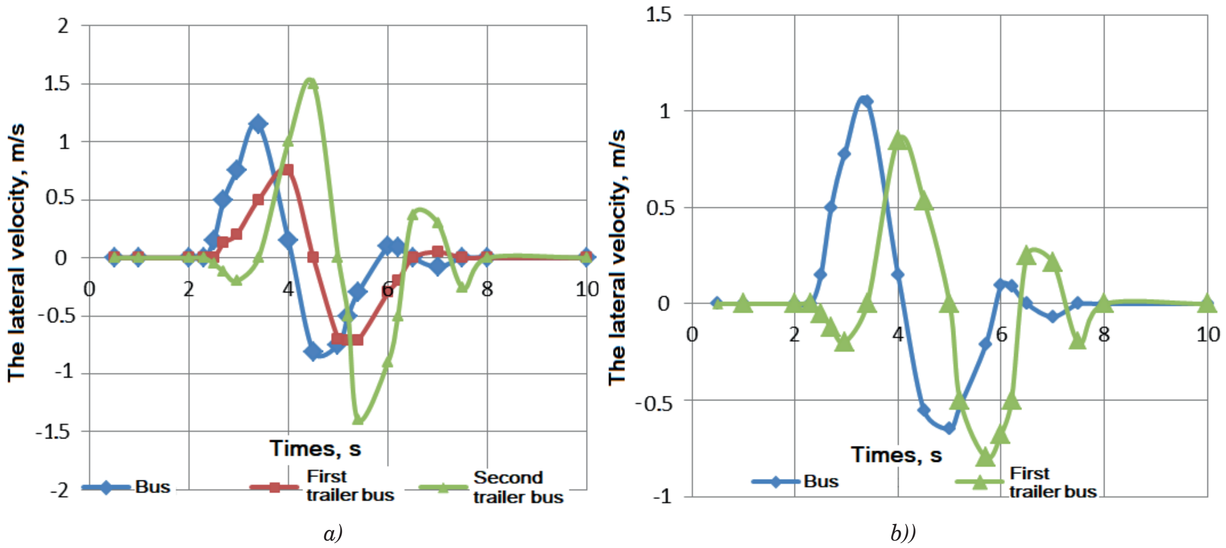


***Figure 2** The lateral velocity of the articulated bus during the "jerk the steering wheel" manoeuver: a - three-section articulated bus; b - two-section articulated bus*

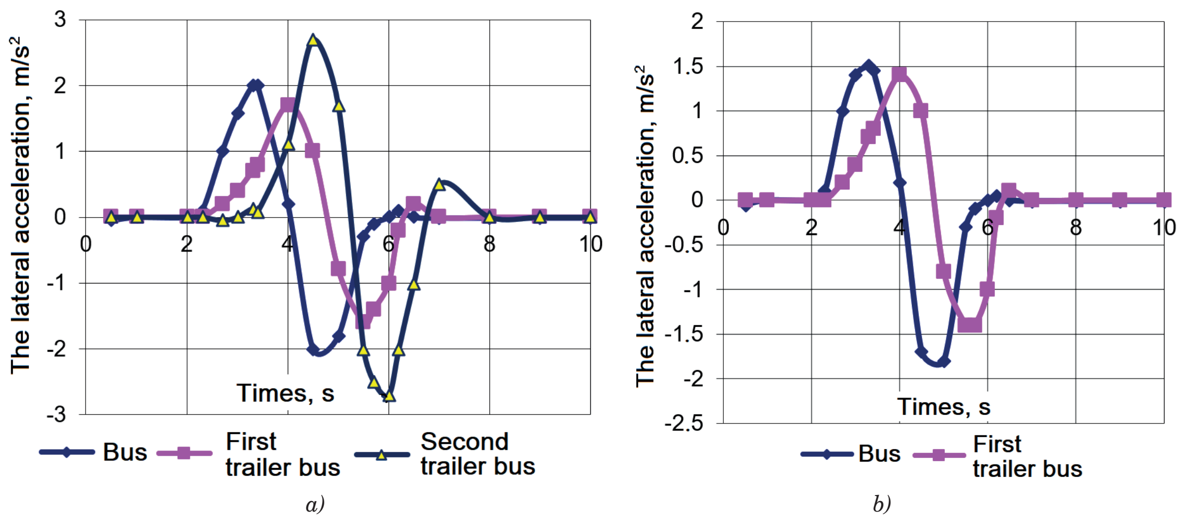
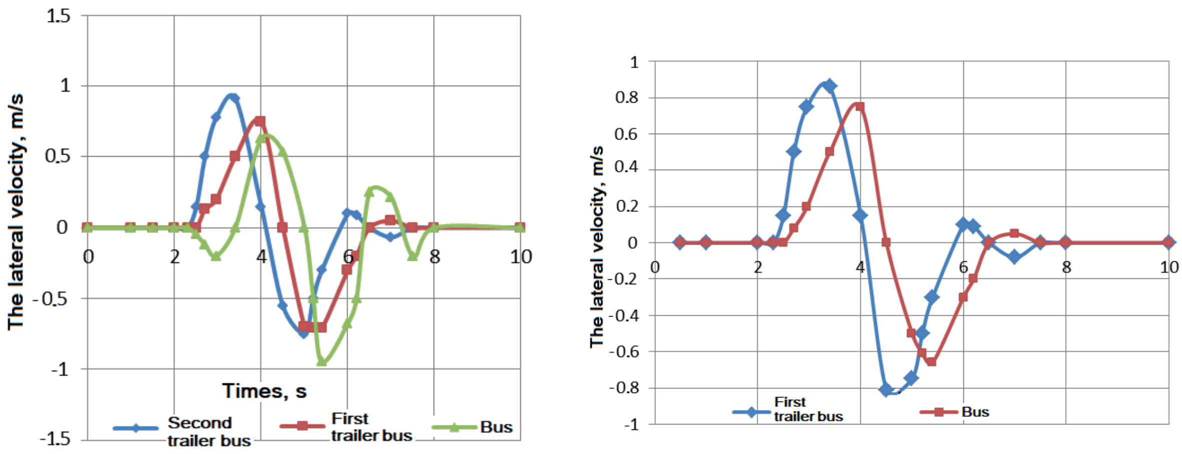
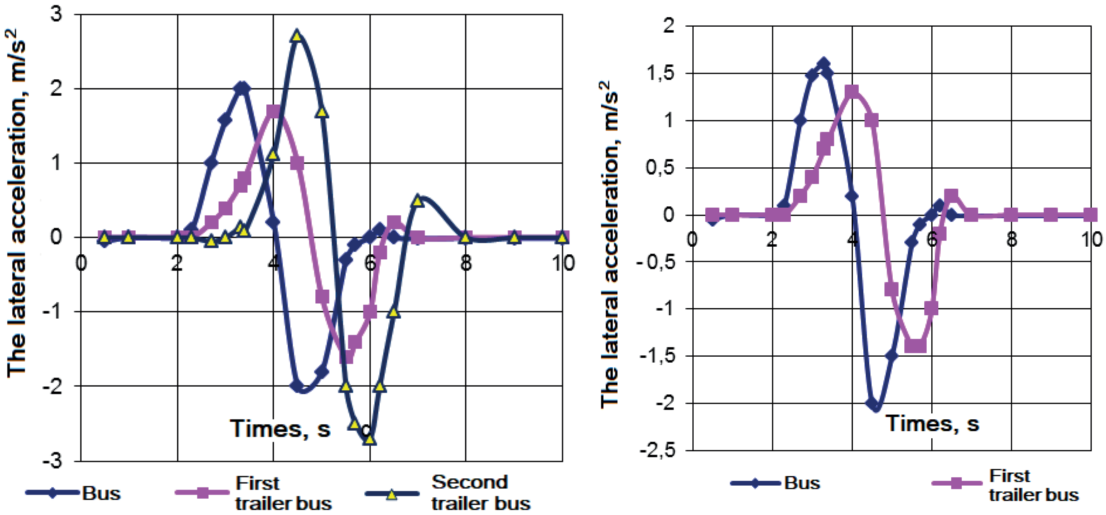


***Figure 3** The lateral acceleration of the mass centers of multi-section articulated bus during the "jerk the steering wheel" manoeuver: a - three-section articulated bus; b - two-section articulated bus*



a) *b)*

***Figure 4** The lateral velocity of the articulated bus during the "displacement" manoeuver: a - three-section articulated bus; b - two-section articulated bus*



a) *b)*

***Figure 5** The lateral acceleration of the mass centers of multi-section articulated bus during the "displacement" manoeuver: a - three-section articulated bus; b - two-section articulated bus*

on separately allocated lanes at a speed of 25-28 m/s, it can be considered that the stability of the rectilinear motion of both two and three-sections articulated buses in Metrobus system is ensured.

The stability of the multi-section articulated bus in the non-stationary modes of motion can be judged by the lateral speed of individual sections of the multi-section articulated bus and the value of lateral accelerations when it performs various maneuvers, in particular, the "jerk of the steering wheel - turning the steering wheel with an angular speed of at least 19 rad/s" and "displacement" Figures 2 to 5.

From the given Figures 2 to 5 it follows that the limiting factor for the lateral speed in the case of a three-section articulated bus is the third trailer bus (its lateral speed is almost 20.5% higher than of the first). In the case of a two-section articulated bus, the lateral speed of the first bus decreases by almost 8.5% compared to the three-section and the second trailer bus increases by almost 9.5% compared to the three-section). However, the stability of motion can be judged to a greater extent by the value of the lateral acceleration acting in the center of mass of the individual sections. According to literary sources, the motion stability can be considered satisfactory if the lateral accelerations at the center of mass of the section do not exceed 0.45g. In addition, the damping nature of the oscillations also indicates the stability of the multi-section articulated bus motion. In this case, the limiting factor, when performing the "jerk of the steering wheel" maneuver, is the mass center acceleration of the second trailer section in the case of a three-section articulated bus and a bus in the case of the two-section articulated bus.

4 Conclusion

An improved mathematical model of the three-section articulated bus is presented, which made it possible to determine the stability indicators of both the two-section articulated bus and the three-section articulated bus. Thus, the critical speed for the three-section articulated was 28.06 m/s and for the two-section articulated bus - 30.89 m/s. Despite the fact that the motion by the multi-section articulated bus in the Metrobus system is carried out on separately allocated lanes at a speed of 25-28 m/s, it can be considered that the stability of the rectilinear motion of both two and three-section buses is ensured. In the non-stationary modes of motion with the two-section articulated buses, the lateral speed of the first bus decreases by almost 8.5% compared to the three-section and the second trailer-bus increases by almost 9.5% compared to the three-section. At the same time, the motion stability of both multi-sections articulated buses can be considered satisfactory, since the lateral accelerations in the center of mass of all sections do not exceed 0.45 g. In addition, the damping nature of the oscillations also indicates the stability of the multi-section articulated bus motion.

5 Discussion of research results

The mathematical model of the plane-parallel movement of the two- and three-lane Metrobuses developed in the work made it possible to achieve the goal of the study. As noted in [18], the kinetic models, to which the developed model belongs, are better in high-precision simulators and in control problems related to «dynamic» tasks (for example, preventing overturning, ensuring stability, braking without sliding, etc.). A reasonable choice of the model should be the result of a compromise between the price of use and efficiency. A complication of the model, as shown in [7], is not always justified, because increasing the degrees of freedom of the system leads to an increase in the number of initial parameters that must be determined with a sure accuracy, which cannot always be achieved. Linearization of the model to determine the maximum speed (critical for stability) did not lead to significant errors. Thus, three-dimensional dynamic models of a car and a trailer were developed in work [12], based on which a dynamic train model was built, which is used to study the linear and non-linear stability of each element and the autotrain as a whole during rectilinear movement. numerical results show that for the nonlinear and linear models, the critical velocities are a little different from each other. However, when studying the stability of the Metrobus in transient traffic modes, it is necessary to use the three-dimensional models, which will be the subject of further research.

References

[1] OMELNICKIJ, O. E. Analysis of the design of metro buses. *Scientific and Industrial Journal Highway Truck of Ukraine / Naukovo - Vyrobnychyy Zhurnal Avtoshliakhovyk Ukrayiny*. 2018, **3**, p. 7-11. ISSN 0365-8392.

[2] PEKEL, E., KARA, S. S. Simulation-based fleet scheduling in the Metrobus. *International Journal of Simulation and Process Modelling* [online]. 2016, **11**(3-4), p. 326-333. ISSN 1740-2123. Available from: https://doi.org/10.1504/IJSPM.2016.078523

[3] ORTEGON-SANCHEZ A., TYLER N. Towards multi-modal integrated mobility systems: views from Panama City and Barranquilla. *Research in Transportation Economics* [online]. 2016, **59**, p. 204-217. ISSN 1740-2123. Available from: https://doi.org/10.1016/j.retrec.2016.03.001

[4] AVALOS-BRAVO, V., ARELLANO, C. C., PEREZ, D. A. P., CRUZ, M. H. Forecasting people's influx on Mexico City metrobus line 1 using a fractal analysis. In: *Communications in Computer and Information Science* [online]. MATA-RIVERA, M. F., ZAGAL-FLORES, R., ARELLANO VERDEJO, J., LAZCANO HERNANDEZ, H. E. (eds.). Vol. 1276. Cham: Springer, 2016. ISBN 978-3-030-59871-6, eISBN 978-3-030-59872-3, p. 92-105. Available from: https://doi.org/10.1007/978-3-030-59872-3_7

[5] DIZO, J., BLATNICKY, M. Investigation of ride properties of a three-wheeled electric vehicle in terms of driving safety. *Transportation Research Procedia* [online]. 2019, **40**, p. 663-670. ISSN 2352-1457. Available from: https://doi.org/10.1016/j.trpro.2019.07.094

[6] DIZO, J., BLATNICKY, M. Evaluation of vibrational properties of a three-wheeled vehicle in terms of comfort. *Manufacturing Technology* [online]. 2019, **19**(2), p. 197-203. ISSN 1213-2489. Available from: https://doi.org/10.21062/ujep/269.2019/a/1213-2489/MT/19/2/197

[7] STASTNIAK, P. Freight long wagon dynamic analysis in S-curve by means of computer simulation. *Manufacturing Technology* [online]. 2015, **15**(5), p. 930-935. ISSN 1213-2489. Available from: https://doi.org/10.21062/ujep/x.2015/a/1213-2489/MT/15/5/930

[8] GECHEV, T., MRUZEK, M., BARTA, D. Comparison of real driving cycles and consumed braking power in suburban Slovakian driving. *MATEC Web of Conferences* [online]. 2017, **133**, 02003. ISSN 2261-236X. Available from: https://doi.org/10.1051/matecconf/201713302003

[9] DIZO, J., BLATNICKY, M., BARTA, D., MELNIK, R. Application of simulation computations in investigation of vibration properties of a tricycle. *Diagnostics / Diagnostyka* [online]. 2019, **20**(3), p. 97-104. ISSN 1641-6414. Available from: https://doi.org/10.29354/diag/111827

[10] SCHMID, I. Engineering approach to truck and tractor train stability. *SAE Transactions* [online]. 1967, **76**, p. 1-26. ISSN 0148-7191. Available from: https://doi.org/10.4271/670006
[11] KOLESNIKOVICH, A. N., VYGONNYJ, A. G. Stability of a trailer road train of increased length (25.25 m) in straight-line motion. *Actual Problems of Mechanical Engineering*. 2018, **7**, p. 96-100. ISSN2306-3084.
[12] LUO, R., ZENG, J. Hunting stability analysis of train system and comparison with single vehicle model. *Jixie Gongcheng Xuebao/Chinese Journal of Mechanical Engineering* [online]. 2008, **44**(4), p. 184-188. ISSN 0577-6686. Available from: https://doi.org/10.3901/JME.2008.04.184
[13] HALES, F. D. *The vertical motion lateral stability of road vehicle trains*. Ukraine: Transport and Road Research Laboratory (TRRL) Technology University, 1972.
[14] GALLUPPI, O., FORMENTIN, S., NOVARA, C., SAVARESI, S. M. Nonlinear stability control of autonomous vehicles: a MIMO D2-IBC solution. *IFAC-PapersOnLine* [online]. 2017, **5**(1), p. 3691-3696. ISSN 2405-8963. Available from: https://doi.org/10.1016/j.ifacol.2017.08.563
[15] CHEN, H.-J., SU, W.-J., WANG, F.-C. Modelling and analyses of a connected multi-car train system employing the inerter. *Advances in Mechanical Engineering* [online]. 2017, **9**(8), p. 1-13. ISSN 1687-8132. Available from: https://doi.org/10.1177/16878140177017
[16] SAHNO, V., MYROVANY, I., POLJAKOV, V., MISKO, E. To comparative assessment of three-unit lorry passenger trains. *Advances in Mechanical Engineering and Transport.* 2019, **2**(13), p. 146-155. ISSN 2313-5425.
[17] HREBET, V. G., MISKO, E. M., VERBICKIJ, V. G. To the experimental determination of the dependences of the lateral slip forces of a two-axle vehicle in the oversteer curves. In: 10th International Scientific and Practical Conference Modern Technologies and Prospects for the Development of Road Transport: proceedings. 2017. ISBN 978-966-641-707-0, p. 126-128.
[18] ZHANG, Y., KHAJEPOUR, A., HUANG, Y. Multi-axle/articulated bus dynamics modeling: a reconfigurable approach. *Vehicle System Dynamics* [online]. 2018, **56**(9), p. 1315-1343. eISSN 1744-5159. Available from: https://doi.org/10.1080/00423114.2017.1420205