OPTIMIZING PATIENT TRANSPORT UNDER INFECTION CONTROL CONSTRAINTS: A MILP-BASED DIAL-A-RIDE APPROACH

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Resume

With aging populations and rising healthcare demands, efficient patient transportation has become a critical challenge, particularly in the context of infection control. In this paper an extended mixed-integer linear programming (MILP) model for optimizing patient transport in urban environments is presented, with a focus on the separate transportation of infectious and non-infectious individuals. The model incorporates time windows, maximum allowable ride durations, and mandatory vehicle disinfection requirements. Experimental results obtained using CPLEX demonstrate that incorporating infection control measures significantly influences both route planning and computational complexity. The proposed approach provides a scalable foundation for future multi-vehicle extensions and cost-based optimization strategies.

Article info

Received 3 October 2025
Accepted 18 November 2025
Online 16 January 2026

Keywords:

Dial-a-Ride problem
patient transportation
mathematical modeling
mixed-integer linear programming

Available online: https://doi.org/10.26552/com.C.2026.008

ISSN 1335-4205 (print version)
ISSN 2585-7878 (online version)

1 Introduction

Modern transportation systems have undergone the rapid transformation over past decade, primarily driven by the spread of digital platforms and the rise of real-time service coordination. In both passenger and freight sectors, technological advancements have enabled unprecedented levels of flexibility, speed, and user-centered customization. Same-day delivery services, app-based ride-hailing platforms, and real-time tracking have become standard features across much of the logistics landscape. These developments have not only improved operational efficiency but have also reshaped customer expectations regarding accessibility and responsiveness.

While many aspects of transportation have advanced significantly, the mobility of nonemergency patients, particularly in urban healthcare environments remains a complex, evolving area with ongoing challenges. While the ambulances and specialized vehicles are available for critical or long-distance cases, patients attending routine appointments often rely on public transport or private vehicles. However, these alternatives are frequently unsuitable for elderly individuals or those with reduced mobility, as they may involve multiple transfers and long walking distances. Furthermore, private vehicle use is not always a viable option for older adults due to physical or cognitive limitations, and services such as taxis or ride-hailing platforms (e.g., Uber) can be prohibitively expensive for regular use.

In this study a door-to-door, ride-sharing-based patient transportation model that operates with a single vehicle is introduced. The model builds upon the well-established Dial-a-Ride Problem (DARP) framework, which is widely used in demand-responsive transportation systems [1]. It is designed to coordinate multiple patient pickups and drop-offs, while respecting strict time windows and compatibility constraints. The objective was to minimize the total distance traveled by a vehicle, while maximizing its utilization [2].

In the previous work [3] authors studied a problem in which patients are transported to the same hospital by a single vehicle. Each patient specifies their desired arrival time, earliest possible pickup time, and a realistically acceptable maximum travel duration. The model also takes into account the number of passengers at each pickup location, as well as their specific mobility-related requirements.

Four MILP models of the problem were proposed and tested on 5 different sized problems. The largest problem contained 30 pickup points and 60 patients. The best model was able to find the optimal solution in all of the problems in less than 60 seconds.

In this article, the previously examined patient transportation problem is extended by incorporating two real-world features that are essential for practical implementation: (1) the incompatibility between infectious and noninfectious passengers, who must not share the same route segment; and (2) mandatory vehicle disinfection after the transportation of infectious patients, which requires the vehicle to return to the depot before continuing its route. These additional constraints are motivated by infection prevention protocols, particularly those introduced during the COVID-19 pandemic, and they are critical for designing safe and regulation-compliant patient logistics systems.

To address the extended problem, a mixed-integer linear programming (MILP) model is proposed that incorporates infection-aware constraints, such as passenger incompatibility and mandatory vehicle disinfection. The model is tested on various infection scenarios to evaluate its efficiency and scalability. By embedding these constraints into an exact optimization framework, this study contributes both to the theoretical advancement of compatibility-sensitive DARP models and to the practical development of patient transportation systems that comply with modern public health standards.

2 Literature overview

The Dial-a-Ride Problem (DARP) is a well-established framework in transportation research, particularly suited for modelling the demand-responsive services where users specify pickup and drop-off locations, time windows, and service preferences. In its classical form, a fleet of vehicles departs from a depot to fulfill transport requests while minimizing total travel costs or distance, subject to vehicle capacities and temporal constraints [4].

Over the past decade, DARP has been adapted for a variety of real-world applications. Bongiovanni et al. [5] addressed an electric autonomous DARP variant, taking into account energy constraints and recharging infrastructure, by applying a branch-and-cut algorithm. Similarly, Agra et al. [6] explored the use of a new branching algorithm to solve a continuous-time inventory routing problem involving both pickups and deliveries, showcasing the flexibility of the DARP model in logistics domains.

Within the healthcare sector, patient transportation presents additional challenges, including medical requirements, service reliability, and personnel scheduling. Lim et al. [7] proposed a methaheuristic to solve a patient transportation problem in Hong Kong that includes staff scheduling as well. Luo et al. [8] further extended this work by including constraints for the staff breaks and rest periods, and developed a two-phase branch-and-price-and-cut algorithm. In rural Austrian contexts, Armbrust et al. [9] investigated a dynamic deterministic DARP in which the goal was to minimize a linear combination of the total kilometer travelled by the vehicles, the number of the vehicles and the number of the unmet requests. Their hybrid approach combined MILP and large neighborhood search techniques.

One of the most relevant themes for this study is passenger compatibility, especially in the context of infection risk. Molenbruch et al. [10] proposed a bi-objective DARP formulation in which the compatible patients were grouped based on medical needs. Schulz [11] introduced a more refined compatibility model, defining customer types based on their ability or willingness to share a vehicle. Their branch-and-cut algorithm ensured that mutually incompatible passengers would not be transported together. Further advancements were made by Lokhandwala et al. [12], who introduced a graded preference scale for ride-sharing, ranging from those who prefer to ride alone to those who prefer shared travel. The authors applied a column generation approach to address the problem.

The issue of incompatibility also arises in freight and postal delivery contexts. Colombi et al. [13] studied the rural postman problem with incompatible deliveries, modelling route penalties based on pairing constraints. Bernardino et al. [14] applied iterated local search to solve the family Travelling Salesman Problem (TSP) with incompatibility constraints. In the freight sector, Manerba et al. [15] and Gendreau et al. [16] developed multivehicle models to address pairwise incompatibility between transported products, leveraging exact methods including branch-and-cut and hybrid column generation. Factorovich et al. [17] studied the pickup and delivery problem with incompatibilities. In their work, a single vehicle is used to transport the goods, but the problem does not contain time windows or maximum riding times.

Despite the breadth of research on compatibility-aware routing, this literature review found no prior studies that explicitly integrate the infection-based patient incompatibility and mandatory vehicle disinfection procedures into a MILP-based DARP model. This represents a significant research gap, especially in the post-pandemic context where infection control measures are crucial in public health transportation. The present study addresses this gap by developing a model that combines infection-aware constraints with exact optimization techniques, making it directly applicable to real-world healthcare logistics planning.

3 Problem description

Let $G = (V, E)$ be a directed graph whose vertices are labeled from 0 to $N + 1$, where N is the number of the pickup places. The depot is represented by point 0, the hospital is represented by point $N + 1$, and the pickup points are represented by the vertices 1, 2, … N. Every arc (i, j) of the graph has two nonnegative weights: the traveling distance between points i and j and the traveling time from point i to point j. Starting from a depot a single vehicle visits the pickup points and transports the patients to the hospital. After all the patients are transported to the hospital, the vehicle returns to the depot. There is a predefined time window during which the vehicle must start and complete its tour. In the examined scenarios, the operating company allows vehicle operations only between 6:00 AM and 6:00 PM. This constraint limits both the earliest departure from the depot and the latest return, ensuring that all the patient transport activities occur within standard working hours. Furthermore, the vehicle has two different capacities, one for the number of the patients, and another one for the number of the mobility-impaired patients that can be transported by a vehicle. For each pickup point i a list containing the following key parameters is known:

- The number of patients at point i,
- The number of mobility-impaired patients at point i,
- The earliest time the vehicle can pick up the passengers at point i,
- An upper bound on the riding time of the passengers at point i,
- A due date at which the passengers at point i have to arrive at the hospital,
- The boarding time of the patients at point i on the vehicle, and
- The possible presence of infectious patients among the passengers at point i.

The vehicle has to visit each pickup point exactly once, which implies that for each pickup point, neither the number of the patients nor the number of the mobility-impaired patients exceeds the corresponding capacity of the vehicle. Since the goal is to protect patients from infection, infectious and noninfectious patients cannot be transported together. Furthermore, after the vehicle arrives at the hospital with infectious patients, it has to go back to the depot and undergo a disinfection procedure before continuing its tour. Finally, once the vehicle arrives at the hospital all the patients must get off the vehicle. The goal is to find the optimal route for a vehicle, which satisfies all of the constraints and minimizes the total distance run by the vehicle.

4 Mathematical formulation

To formulate the problem as a MILP model, the problem is first transformed into a Traveling Salesman Problem (TSP) with additional constraints.

To model the hospital connections, we define an extended graph $G' = (V', E')$, where each pickup point i has a corresponding hospital point $i + N$. The G' is a directed graph whose vertices are numbered from 0 to $2N + 1$. Vertex 0 is the depot, the vertices labeled as 1, 2, … N are representing the pickup points. For each pickup point i there is a corresponding 'hospital point in G' denoted by $i + N$. For each pickup point i patients at point i have to be transported to the corresponding hospital point $i + N$. Thus, the new graph G' has N hospital points. In G', the traveling time and the distance between points i and j are defined as follows:

- if i is a pickup point or the depot, and j is a pickup point, then the traveling time and the distance between points i and j in G' are the same as in G;
- if i is a pickup point or the depot, and j is a hospital point (i.e., $N + 1 < = j <= 2N + 1$), then the traveling time and the distance between points i and j in G' are the traveling time and the traveling distance between i and the hospital in G;
- if i and j are both hospital points, then the traveling time and the distance between points i and j are 0 in G';
- if i is a hospital point, then the traveling time and the distance between point i and point 0 (i.e. the depot) in G' is the traveling time and the distance between the hospital and the depot in G;
- if i is a hospital point corresponding to a pickup point with noninfectious patients and j is a pickup point, then the traveling time and the distance between points i and j in G' are the traveling time and the distance between the hospital and j in G;
- if i is a hospital point corresponding to a pickup point with infectious patients and j is a pickup point, then the distance between i and j in G' is the sum of the distance between the hospital and the depot in G and the distance between the depot and j in G (i.e., $D_{ij} = D_{i0} + D_{0j}$);
- finally, if i is a hospital point corresponding to a pickup point with infectious patients and j is a pickup point, then the traveling time from i to j in G' is set to the sum of the traveling time from the hospital to the depot in G, the traveling time from the depot to j in G and the time of the disinfection procedure at the depot (i.e., $T_{ij} = T_{i0} + T_{0j} + disinf$).

It can be easily seen that a tour in G' yields a tour in G. However, a tour in G' may not yield a feasible solution of the original problem. To show this, a problem with 4 pickup points is considered. In this case, vertex 0 is the depot, vertices 1/2/3/4 are the pickup points and vertices 5/6/7/8 are the hospital points in G'. The tour 0-1-6-4-3-2-5-7-8-0 is not a feasible tour since the patients at a pickup point 2 are picked up after that vehicle visited the corresponding hospital point 6. Furthermore, the tour 0-1-2-3-5-6-4-8-7-0 is not a feasible tour either, because patients at pickup point 3 are transported to the hospital but they do not get off the vehicle there (the vehicle visits pickup point 3, then it travels to hospital points 5 and 6, but before reaching hospital point 7 (the delivery point of pickup point 3) it visits pickup point 4). So, in order that the solution of the TSP in *G'* yields the optimal solution of the original problem, the model has to ensure that:

- the constraints associated with the vehicle (capacities of the vehicle (patients, mobility-impaired patients) and the passengers (maximum riding time, earliest pickup time, maximal arrival time, incompatibility restrictions) are satisfied;
- each pickup point i has to be visited earlier by the vehicle than the corresponding hospital point $i+N$;
- if the vehicle visits the pickup points i_1, i_2, ...i_k before traveling to a hospital point, then the vehicle has to continue its tour by visiting the hospital points $i_1 + N$, $i_2 + N$, ..., $i_k + N$ in the same order.

The following notations are used in the model:

Parameters:

- N: total number of pickup points,
- $D_{i,j}$: distance between points i and j where $0 \leq i, j \leq 2N$,
- $T_{i,j}$: time required to travel from point i to j, where $0 \leq i, j \leq 2N$,
- C_1: maximum capacity of the vehicle for regular patients,
- C_2: maximum capacity of the vehicle for mobility-impaired patients,
- p_i: number of regular patients at pickup point i; $1 \leq i \leq N$,
- dp_i: number of mobility-impaired patients at pickup point i; $1 \leq i \leq N$,
- ip_i: indicator of the presence of an infectious patient at point i.; $i_{pi} = 1$ if infectious, otherwise 0,
- I_i: maximum allowable travel time for patients at point i; $1 \leq i \leq N$,
- a_i: time required for boarding patients at point i; $1 \leq i \leq N$,
- tr_i: earliest time patients can be picked up at point i; $1 \leq i \leq N$,
- ta_i: latest allowable arrival time at the hospital for patients from point i; $1 \leq i \leq N$,
- A_0: opening time of the depot,
- B_0: closing time of the depot.

Continuous variables:

- pn_i: number of regular patients on the vehicle after visiting point i; $0 \leq i \leq 2N$,
- dpn_i: number of mobility-impaired patients in the vehicle after visiting point i; $0 \leq i \leq 2N$,
- m_i: arrival time of the vehicle at point i; $0 \leq i \leq 2N$.
- Integer variables
- u_i; $0 \leq i \leq 2N$.

The integer variable u_i is an auxiliary variable used in the Miller–Tucker–Zemlin (MTZ) formulation to eliminate subtours [18]. It represents the order or position of vertex i in the route, ensuring that the solution forms a single continuous tour visiting all nodes exactly once. For the depot, $u_0 = 0$.

Binary variables:

- $x_{i,j}$: equals 1 if the vehicle travels from point i to point j; $0 \leq i, j \leq 2N$.

5 Constraints of the model

The mathematical formulation of the problem includes several constraints, which define the structure of the vehicle's route and the conditions for transporting patients. These constraints ensure the feasibility and efficiency of the solution.

$$\sum_{i=0}^{2N} x_{i,j} = 1, 0 \leq j \leq 2N, \tag{1}$$

$$\sum_{j=0}^{2N} x_{i,j} = 1, 0 \leq i \leq 2N, \tag{2}$$

$$u_0 = 1, \tag{3}$$

$$2 \leq u_i \quad 1 \leq i \leq 2N, \tag{4}$$

$$u_i \leq 2N + 1, 1 \leq i \leq 2N, \tag{5}$$

$$u_i + 1 \leq u_j + 2N^*(1 - x_{i,j}), 1 \leq i, j \leq 2N, \tag{6}$$

$$u_i + 1 \leq u_{i+N}, 1 \leq i \leq N, \tag{7}$$

$$x_{i,j} = x_{j+N, i+N}, 1 \leq i, j \leq N, \tag{8}$$

$$pn_0 = 0, N + 1 \leq i \leq 2N, \tag{9}$$

$$pn_i = 0, N + 1 \leq i \leq 2N, \tag{10}$$

$$dpn_0 = 0 \quad N + 1 \leq i \leq 2N, \tag{11}$$

$$dpn_i = 0, N + 1 \leq i \leq 2N, \tag{12}$$

$$pn_i \leq Cp, 1 \leq i \leq 2N, \tag{13}$$

$$dpn_i \leq Cdp, 1 \leq i \leq 2N, \tag{14}$$

$$pn_i + p_j + 2 * Cp * (x_{i,j} - 1) \leq pn_j, \\ 0 \leq i \leq 2N; 1 \leq j \leq N, \tag{15}$$

$$dpn_i + dp_j + 2 * Cdp * (x_{i,j} - 1) \leq dpn_j, \\ 0 \leq i \leq 2N; 1 \leq j \leq N, \tag{16}$$

$$m_0 + T_{0,i} + M^*(x_{0,1} - 1) \leq m_i, \ 0 \leq i \leq N, \tag{17}$$

$$m_i + T_{i,j} + a_i + M^*(x_{i,j} - 1) \leq m_j, \\ 1 \leq i \leq N; 1 \leq j \leq 2N, \tag{18}$$

$$m_i + T_{i,j} + M^*(x_{i,j} - 1) \leq m_j, \\ N + 1 \leq i \leq 2N; 1 \leq j \leq 2N, \tag{19}$$

$$tr_i \leq m_i, 1 \leq i \leq N, \tag{20}$$

$$m_{i+N} - m_i \leq I_i \quad 1 \leq i \leq N, \tag{21}$$

$$m_{i+N} \leq ta_i, 1 \leq i \leq N, \tag{22}$$

$$A_0 \leq m_0, \tag{23}$$

$$m_{i+N} + T_{i+N,0} \leq B_0, 1 \leq i \leq N, \tag{24}$$

$$x_{i,j} = 0 \text{ and } x_{j,i} = 0, \ 1 \leq i, j \leq N, ta_i \leq tr_j, \tag{25}$$

$$x_{i,i} = 0, 0 \leq i \leq 2N, \tag{26}$$

$$u_i + 2 \leq u_j, \ 1 \leq i, j \leq N, ta_i \leq tr_j, \tag{27}$$

$$x_{i,j} = 0 \text{ and } x_{j,i} = 0, 1 \leq i; j \leq N; ip_i \neq ip_j, \tag{28}$$

$$\sum_{i=0}^{2N} \sum_{j=0}^{2N} x_{i,j} * D_{i,j} \to \min. \tag{29}$$

Equations (1) and (2) ensure that each point of G' is visited by the vehicle exactly once. Constraints in Equations (3) to (6) are the subtour elimination constraints of the MTZ model of the TSP problem. Constraint in Equation (7) states that each pickup point is visited earlier by the vehicle than the corresponding hospital point. Equation (8) states that the vehicle travels from a pickup point i to a pickup point j if the vehicle travels from hospital point $j + N$ (corresponding to a pickup point j) to hospital point $i + N$ (corresponding to a pickup point i). This implies that if the vehicle visits pickup points i_1, i_2 … i_l, before going to a hospital point, the vehicle visits the corresponding hospital points in the reverse order, i.e., after i_1 the vehicle visits the hospital points i_{l+N}, i_{l-1+N}, … i_{1+N}. Constraints in Equations (9) to (12) ensure that after leaving the depot or a hospital point, there is no patient or mobility-impaired patient in the vehicle. Constraints in Equations (13) and (14) state that the number of patients or mobility-impaired patients cannot exceed the capacity of the vehicle. Constraints in Equations (15) and (16) ensure, that if the vehicle travels from point i to point j, the number of patients (mobility-impaired patients) in the vehicle after leaving point j cannot be less, than the number of patients (mobility-impaired patients) in the vehicle after leaving point i plus the number patients (mobility-impaired patients) at point j. Constraints in Equations (17) to (19) imply that if the vehicle visits point j after points i, the arriving time at point j cannot be less than the arriving time at point i plus the traveling time from point i to point j plus the boarding time of the patients (if there are any patient) at point i. Constraint in Equation (20) states that the vehicle cannot arrive earlier at a pickup point than the earliest pickup time of the patients at that pickup point. Inequality in Equation (21) ensures that the maximal riding time constraints of the passengers are satisfied. Constraint in Equation (22) states that all of the patients arrive at the hospital before their due date. Constraints in Equations (23) and (24) state that the vehicle starts and finishes its tour in the opening time of the depot. Constraints in Equations (25) and (27) imply that if the latest arrival time at the hospital for patients at pickup point i is less or equal than the earliest pickup time of patients at point i, the vehicle can travel neither from i to j nor from j to i. Constraint in Equation (26) states that the vehicle has to move from a point to another point. Constraint in Equation (28) ensures that patients and infectious patients cannot be transported together. Finally, Equation (29) states the total distance travelled by the vehicle has to be minimized.

6 Computational results

To evaluate the performance and scalability of the proposed MILP model, a series of computational experiments were conducted using IBM ILOG CPLEX Optimization Studio (version 22.1.0). The tests were performed on a personal computer equipped with an Intel Core i7-7700HQ CPU, 8 GB of RAM, and a 1 TB SSD. Each test run was subject to a maximum time limit of 3,600 seconds. Except for enabling the strong branching strategy for variable selection, all the other solver parameters were kept at their default settings.

The test instances were generated based on a real-world urban environment in the city of Sopron, Hungary. All the travel distances and travel times between pickups, hospital, and depot locations were determined using Google Maps data under typical traffic conditions. The desired arrival times of patients from each pickup point to the hospital were generated using random number functions in Microsoft Excel to reflect realistic time window constraints. This dataset served as a basis for all computational experiments conducted in this study.

The experimental design included five test groups, categorized by the number of pickup points: 10, 15, 20, 25, and 30. For each group, four infection-related scenarios were examined, varying the proportion of infectious patients across all pickup locations. These scenarios ranged from 0% (no infectious patients) to one-third, two-thirds, and 100% of patients being infectious.

For each scenario, the objective was to minimize the total distance traveled, while satisfying all the constraints related to time windows, vehicle capacity, infection-based compatibility, and mandatory disinfection procedures. The results are summarized in Table 1.

Out of the 20 generated test instances, the MILP model was able to find the optimal solution in all the cases involving 10, 15, and 20 pickup points, typically within just a few seconds of computation time. For the 25-pickup-point cases, the optimal solution was obtained for all four infection scenarios; however, the required runtime was much higher than for the 10–20-point instances, which were solved within seconds. In terms of computation time, for problems with 25 pickup points, the mixed scenarios were easier for CPLEX to solve than the homogeneous ones. The optimal solution was found in 620.2 seconds when one-third of the patients were infectious, and in 392 seconds when two-thirds of the patients were infectious. By contrast, the homogeneous scenarios took considerably longer: 963.04 seconds with no infectious patients and 1001.84 seconds when all patients were infectious.

As expected, the presence of infectious patients resulted in longer total route distances due to additional constraints and the requirement to return to the depot for disinfection. For the 10-pickup point case, the scenario in which all patients were infectious (100%) yielded shorter optimal route lengths than those with mixed groups (e.g., 33% or 66% infectious). For the 25-pickup point case, the scenario in which all patients were infectious also produced shorter optimal route lengths than the mixed case with two-thirds infectious patients. Furthermore, in the 25-pickup point case, this homogeneous scenario converged faster. This is not surprising: when all the patients are infectious, the vehicle must always return to the depot after visiting the hospital, but it can freely travel between any pickup points without compatibility restrictions. In contrast, with mixed groups, transitions are restricted, infectious pickup points can only be followed by another infectious pickup, and similarly for non-infectious points, which significantly reduces the number of feasible routing options, often resulting in longer total distances despite fewer depot returns.

For the largest test group with 30 pickup points, different outcomes were observed depending on the infection composition. In the case without infectious patients, the program was unable to prove optimality within the 3,600-second time limit, but it returned a feasible solution with an objective value of 111.09. At termination, the best lower bound was 93.60, indicating that the reported solution may still be improved. This highlights the substantial computational challenges associated with large-scale instances, even in the absence of infection-related constraints.

By contrast, when infectious patients were included, the problem became unfeasible due to the combined effect of vehicle capacity restrictions, mandatory disinfection times, and tight arrival time windows. In these cases, CPLEX was unable to prove infeasibility within the 3,600-second time limit.

Table 1 summarizes the computational results across all generated test instances. Time values are reported in seconds. Entries marked with „–" denote cases where no feasible solution was found, while „>3600" indicates that the one-hour time limit was exceeded without proving optimality.

Table 1 *Experimental results*

Initial data		Results	
Pick up points	Infectious Points	Time to find Optimum [s]	Total distance of delivery tasks [km]
10	0	1.45	51.9
10	3	1.51	70.3
10	6	1.47	71.1
10	10	1.57	68.9
15	0	1.76	74.4
15	5	1.56	83.2
15	10	1.50	92.2
15	15	1.67	96.1
20	0	3.16	93.3
20	7	4.34	96.2
20	14	8.11	102.1
20	20	6.37	109.1
25	0	963.04	104.6
25	8	620.20	118.35
25	16	392	135.6
25	25	1001.84	125.6
30	0	>3600	111.09 (not proven opt.)
30	10	(n/f)	-
30	20	(n/f)	-
30	30	(n/f)	-

7 Conclusion

In this paper a mathematical optimization model for patient transportation that explicitly incorporates infection control constraints is presented. Building on the classical Dial-a-Ride Problem (DARP) framework, an existing MILP formulation is extended by introducing two critical real-world features: (1) compatibility constraints that prohibit the joint transport of infectious and noninfectious patients, and (2) mandatory vehicle disinfection following the transport of infectious individuals.

The model accommodates a wide range of operational constraints, including passenger-specific pickup and drop-off time windows, maximum allowable ride durations, and vehicle capacity limits differentiated by passenger type (e.g., standard vs. mobility-impaired). To solve the problem, a modified MTZ-based formulation was employed and used IBM ILOG CPLEX to evaluate the model's performance on across multiple scenarios involving varying numbers of pickup locations and different proportions of infectious patients.

The obtained computational results demonstrate that incorporating infection control constraints significantly increases the complexity of route planning, particularly in mixed-infection scenarios. At 25 pickup points, homogeneous scenarios, where either all or none of the patients are infectious, converged slower than mixed scenarios, while for the smaller instances (10, 15, and 20 points) every case was solved within seconds, and for 30 points no feasible solution was returned within the time limit. This suggests that compatibility constraints, while restrictive, can sometimes reduce the solution space in a way that accelerates optimization. Furthermore, the experimental results highlight the limitations of a single-vehicle model when scaling beyond 25 pickup points, thereby motivating the need for more scalable, multivehicle approaches.

The proposed model contributes not only to the theoretical advancement of compatibility-aware DARP formulations but provides practical insights for designing safe, efficient, and regulation-compliant patient transportation systems in urban healthcare settings, as well.

Future work should focus on extending the model to a multivehicle setting, incorporating cost-based objectives, including fuel consumption, labor costs, and addressing return-trip scenarios for patients.

Acknowledgment

The authors received no financial support for the research, authorship and/or publication of this article.

Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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